



## 4D Computer Output

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### ABSTRACT

*The techniques available for the animation of 3D images by computer are summarised, and exemplified by still and animated images. Monocular techniques discussed are: hidden lines and surfaces, depth cueing, background blurring, perspective, surface shading, and temporal parallax. Binocular techniques covered are: parallel stereo pairs, crossed stereo pairs, Anaglyphs, image switching, polarised images, dual displays (Virtual Reality), Lenticular film, Pulfrich effect, Multiplex holograms, and Autostereograms. Polyocular techniques discussed are: volume scanned displays, vibrating mirrors, intersecting beams, layered displays, and full computed holograms. The last may require a processor capable of around  $10^{19}$  operations per second. These are not yet readily available.*

### INTRODUCTION

A number of techniques are at last providing computer professionals and users with 3D spatial images which can be animated. Such animated sequences can be thought of as four dimensional entities. Other methods are available for presenting 4D entities, e.g. using combinations of RGB colour space plus geometric space, but none have the impact of spatial 3D + time, as may be judged by the popularity of Virtual Reality and of Autostereograms with the general public for recreational purposes. The combination of 3D imaging and animation is the best way of presenting complex data [Ware, 1996].

This article is aimed at providing a background to the various techniques which over the years have contributed to our current position [Okoshi, 1976].

### EXAMPLES

The illustrations and movies were all composed using the [NUDES](#) animation system (an acronym for: Numerical Utility Displaying Ellipsoid Solids) [Herbison-Evans, 1978], using [figures](#) composed by the author and his students [Herbison-Evans, 1987]. For the images which are designed for parallel stereo viewing, the dots at the top/bottom of these images should be 3 to 5 centimetres apart for greatest comfort. The reader may need to scale the images depending on the display being used. The animations are all 31 or 63 frame cyclic loops in the mpeg format, and are best shown in continuous repeating mode. The animations where available are obtained by clicking on the appropriate single images. A short commentary (in WAV format) on each animation example can be obtained by clicking on the loudspeaker icon under each image.

## MONOCULAR TECHNIQUES

We obtain much information about the 3D nature of the world that we see by simply using the information in an image at one eye:

### Hidden Surfaces

The opacity of many objects in our world means that a nearer object can obscure parts of a more distant object. This led to the 'Hidden Line' and '[Hidden Surface](#)' problems in computer graphics [Herbison-Evans, 1983]. The many algorithms for the rapid solution of this problem have been a mainstay of computer graphics academic examinations for two decades [Hearn and Baker, 1986]. Simple examples for comparison are shown in Figures 1 and 2, and Animations 1 and 2:

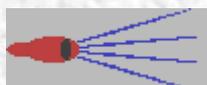
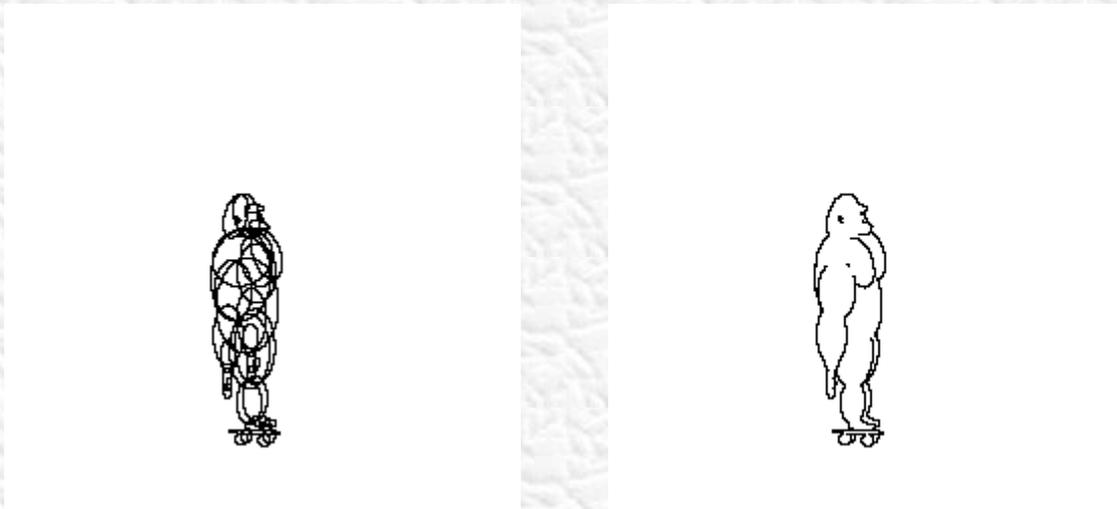


Figure 1

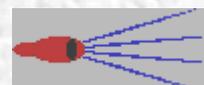


Figure 2

## Chiaroscuro

Most animals have an upper surface that is darker than the underside. This has been interpreted as an attempt at camouflage, because monochromatic objects are brighter on the side closest to a source of illumination. Outdoors, averaging over all normal upright orientations of an animal, the sun is overhead. Thus a simple shading of a computed image of a 3D object from lighter on top to darker underneath is a good way of presenting a solid look to the object. This has been used for many years by artists, under the term [Chiaroscuro](#).



Figure 3

## Depth Cueing

This effect has been used for many years in Chemistry and in Computer animation. In this technique, items that are close to the observer are made more intense or thicker than items further away. This has been common practice in diagrams used in [stereochemistry](#). It is equivalent putting an implicit light source at the observer's eye, rather than say, above the scene. It does have the advantage of requiring no computation of shadows. Except for the shadows, the difference between this and the vertical illumination described above for Chiaroscuro is computationally trivial: just a question of referring the shading to the  $y$  component rather than the  $z$  component of the surface normal [Herbison-Evans, 1980]. However, visually, the impacts are very different. Vertical light gives a simple natural feel to the solidity of objects in a scene, even if no shadows are computed. Light from the observer washes out any shading due to curvature of the objects in a scene. It also gives the feel of being down a mine, with a light on the hat illuminating the scene. For many observers this is associated with a claustrophobic sensation which is usually not what the programmer had in mind. Examples may be seen by comparing Figures 3, and 4.

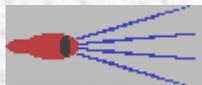
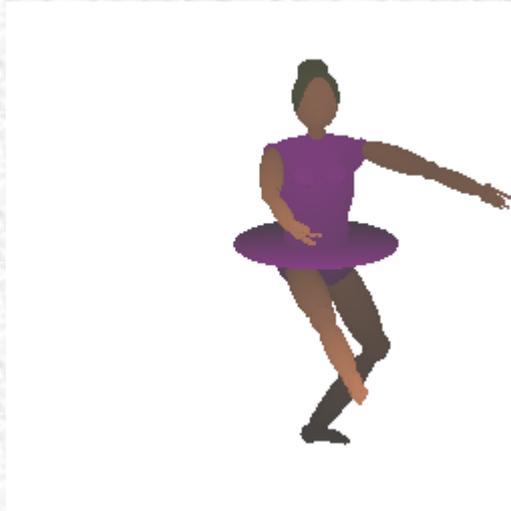


Figure 4

## Background Blurring

An early observation was that backgrounds appear to be [more blurred](#) than the objects in front of them which have our visual attention. This is probably due to both the limited focussing depth of field of the ocular lens, and to the reduced resolution available outside of the 1/2 degree field of the fovea centralis, as well as the effects of mist and dust in the intervening atmosphere ('[aerial perspective](#)'). Examples are shown in Figures 5, 6, and 7: The effect has been widely used by painters, photographers, and cinematographers to give a 3D feel to their images [Rokita, 1996].

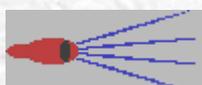


Figure 5

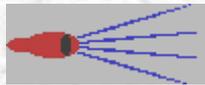


Figure 6

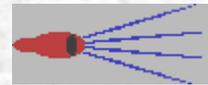


Figure 7

## Motion Parallax

[Temporal or Motion parallax](#) can be obtained using only one eye by means of animation: near objects move faster on the visual field than distant objects. And while this is true in general, more specifically the brain infers 3D character most effectively from the comparison of two views of a scene, taken with the scene rotated a couple of degrees about a vertical axis between the views. This is the basis of binocular 3D imaging, about which more later. But it works even with one eye. The brain can apparently use the same algorithms for depth perception whether the two images arise from different eyes at the same time or from one eye at different times. This effect has been used in computer graphics for many years by having the default state of the display of a 3D entity to be a continuous slow rotation about a vertical axis. The reader may care to compare the apparent solidity of the molecules rotating about the vertical and the horizontal axes in Animation 8: if the theory is correct, the ones rotating about the vertical axis should appear more solid than the ones rotating about the horizontal axis.

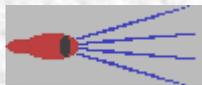
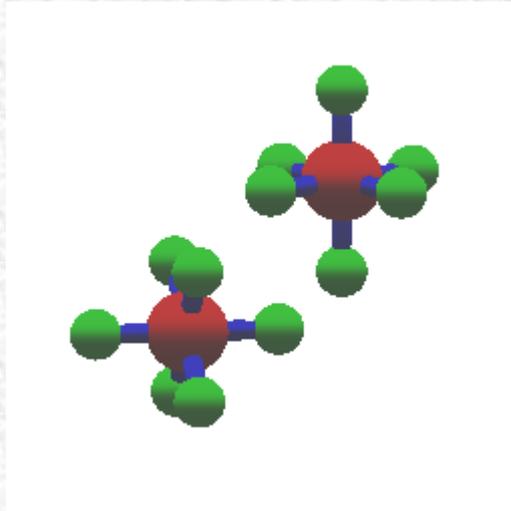
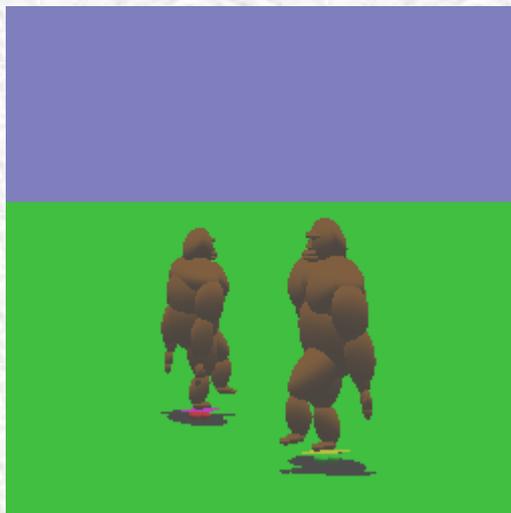


Figure 8

## Perspective

Another early observation was that images of nearer objects appear larger than images of more distant objects. This led to the science of [Perspective](#), epitomised by the projective transformation learned by all computer graphics students. This transformation does have two minor problems however. The first is that the perspective is for viewing from only one point in space. If the observer's eye is placed anywhere else, the image is incorrect, and the 3D effect is strained. The second problem is that the perspective image is projected mathematically onto a flat plane, whereas the retina of the eye is approximately hemispherical. This again leads to a distortion around the periphery of images generated by the flat perspective projection. An example may be seen in the animation at Figure 9:



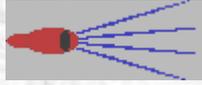


Figure 9

## Texture Gradient

Another 3D effect is called '[texture gradient](#)', where the pattern of some texture such as grass varies with distance. Statistically: the autocorrelation function of the texture contracts as distance increases.

## BINOCULAR TECHNIQUES

The illusion of solidity is particularly strong if the left and right eye images can be separately computed, and each presented to the appropriate eye. Doing this by computer has problems. The eyes can only appreciate the solidity of a scene if the ratio of the nearest to furthest distance is greater than about 2:3, and the left and right scenes differ by a rotation of about 2 or 3 degrees about a vertical axis (clockwise as viewed from above if deriving the right eye view from the left). Given a screen resolution of about 4 pixels/millimetre, for most binocular techniques, this quantises depths to about 16 possible values, which can give an unfortunate layering effect. Printed stereo images, having an order of magnitude better resolution, can give nearly continuous depth perception, as the eye/brain has itself only a resolution of about 250 values.

### Freeview

Also called 'Parallel Viewing', this simple technique is to place the stereo images side by side, left on the left, right on the right. The viewer must then keep the axes of the eyes parallel as if looking at infinity, while focussing on the display. Initially, this is not easy, and some practice may be necessary. This display technique is called [Freeview](#). A restriction is that the images must be placed no further apart than the spacing of the eyes, and hence they can each be no wider than this, and each is limited to using only half the display width. An example is shown at Figure 10:

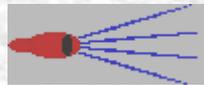
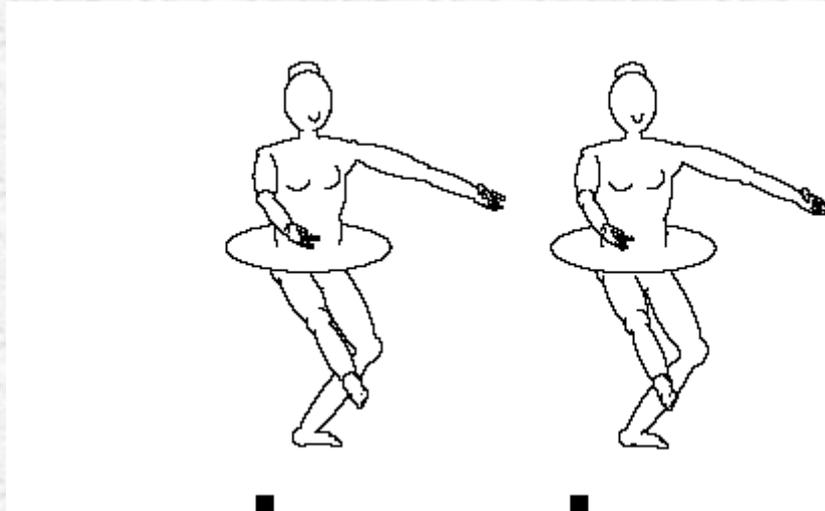


Figure 10

### Crossview

Alternatively, the images can be placed side by side, left on the right and right on the left, commonly called the '[Crossview](#)' or 'Transverse View' technique. To view such images, the viewer must keep the eyes crossed, as if looking at something closer than the screen, while focusing on the screen. This is not initially easy either. The images however are now no longer restricted in their spacing, and can be of arbitrary width. However, again, only half the area of the display can be used for each image, effectively halving the display resolution. An example is shown at Figure 11:

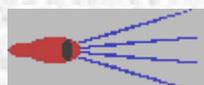
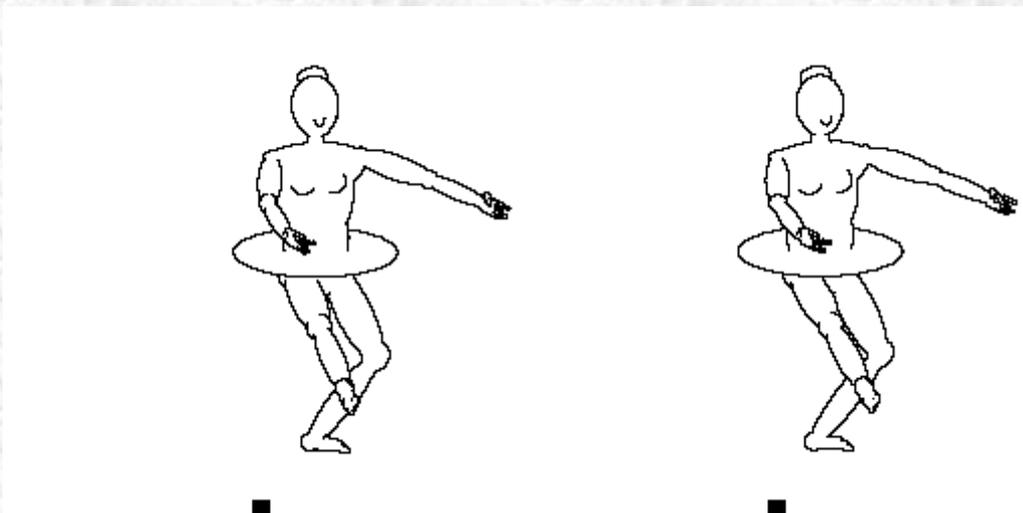


Figure 11

## Anaglyphs

A 3D scene can be displayed in [Anaglyph](#) form [Morgan & Symmes, 1982]. The image for one eye is displayed in red, and the other in green. The observer must wear glasses with one red lens and one green. The major disadvantage of this technique is that only monochrome images can be displayed. Also the observer needs to use special eyewear, which is troublesome for people who normally wear spectacles already. An example may be seen at Figure 12:



Figure 12

## Shuttered Glasses

The left and right images can be displayed alternately at a rate faster than the critical flicker frequency of the eye (about 20 per second). The observer again needs to wear special [Shuttered Glasses](#), this time having lenses that are switched to alternately blacken out each eye [in synchronism with the display](#). The images with this technique can now use the whole screen and be in full colour, but the eyewear problem remains.

## Cross Polarised Images

Two normal displays can be used and their images [Polarised](#) at orthogonally to each other, and then combined with a half silvered mirror. The observer must wear glasses with cross

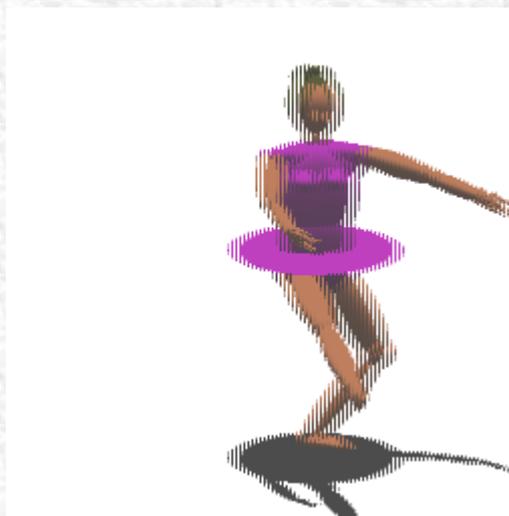
polarised lenses. This technique is the one used most often for '3D Movies'. The complexity of the display and the glasses are problems.

## Virtual Reality

Two tiny displays, one for each image, can be mounted on a headset, and mirrors used to present the images to each eye. This is the technique normally used in [Virtual Reality](#) systems [Pimentel & Teixeira, 1993]. Again the headset is a problem.

## Lenticular Film

A [Lenticular](#) film composed of a set of thin vertical cylindrical lenses (each typically with a width of 1/4 millimeter) can be placed over the screen, and the two stereo images dissected into vertical stripes and interlaced so that the lenses present each stripe to the appropriate observer's eye. This technique has been used on picture postcards for many years.



A pair of stereo images with the left and right images interlaced.  
A suitable lenticular film must be placed over the image to see the 3D effect.

Figure 13

It has the advantage of not needing the observer to train the eyes or to wear special gear. Its main disadvantages are the reduced horizontal resolution, and the difficulty of aligning the images stripes with the lenses.

## Pulfrich Effect

The [Pulfrich](#) effect can be used to give some 3D illusion. This depends on the fact that the eye requires longer to process dim images than bright ones. Thus if animation is created of a scene rotating about a vertical axis, and the appropriate eye covered with a dark filter, the

scene will appear in 3D [Watkins & Mallette, 1996]. If the left eye is the one darkened then the scene must be rotating clockwise at about 40 degrees per second for the effect to occur. An interesting variant is the possibility that if one eye is dominant, it may process an image faster than the other eye, in which case no special glasses need be used. Unfortunately, people having a dominant left eye would require the scene to be rotating in the opposite direction to those with a dominant right eye. Examples may be compared in the two molecules in Animation 14:

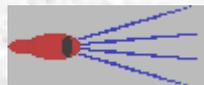
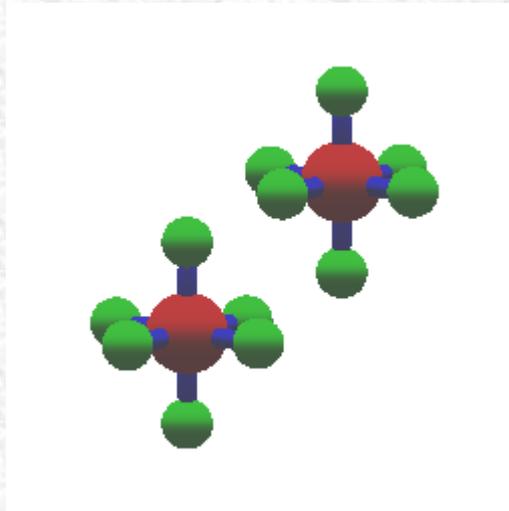


Figure 14

## Multiplex Hologram

A Multiplex or [Rainbow](#) hologram can offer a 3D monochrome image, albeit multicolored [American Banknote Company 1984]! It is composed of a series of thin vertical holograms of flat images of the scene, each generated with the scene rotated a degree or so from the last one about a vertical axis. Each eye of an observer sees a different hologram stripe, each reconstructing the scene at the appropriate angle for that eye as if the scene were solid. The generation of a Multiplex hologram takes some time, so no interaction with the image is possible, but limited animation is possible in the precomputation of the set of flat images. The horizontal resolution is restricted, as well as the scene having to be monochromatic. However, observers need no special eyewear, and multiple simultaneous observers are easily accommodated. A number of commercial firms offer the service of custom making Multiplex holograms of virtual objects.

## Autostereograms

The latest technique is to overlap the left and right images, and indeed to overlap multiple copies of each. The result is called an [Autostereogram](#) [Tyler, 1994]. The left and right

images can use random dots or indeed any arbitrary pattern, for displaying the depth, with no coloring information (making a Random Dot Stereogram or Single Image Stereogram), or be in full shaded colour (resulting in a Wallpaper Stereogram). Possibly a better technique is to combine the shaded colour at each pixel 50/50 with a random value, giving a sort of speckled autostereogram. This combines the proper colour shading of the wallpaper stereogram with the depth precision of the random dot technique. The Random Dot stereograms pose a problem for some systems because they are impossible to compress. When using random numbers, the same set of random numbers can be used for every frame, which gives peculiar effects for the observer, because some of the lines change numbers and some do not, so patches of the image appear not to animate. Alternatively, different random numbers can be used for each frame, which gives a sparkling effect to the images, as though the objects were covered in sequins. An example of a Wallpaper stereogram may be seen at Figure 14:

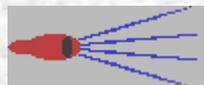
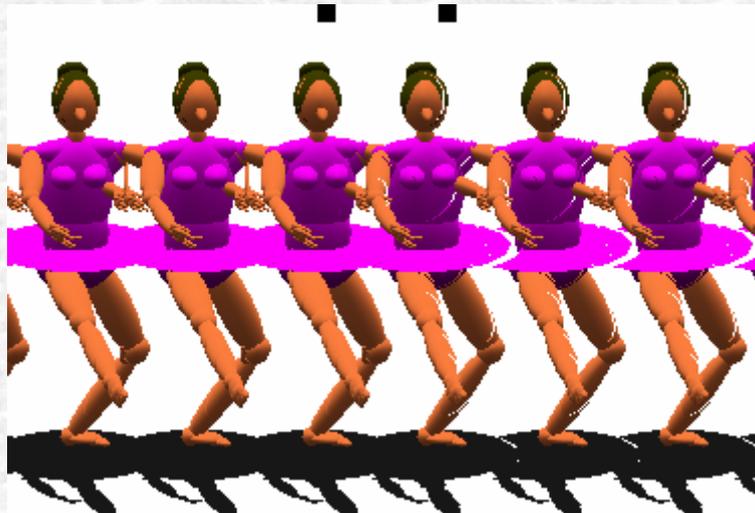
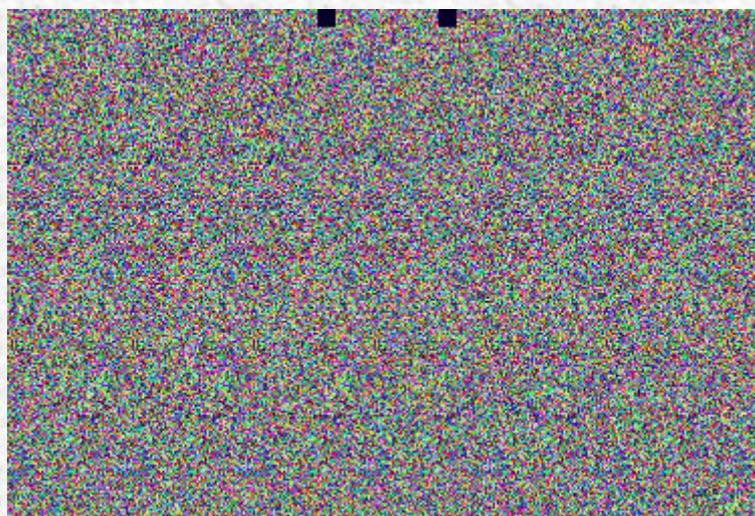


Figure 15

An example of a Random Dot Stereogram may be seen at Figure 15:



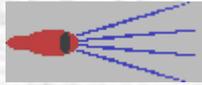


Figure 16

These are very hard to see, so a hybrid one with the same figure but a plain background is shown at Figure 16:

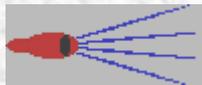
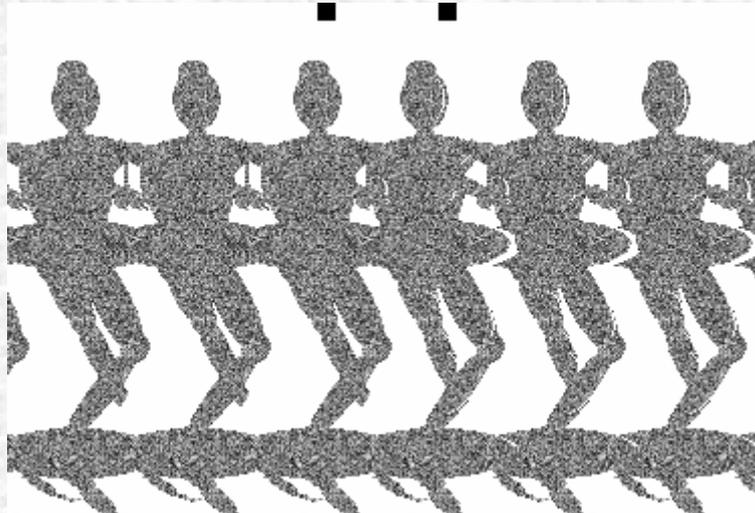


Figure 17

Another type of hybrid speckled stereogram may be seen at Figure 17:

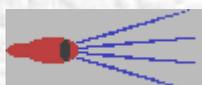
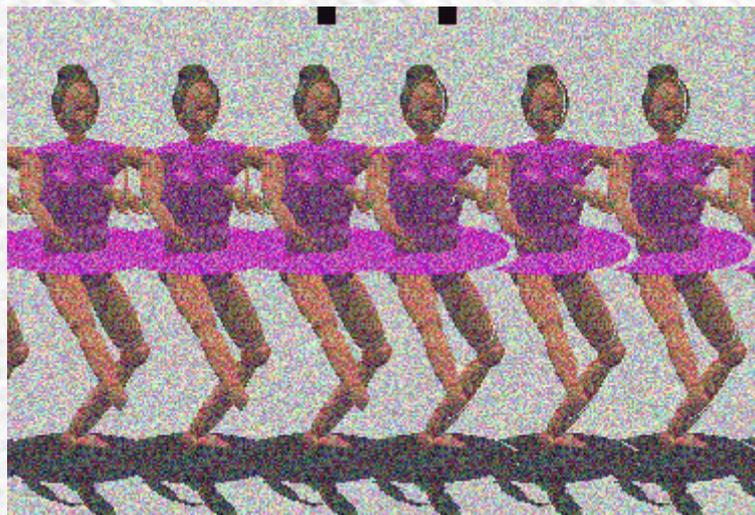


Figure 18

# POLYOCULAR TECHNIQUES

These create an image of solid appearance which is viewable by any number of eyes, and so, for example, are viewable by several people at once. Various methods have been tried for this.

## Volume Scanning

A display screen can be driven mechanically to scan a volume, preferably faster than the critical flicker frequency of the eye. For example, a cathode ray tube can be constructed so that the screen oscillates or rotates inside the vacuum. Alternatively, a scannable laser beam can be arranged [to shine on a rotating screen](#), so as to scan out volume. A [Helically Shaped Screen](#) has been suggested for this. Also, a Rotating Array of LED's has been tried. With any of these techniques, the points on the image are painted in appropriate synchronism with movement of the screen so that they appear at the required point in the 3D space scanned by the screen.

## Vibrating Mirror

Another technique is to view a normal CRT via a [Vibrating Mirror](#). This technique can give a large virtual scanned volume by making the mirror curved, so magnifying the CRT screen image. If the mirror is driven by a loudspeaker mechanism, its focal length can be oscillated, and so a virtual image of considerable depth created [Traub,1967].

## Double Beam Display

An optically non-linear material can be used to create Double Quantum transitions. Two scannable invisible infra-red beams are shone into the material at an angle to each other. [Where they intersect](#), visible light is emitted, so that the beams can be scanned to draw 3D shapes in the medium.

## Layered Displays

A [set of transparent flat surface displays](#) can be placed on top of each other to make a solid volume [Sullivan, 2005]. These layers do not have to fill the volume as the eye interpolates a pair of dots on adjacent screens as falling in between them. A resolution of 20 depths per pair of layers can be easily achieved by using various proportions of brightness divided between two successive layers.

## Holograms

The ultimate 3D display may be the computed projected full [Hologram](#) [Firth, 1972]. The computational problems however are immense. For example, a 10 cm. square hologram would require approximately  $10^{12}$  pixels, the value at each being a Fresnel transform of the light coming from the scene. For interactive animation, these  $10^{12}$  transforms would need to be computed faster than the eye's critical flicker frequency. Suitable materials exist which change their optical properties when an appropriate voltage is placed across them, and this can be done using transparent electrodes. Thus a hologram could be made from such a material, and the electrodes can be striped and painted at right angles across opposing faces, so that a tiny volume of material can be addressed where the stripes cross. Thus suitable displays seem quite possible. The problem is that the value at each output pixel is an integral (approximated presumably as a finite sum) over all the input pixels in a depth map of the scene. So the problem appears to require a processor capable of approximately  $10^{25}$  multiplications and additions per second. This does seem to pose a difficult problem. One of the best hopes for reducing this might be to compose the scene from flat triangles, and to approximate the Fresnel transform of each by its Fourier transform multiplied by a quadratic phase factor representing the depth of its centre [Vaughan-Taylor, 1994]. If the Fourier transforms are precalculated and held to appropriate accuracy in tables, a scene of say 10,000 triangles (enough for a human figure given current screen resolutions) would only need  $10^{19}$  operations per second. Further research may lead to improved ways of cheating.

## ACKNOWLEDGEMENTS

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## CONCLUSIONS

An animated image of solid appearance viewable by any number of unencumbered observers is still a dream popularised by such images as that of Princess Leia in the movie Starwars, and Selma in the TV series Timetrax. A number of techniques are closing in on this dream, many requiring rather specialised types of hardware. With hardware speeds currently reaching 1 Teraflop ( $10^{12}$  floating operations per second) and improving at approximately a factor of 2 every 18 months, [Bell,1996] it may take only 37 years for this dream to become possible.

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