RiskRank: Measuring interconnected risk

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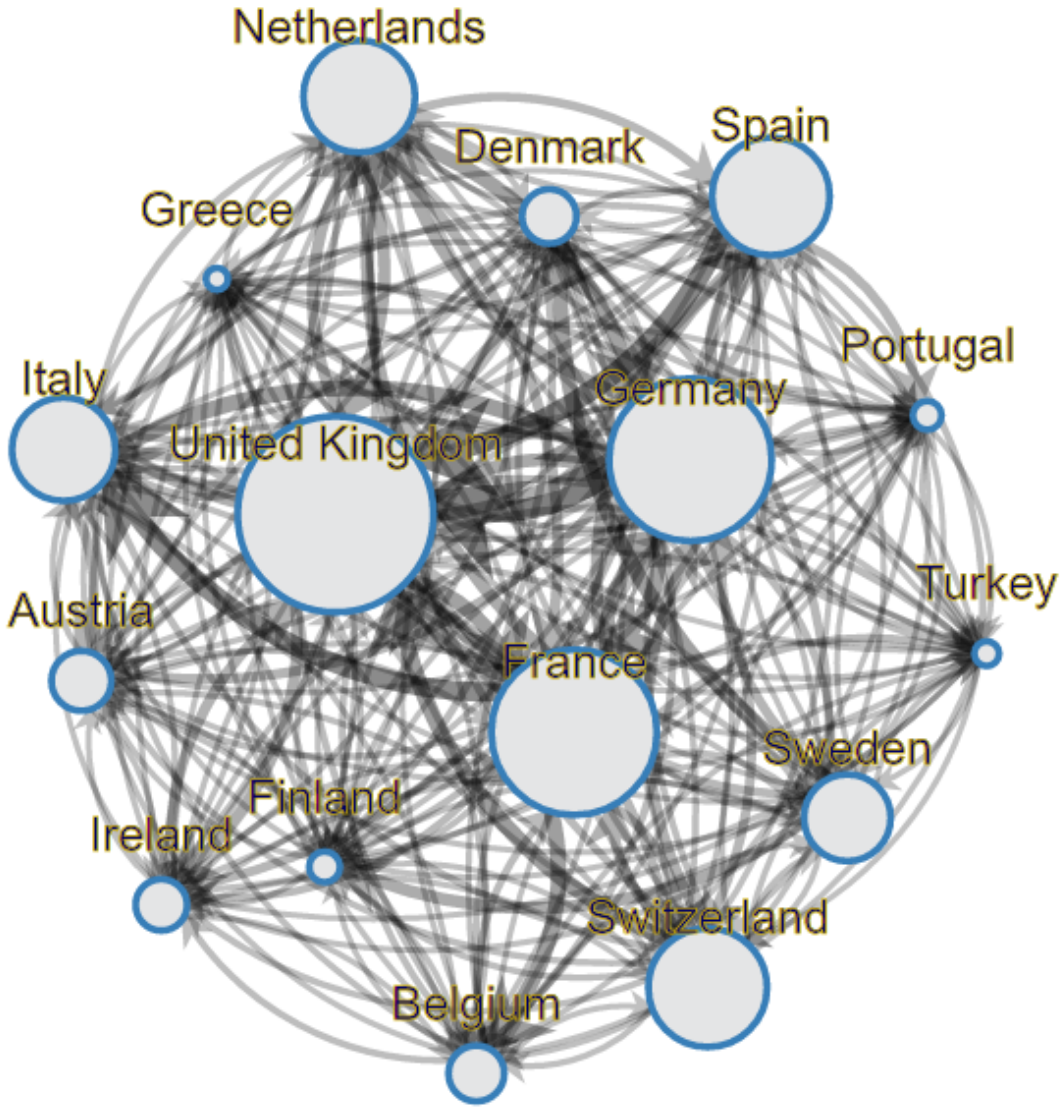
Netherlands
Denmark
Spain
Greece
Portugal
Italy
Germany
United Kingdom
Turkey
Austria
France
Sweden
Finland
Ireland
Switzerland
Belgium
2000
2001
2002
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2012
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2014
2015

Motivation

- An acute interest in new approaches to assess systemic risk
- Financial crises triggered by various shocks (unpredictable)...
- ...but widespread imbalances build-up ex ante (identifiable)
- Analytics to identify systemic risk at early stages
  - Cyclical: Build-ups of widespread macro-financial imbalances
  - Cross-sectional: Interconnected nature of the financial system
- How to assess vulnerability when risk is interconnected?

Related literature

Early-warning models

- **Univariate:** Kaminsky & Reinhart 1999, El-Shagi et al 2013
- **Logit:** Lo Duca & Peltonen, 2013, Betz et al 2014
- **Machine learning:** Holopainen & Sarlin 2015

Interconnectedness

- **Estimated:** Poon et al 2005,Billio et al 2011,Diebold&Yilmaz 2014
- **Real:** Upper & Worms 2004, van Lelyveld & Liedorp 2006, Poledna et al, 2015

Early-warning models & interconnectedness

- **Central:** Rose&Spiegel 2009, Minoiu et al 2013, Rancan et al 2015
- **Pass-through:** Peltonen et al 2015, Hale et al 2015

Systemic risk aggregation

- From risk indicators to probability
  - Signaling: Monitor univariate indicators
  - Non/linear approaches for combining indicators
  - Ensemble learning for model aggregation
- From interlinkages to centrality
  - In, out & total strength/degree
  - Betweenness, closeness & eigenvector centrality
  - DebtRank (Battiston et al 2012)
- How to combine probabilities and links?

Contribution

Measuring interconnected risk

- EWMs aggregate indicators & network measures connectivity
- We assume a hierarchical & interconnected system of nodes
- RiskRank: Likelihood & impact in an interconnected system

Key features of RiskRank

- A general-purpose measure of interconnected risk
- Allows disentangling individual, direct & indirect effects
- Allows multiple indirect effects and feedback loops
- Beyond entities, aggregates upward in the hierarchy

Aggregation operators

- Conventional aggregation operators
  - Min/max: con-/disjunctive operators
  - Weighted mean: fix trade-off & compensatory
  - Quadratic/geometric/harmonic/power α mean
  - Ordered Weighted Average (OWA) (Yager 1988)
- Choquet (1953) integral as a general aggregation operator
  - Includes the above (and more) as special cases
  - Generalizes to non-linearity and non-additivity
  - Extends conventional operators with interactions

Choquet integral

Definition

Fuzzy measure μ on the finite set $N = \{1, 2, ..., n\}$ is a set function $\mu : P(N) \to [0, 1]$ (where $P(N)$ is the power set of N) satisfying the following two conditions:

- $\mu(\phi) = 0$, $\mu(N) = 1$;
- Monotonic, non-decreasing: $A \subseteq B$ implies that $\mu(A) \leq \mu(B)$.

Definition

Discrete Choquet integral with respect to a monotone measure μ is

$$C_\mu(x_1, \ldots, x_n) = \sum_{i=1}^{n} \left(\mu\left(C_{(i)}\right) - \mu\left(C_{(i+1)}\right)\right) x_{(i)}$$

where $x_{(i)}$ denotes a permutation of the x_i values such that $x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(n)}$ and $C_{(i)} = \left\{c_{(i)}, c_{(i+1)}, \ldots, c_{(n)}\right\}$.

Additive Choquet integral

The additive Choquet integral is the weighted sum

$$C_\mu(x_1, \ldots, x_n) = \sum_{i=1}^{n} \mu\left(c_{(i)}\right) x_{(i)}$$

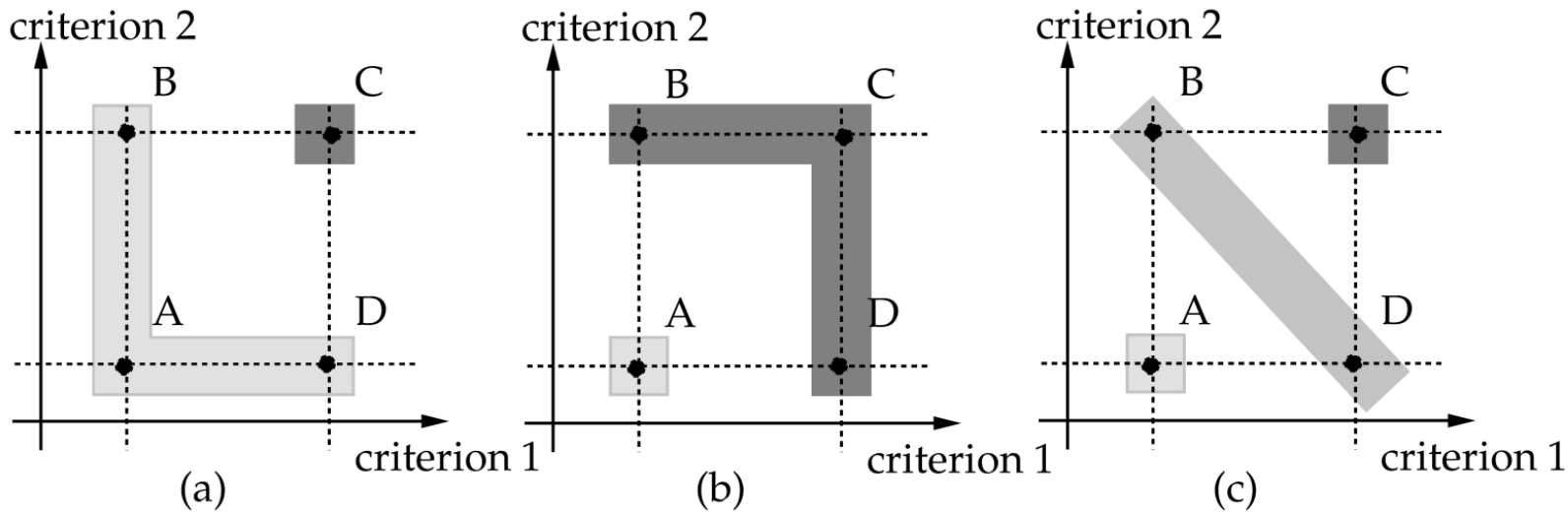
2-additive case covers pairwise interactions and individual effects

$$C_\mu(x_1, \ldots, x_n) = \sum_{i=1}^{n}(v\left(c_i\right) - \frac{1}{2}\sum_{j \neq i} I(c_i, c_j))x_i + \sum_{I(c_i,c_j)>0} I(c_i, c_j)\min(x_i, x_j) + \sum_{I(c_i,c_j)<0} \mid I(c_i, c_j) \mid \max(x_i, x_j)$$

where $v(c_i)$ stands for the Shapley-index (average contribution of fixed element x_i in any subset) and $I(c_i, c_j) \in [-1, 1]$ for the interaction. This relies on the Möbius transformation of μ and that it equals 0 on any subset with cardinality above 2 (Grabisch, 1997)

Case of utility theory

$$C_\mu(x_1,\ldots,x_n)=\sum_{i=1}^{n}(v(c_i)-\frac{1}{2}\sum_{j\neq i}I(c_i,c_j))x_i+\sum_{I(c_i,c_j)>0}I(c_i,c_j)\min(x_i,x_j)+$$
$$\sum_{I(c_i,c_j)<0}\mid I(c_i,c_j)\mid \max(x_i,x_j)$$



Positive interaction	Negative interaction	No interaction
Complements	Substitutes	Independent
$I(c_i,c_j)>0$	$I(c_i,c_j)<0$	$I(c_i,c_j)=0$
min operator	**max** operator	no operator

From Choquet to RiskRank

For risk levels x_i and links $I(c_i, c_j)$, 2-additive RiskRank is

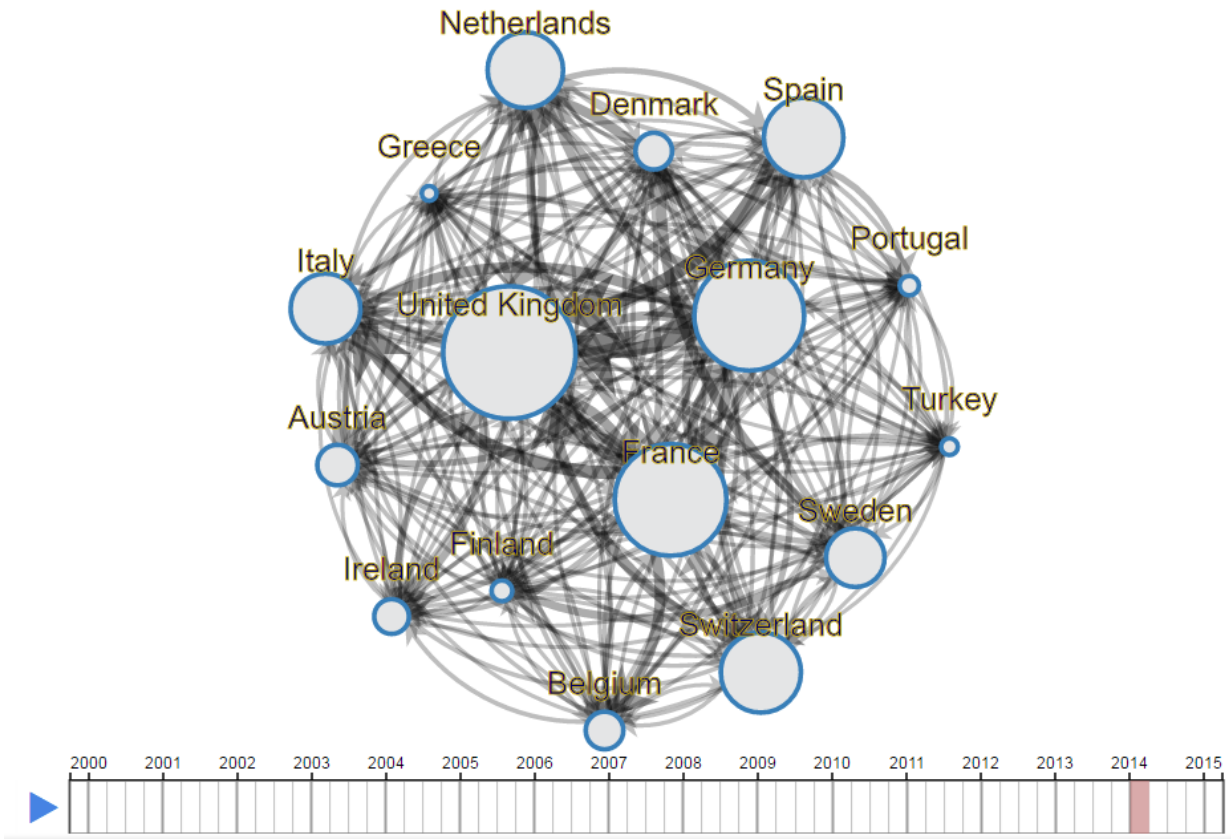
$$RR_c = \underbrace{w(c)x_c}_{\text{Individual effect of entity } c} + \underbrace{\sum_{i=1}^{n}(v(c_i) - \frac{1}{2}\sum_{j\neq i}^{n} I(c_i, c_j))x_i}_{\text{Direct effects of entity } i \text{ on } c} + \underbrace{\sum_{i}^{n}\sum_{j\neq i}^{n} I(c_i, c_j)\prod(x_i, x_j)}_{\text{Indirect effects of } j \text{ via } i \text{ on } c}$$

which

- separates effects
- allows multiple indirect effects via k-additivity
- allows simulated feedback through dynamic iteration

Real-world example

- EU early-warning model & BIS exposures
  - Sample: 15 EU countries, 1980Q1-2015Q1
  - Domestic risk: Logit model, 14 macro-financial indicators
  - Linkages: BIS foreign claims, immediate borrower
  - Output: Country & EU-level measures of interconnected risk

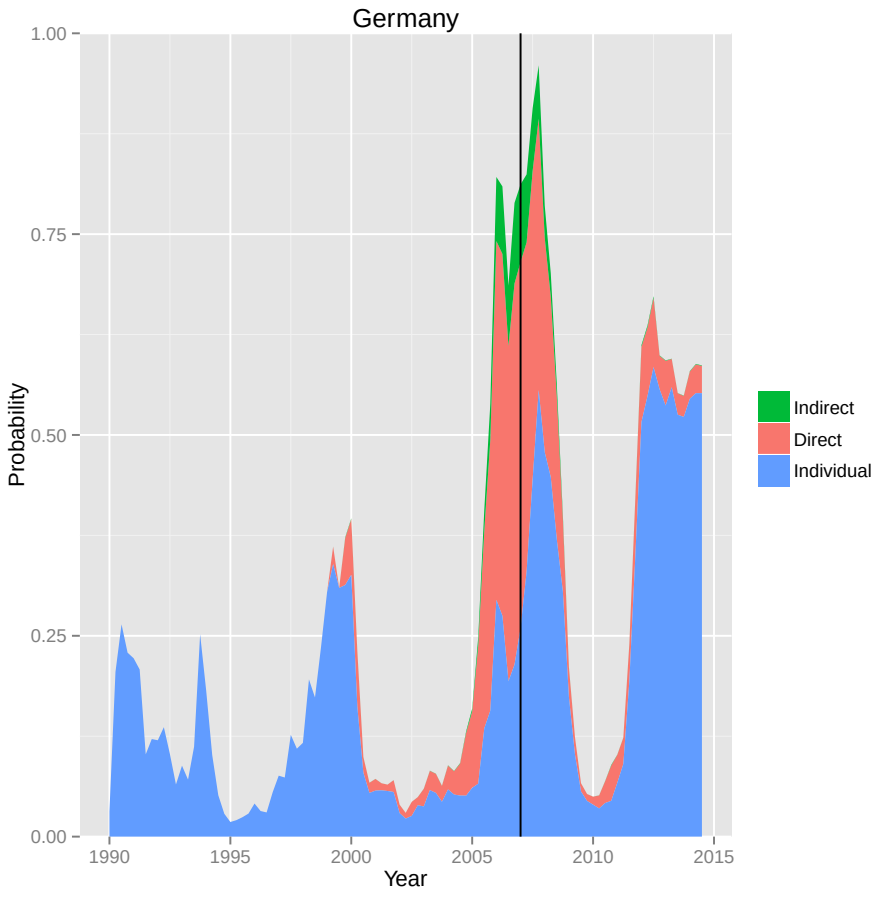


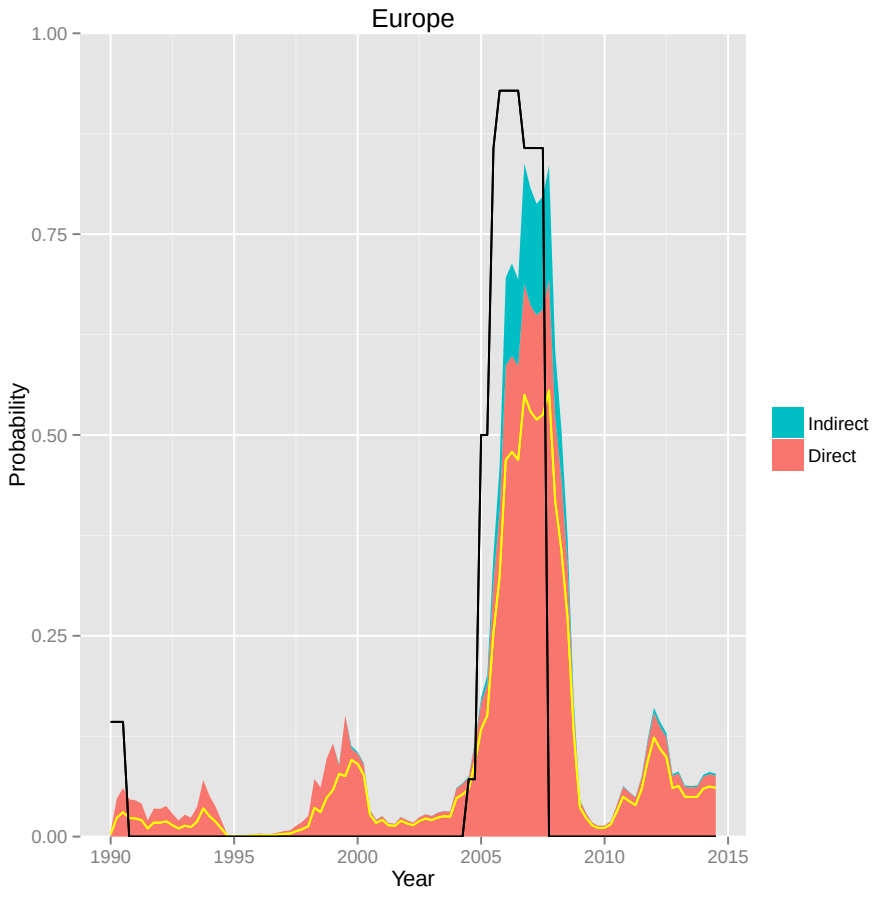
Real-world example

- $U_r(\mu = 0.8)$: Model usefulness with imbalanced preferences
- AUC: Model usefulness for all μ

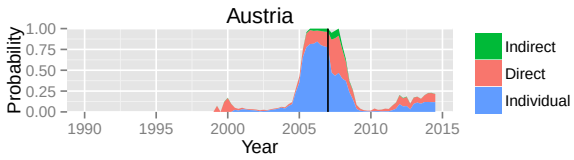
	Individual		RiskRank	
μ	$U_r(\mu)$	AUC	$U_r(\mu)$	AUC
0.0	0 %	0.915	0 %	0.934
0.1	-6 %	0.915	1 %	0.934
0.2	-3 %	0.915	3 %	0.934
0.3	6 %	0.915	14 %	0.934
0.4	12 %	0.915	28 %	0.934
0.5	15 %	0.915	37 %	0.934
0.6	25 %	0.915	47 %	0.934
0.7	44 %	0.915	59 %	0.934
0.8	60 %	0.915	69 %	0.934
0.9	73 %	0.915	78 %	0.934
1.0	0 %	0.915	0 %	0.934

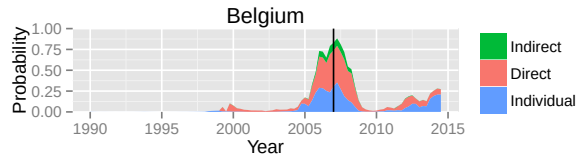
Real-world examples

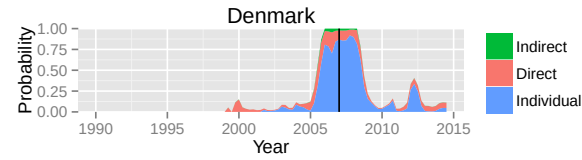


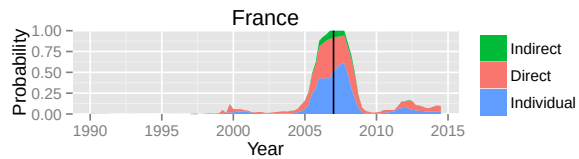


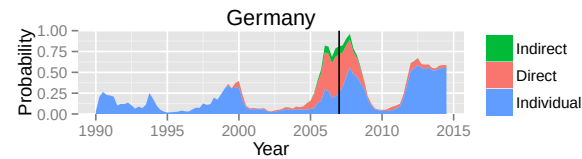
Real-world examples

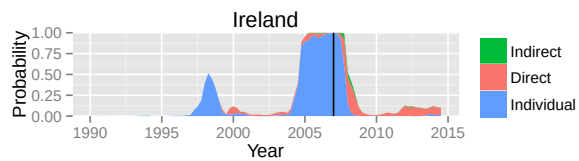


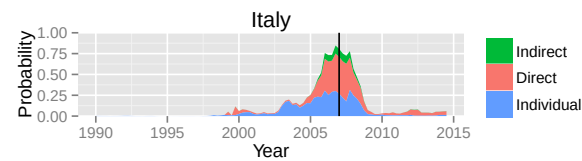


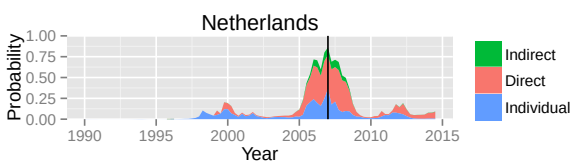


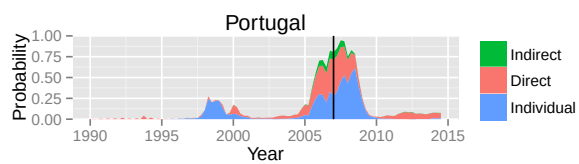


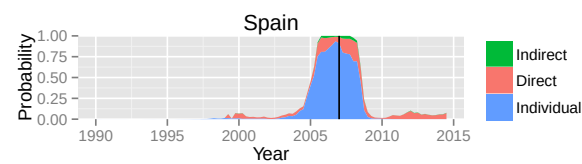


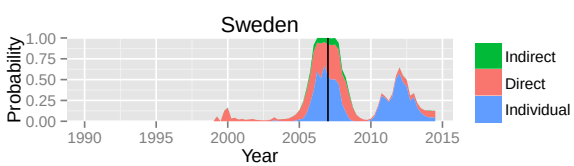


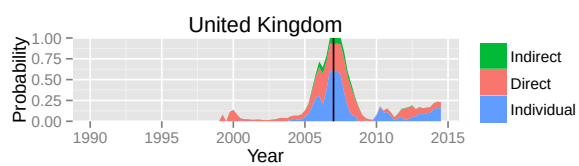












Conclusion

- RiskRank as a measure of systemic risk
  - EWM: From indicators to probability
  - Centrality: From interlinkages to centrality
  - RiskRank aggregates probabilities over links
- The properties of RiskRank are general in nature
  - Allows disentangling individual, direct & indirect effects
  - Allows multiple indirect effects and feedback loops
  - Beyond entities, aggregates upward in the hierarchy
  - Could be used for measuring any interconnected risk

Thanks for your attention!