Firm-Level Dispersion in Productivity: Is the Devil in the Details?

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An important finding in the empirical literature on firm-level productivity is large dispersion across firms in measured total factor productivity (TFP), even within narrowly defined industries.[1] Since micro datasets rarely contain information on product prices or quantities, much of the evidence is based on revenue productivity measures. Researchers have developed a variety of methods for estimating revenue productivity and, as Syverson (2011) highlights, the finding of large dispersion in revenue productivity across firms is robust across methods. This and related findings have been interpreted as suggesting the estimation method is not critical for key findings. However, this paper highlights that the alternative estimation methods yield conceptually different measures.[2]

We show the importance of drawing out these distinctions in light of the insights of Hsieh and Klenow (2009) (hereafter HK). They show that dispersion in a specific type of revenue productivity measure reflects only distortions under strict assumptions about technology and demand. This insight has led to a burgeoning literature constructing measures of distortions across countries, industries and time periods to assess the degree of misallocation. We explore the generality of this insight in the context of commonly used revenue productivity measures and show that alternative measures do not yield the same implications even if the strict assumptions made by HK hold. Moreover, we show the differences in implications can be used to make inferences about the nature of distortions and other sources of dispersion in firm-level productivity.

A useful starting point is Foster, Haltiwanger and Syverson (2008) (hereafter FHS) who introduce two distinct productivity concepts: physical productivity (TFPQ) and revenue productivity (TFPR). The relationship between the two is $\mathrm{TFPR}_i = \mathrm{P}_i {*} \mathrm{TFPQ}_i$, where i indexes firms and P_i denotes the firm's product price. TFPR confounds demand side and supply side factors but also allows the exact decomposition of these margins because it is additive in logs so that $\mathrm{tfpr}_i = \mathrm{p}_i + \mathrm{tfpq}_i$.[3] Under additional assumptions of iso-elastic demand, Cobb-Douglas technology, common factor prices, and constant returns to scale (CRS), HK show that TFPR_i will be equalized across firms in the absence of frictions

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[1]See Syverson (2011) for a survey.

[2]Our discussion here draws upon insights from Haltiwanger (2015).

[3]We use lower case letters to denote logs throughout the paper.

and distortions. While this strong inference is sensitive to assumptions, it reflects the more general insight that there will be an inverse relationship between plant-level prices and TFPQ if firms face downward sloping demand curves. Under the assumptions made by HK, high TFPQ firms have an exactly offsetting decline in prices so that TFPR remains constant. Given this insight, HK argue that any observed dispersion in TFPR_i must reflect distortions that adversely impact aggregate productivity via misallocation.

With these remarks as a background, we focus on the relationship between conceptual TFPR_i and commonly used empirical revenue productivity indices. We show one typical measure, namely revenue per unit composite input, corresponds to TFPR_i under CRS; however, the correspondence breaks down if returns to scale depart from CRS. We also show that estimation of the revenue function as is commonly done using proxy methods does not yield a residual equal to TFPR_i; this finding is independent of returns to scale. We use these distinctions between empirical productivity measures and TFPR_i to draw inferences about the role of distortions, technical efficiency and demand shocks in accounting for the observed within-industry dispersion in revenue productivity measures using plant-level data from the U.S.

I. Revenue-based productivity measures

Revenue productivity is typically estimated assuming log-linear production technology and log-linear demand functions so the revenue function is log-linear. There are two commonly used types of estimation procedures. The first type relies on first order conditions derived from the cost minimization problem of the firm. The second type is based on regression techniques.

In the first approach, under Cobb-Douglas technology and CRS, cost-shares (of total costs) of respective production inputs are valid estimates of factor elasticities. An important advantage of this framework is that factor elasticities and therefore productivity residuals can be calculated from data without further assumptions. To be specific, the residual of the log revenue function for firm i, which we will denote by tfpr_i^{cs}, can be written as $\text{tfpr}_i^{cs}{=}p_i{+}q_i\text{-}\sum_j \widehat{\alpha}_j x_{ij}{=}p_i{+}\text{tfpq}_i$, where q_i and p_i denote firm i's log output and product price level, $\widehat{\alpha}_j$ denotes the factor elasticity estimate for input factor x_{ij} and tfpq_i denotes log physical total factor productivity. Under CRS and Cobb-Douglas technology, the residual calculated in this manner is tfpr_i. If we deviate from these assumptions, then cost shares are not valid estimates of factor elasticities and tfpr_i^{cs} is a biased estimate of tfpr_i.[4]

Regression approaches, the second type of method, do not impose restrictions on returns to scale. An advantage of these techniques is that they may be estimated using data on proxies or direct measures of demand variables. However, revenue function coefficents are not factor elasticities without data on, or

[4] Cost minimization can be combined with estimation of returns to scale to recover factor elasticities.

further assumptions about, demand. Therefore, the implied revenue residual, tfpr_i^{rr}=p_i+q_i−$\sum_j \widehat{\beta}_j x_{ij}$, is not equal to either tfpr_i or tfpr_i^{cs}. The key point is that $\widehat{\beta}_j$ reflects both factor elasticities and demand parameters.

Without data on firm-level prices or quantities, we need additional assumptions about demand to make inferences about factor elasticities, returns to scale or TFPR. Many studies postulate that demand is iso-elastic.[5] Although this assumption is admittedly restrictive, it helps with analytical tractability and supports joint estimation of revenue elasticities and the price-elasticity of demand. The latter property is important because under these assumptions, revenue elasticities and the price elasticity parameter jointly determine factor elasticities. We adopt the iso-elastic assumption in our analysis, but also extend it by explicitly modeling the idiosyncratic component of product demand. Specifically, demand is given by P_i=$\text{P}(\text{Q}/\text{Q}_i)^{1-\rho}\xi_i$, where ξ_i denotes shocks to the plant's product demand. This demand structure implies we can write the revenue residual as tfpr_i^{rr}=ρtfpq_i+$\ln\xi_i$+p. This makes explicit that tfpr_i^{rr} is a function of tfpq_i, ξ_i, a parameter that determines the price elasticity of demand (ρ), and the aggregate price level (p). As mentioned above, tfpr_i^{rr} is not equal to tfpr_i^{cs} or tfpr_i in general. This result does not depend on assumptions about returns to scale.

II. Implications for dispersion

The distinction between these alternative measures is important in the context of the insights of HK. To make this point, we consider an environment such as in HK where market inefficiencies may distort input and output allocations. In addition, we assume firms face iso-elastic demand conditions in which they experience idiosyncratic shocks to consumer preferences. Unlike HK, we allow for departures from CRS. Under these circumstances, the dispersion of TFPR depends on industry-level variables, distortions, TFPQ, and idiosyncratic demand shocks as given by:

$$\delta_{\text{tfpr}} = 1/(1-\rho\gamma)\left((1-\gamma)\left(\delta_{\ln\xi}+\rho\delta_{\text{tfpq}}\right)+(1-\rho)\sum_j \alpha_j\delta_{\ln\kappa_j}\right), \tag{1}$$

where κ_j and γ respectively denote the combined effect of input and output distortions and returns to scale, and δ_y denotes the distance between the 75th and 25th percentiles of the tfpr distribution measured by variable y. Under CRS (γ=1), the only source of dispersion in tfpr is distortions; this is the insight of HK. However, if we deviate from CRS, the variation in TFPR_i is affected by both demand shocks and TFPQ.

The two commonly used estimation procedures have distinct implications in this context. First, recall that under CRS, tfpr_i^{cs}=tfpr_i. Thus, under CRS and

[5]See Appendix for discussion of the literature and more details.

additional assumptions here, variation in tfpr_i^{cs} should only reflect variation in distortions. Second, consider the relationship between tfpr_i^{rr} and tfpr_i which is given by: $\text{tfpr}_i{=}\lambda{+}(1{-}\gamma)/(1{-}\rho\gamma)\text{tfpr}_i^{rr}{+}(1{-}\rho)/(1{-}\rho\gamma)\sum_j \alpha_j \ln\kappa_{ij}$, which shows that the sign of the correlation between tfpr and tfpr_i^{rr} is determined by γ and ρ. Under CRS, tfpr_i^{cs} and tfpr_i^{rr} are not inherently related. Any relationship would depend on distortions being correlated with tfpq shocks and demand shocks.

With this structure as background, it is useful to discuss the implications of the recent findings in Foster et al. (2015) exploring the relationship between empirical estimates of tfpr_i^{cs} and tfpr_i^{rr}. Using plant-level data for the U.S., they find these measures are highly correlated.[6] They also find corresponding dispersion measures for tfpr_i^{cs} and tfpr_i^{rr} are between .24 and .4. These findings indicate similarity between tfpr_i^{cs} and tfpr_i^{rr} despite their conceptual differences. The only way to reconcile these assumptions for HK, including CRS, with these findings is to assume distortions are highly correlated with technical efficiency and demand shocks (the determinants of tfpr_i^{rr}) and distortions have about the same dispersion as technical efficiency and demand shocks. Correlated distortions may be a plausible explanation but it is not clear this is reasonable in the U.S. We return to this point in the next section.

An alternative interpretation is that the HK assumptions do not hold. One assumption that may not hold is CRS. We explore this further in the next section. Other assumptions may also be violated. For example, FHS show that under linear demand and CRS, $\text{tfpr}_i{=}\text{tfpr}_i^{cs}$ and are a function of tfpq and demand shocks. This naturally yields a high correlation and similar dispersion for tfpr^{cs} and tfpr^{rr}. However, in this case tfpr_i estimates do not inherently correspond to distortions. Alternative interpretations are discussed in the next section.

III. Determinants of TFPR dispersion - an illustrative example

We now consider an exploratory empirical exercise using cross-industry moments constructed from U.S. plant-level data to characterize the dispersion variables in equation (1). Under our assumption about demand, $\widehat{\alpha}$-s can be recovered from joint estimation of $\widehat{\beta}$-s and $\widehat{\rho}$.[7] This approach permits constructing an estimate of conceptual tfpr. Along with estimates of tfpr^{rr}, we can construct estimates of distortions using the decomposition in equation (1). We conduct this exercise using a sample of 50 U.S. Manufacturing industries.[8] Returns to scale estimates are generally larger than 1 and show significant cross-industry variation. Demand elasticity estimates tend to be above one ($\widehat{\rho}$ is below one) and show considerable cross-industry variation. We are primarily interested in the properties of tfpr^{rr} dispersion and distortion dispersion. These two variables exhibit similar dispersion (around .2 and .3 in the average industry), which is comparable to, but

[6]The correlation is between .5 and .8 depending on estimation method.

[7]See Appendix C for a short description and Klette and Griliches (1996) for more details.

[8]More details on the sample can be found in Foster et al. (2015). Further details of the analysis are in Appendix D. The discussion in the text is based on estimators described in Olley and Pakes (1996).

smaller than, the dispersion in tfpr. Consistent with our finding that $\text{tfpr}_i^{rr} \neq \text{tfpr}_i$, we find tfpr^{rr} and tfpr are highly positively correlated but the correlation is lower than 1. In addition, distortions and tfpr are highly correlated. However, distortions are also highly correlated with tfpr^{rr}. This latter finding is consistent with distortions that are more likely at firms with high realizations of either tfpq or product demand.

While caution needs to be used with this illustrative example, it yields conclusions similar to those implied by the estimates of Foster et al. (2015). Specifically, our findings suggest a high correlation between tfpr^{rr} (which reflects tfpq and demand shocks) and the derived estimate of distortions. Since our estimates and those of Foster et al. (2015) reflect the U.S. where distortions are arguably low, an alternative and perhaps more reasonable interpretation is that the correlation between tfpr^{rr} and so-called distortions reflects adjustment frictions. Plants with high draws of tfpq and demand take time to adjust their factors of production implying high tfpr^{rr} plants have high estimates of measured distortions that in fact reflect adjustment frictions. This alternative interpretation raises additional questions about interpretation of estimates of dispersion in tfpr in other countries. More generally, our analysis suggests caution needs to be used in interpreting dispersion in revenue productivity measures as reflecting distortions.

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