

California Tutorials are designed specifically for the California Common Core State Standards and the California Next Generation Science Standards to prepare students for the Smarter Balanced Assessment Consortium (SBAC) exams and the California Science Tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

## Unit 1: Expressions, Equations, and Inequalities

### • LITERAL EQUATIONS

- A-CED.1: Algebra Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.
- A-CED.4: Algebra Creating Equations Create equations that describe numbers or relationships Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

### • FORMULATING AND SOLVING EQUATIONS FROM WORD PROBLEMS

- A-CED.1: Algebra Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.

### • FORMULATING AND SOLVING INEQUALITIES FROM WORD PROBLEMS

- A-CED.1: Algebra Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.

- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

- **SUMS OF GEOMETRIC SEQUENCES**

- A-SSE.4: Algebra Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

## Unit 2: Functions

- **FUNCTIONS AND RELATIONS**

- F-IF.7.b: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

- **DOMAIN AND RANGE**

- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

- **INVERSE FUNCTIONS**

- F-BF.4.a: Functions Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.

## Unit 3: Linear Functions, Equations, and Inequalities

- **SLOPE**

- F-IF.6: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- **GRAPHING AND ANALYZING LINEAR FUNCTIONS**

- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key

features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- F-IF.6: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

- **GRAPHING AND MANIPULATING  $Y = MX + B$**

- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.6: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

#### Unit 4: Exponential Equations, Functions, and Inequalities

- **EXPONENTIAL FUNCTIONS**

- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.8: Functions Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- F-IF.6: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-IF.7.e: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- **EXPONENTIAL GROWTH AND DECAY**

- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- **SOLVING EXPONENTIAL EQUATIONS**

- F-IF.8: Functions Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- F-BF.4.a: Functions Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
- F-LE.4: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems For exponential models, express as a logarithm the solution to  $ab$  to the  $ct$  power =  $d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.7.e: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

- **SOLVING EXPONENTIAL INEQUALITIES**

- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- A-CED.1: Algebra Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.
- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

## Unit 5: Logarithmic Expressions, Equations, and Functions

- **LOGARITHMIC FUNCTIONS**

- F-IF.8: Functions Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- F-BF.4.a: Functions Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
- F-LE.4: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems For exponential models, express as a logarithm the solution to  $ab$  to the  $ct$  power =  $d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-IF.7.e: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- **EVALUATING LOGARITHMIC EXPRESSIONS**
  - F-LE.4: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems For exponential models, express as a logarithm the solution to  $ab$  to the  $ct$  power =  $d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.
  - F-LE.4.3: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.
  - F-LE.4: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems For exponential models, express as a logarithm the solution to  $ab$  to the  $ct$  power =  $d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.
  - A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - F-LE.4: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems For exponential models, express as a logarithm the solution to  $ab$  to the  $ct$  power =  $d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.

- F-LE.4: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems For exponential models, express as a logarithm the solution to  $ab$  to the  $ct$  power =  $d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.
- F-LE.4.2: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Use the definition of logarithms to translate between logarithms in any base.
- **SOLVING LOGARITHMIC EQUATIONS**
  - F-LE.4.2: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Use the definition of logarithms to translate between logarithms in any base.
  - F-LE.4.3: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.
  - F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - F-BF.4.a: Functions Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.

## Unit 6: Polynomials

- **POLYNOMIAL BASICS**
  - A-SSE.1.a: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- **ADDITION AND SUBTRACTION OF POLYNOMIALS**
  - A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - A-APR.1: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- **MULTIPLICATION OF POLYNOMIALS**
  - A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.

- A-APR.1: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- **DIVISION OF POLYNOMIALS**
- A-APR.2: Algebra Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .
- A-APR.6: Algebra Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- **ARITHMETIC OPERATIONS ON FUNCTIONS**
- F-BF.1.b: Functions Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations.

## Unit 7: Graphs and Representations of Quadratic Functions

- **ANALYZING GRAPHS OF QUADRATIC FUNCTIONS**
- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.



- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- F-IF.9: Functions Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

- **REPRESENTATIONS OF QUADRATIC FUNCTIONS**

- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- F-IF.8: Functions Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- A-APR.3: Algebra Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.7.c: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**

- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- F-IF.9: Functions Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).



## Unit 8: Solving Quadratic Equations

### • SOLVING QUADRATIC FUNCTIONS WITH FACTORING

- A-APR.4: Algebra Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- F-IF.8: Functions Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

### • QUADRATIC FORMULA

- N-CN.7: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

### • COMPLEX NUMBERS AND QUADRATIC FUNCTIONS

- N-CN.7: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
- N-CN.1: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
- N-CN.2: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

## Unit 9: Conic Sections

### • COMPLETING THE SQUARE

- G-GPE.3.1: Geometry Expressing Geometric Properties with Equations Translate between the geometric description and the equation for a conic section Given a quadratic equation of the form  $ax^2 + by^2 + cx + dy + e = 0$ , use the method for completing the square to put the

equation into standard form; identify whether the graph of the equation is a circle, ellipse, parabola, or hyperbola, and graph the equation.

- **CIRCLES**

- G-GPE.3.1: Geometry Expressing Geometric Properties with Equations Translate between the geometric description and the equation for a conic section Given a quadratic equation of the form  $ax^2 + by^2 + cx + dy + e = 0$ , use the method for completing the square to put the equation into standard form; identify whether the graph of the equation is a circle, ellipse, parabola, or hyperbola, and graph the equation.

- **PARABOLAS**

- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

## Unit 10: Factoring Polynomials and the Factor Theorem

- **FACTORING SPECIAL CASES**

- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- A-APR.4: Algebra Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- A-SSE.1.b: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- A-APR.3: Algebra Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

- **FACTORING CUBIC POLYNOMIALS**

- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- A-APR.4: Algebra Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- A-SSE.1.b: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- A-APR.3: Algebra Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when

suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

- **FACTORIZING HIGHER-ORDER POLYNOMIALS**

- A-SSE.1.b: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- A-APR.3: Algebra Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- A-APR.4: Algebra Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.

- **FACTOR THEOREM AND REMAINDER THEOREM**

- A-APR.6: Algebra Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
- A-APR.2: Algebra Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .
- A-APR.6: Algebra Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

## Unit 11: Polynomial Functions and Polynomial Identities

- **GRAPHS OF POLYNOMIAL FUNCTIONS**

- A-APR.3: Algebra Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key

features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- F-IF.7.c: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

### • COMPLEX NUMBERS

- N-CN.1: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
- N-CN.2: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

### • POLYNOMIAL IDENTITIES

- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- A-APR.4: Algebra Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- A-APR.5: Algebra Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascals Triangle.

### • POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS

- N-CN.8: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Extend polynomial identities to the complex numbers.
- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- A-APR.4: Algebra Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- N-CN.7: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.

- N-CN.9: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

## Unit 12: Radical and Rational Expressions, Equations, and Functions

### • ANALYZING GRAPHS OF SQUARE ROOT FUNCTIONS

- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- F-IF.7.b: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- F-BF.4.a: Functions Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

### • SOLVING SQUARE ROOT EQUATIONS

- A-REI.2: Algebra Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

## Unit 13: Rational Expressions, Equations, and Functions

### • OPERATIONS WITH RATIONAL EXPRESSIONS

- A-APR.7: Algebra Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- **ANALYZING GRAPHS OF RATIONAL FUNCTIONS**
  - F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
  - F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **SOLVING RATIONAL EQUATIONS**
  - A-REI.2: Algebra Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
  - A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **MODELING SITUATIONS WITH RATIONAL FUNCTIONS**
  - A-SSE.1.b: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
  - A-REI.2: Algebra Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

## Unit 14: Nonlinear Functions

- **LINEAR VERSUS NONLINEAR FUNCTIONS**
  - F-IF.6: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented

symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

### • ABSOLUTE VALUE FUNCTIONS

- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-IF.7.b: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

### • RADIANS AND THE UNIT CIRCLE

- F-TF.1: Functions Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- F-TF.2: Functions Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

### • TRIGONOMETRIC FUNCTIONS

- F-IF.7.e: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- F-TF.5: Functions Trigonometric Functions Model periodic phenomena with trigonometric functions Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- F-TF.8: Functions Trigonometric Functions Prove and apply trigonometric identities Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.
- F-TF.2: Functions Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.



- F-TF.8: Functions Trigonometric Functions Prove and apply trigonometric identities Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.
- F-TF.2.1: Functions Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Graph all 6 basic trigonometric functions.

## Unit 15: Parent Functions and Transformations

### • PARENT FUNCTIONS

- F-IF.7.c: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- F-IF.7.b: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- F-IF.7.e: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

### • TRANSFORMATIONS OF PARENT FUNCTIONS

- F-IF.7.b: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple

cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

- F-IF.7.c: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- F-IF.7.e: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- **MULTIPLE TRANSFORMATIONS OF PARENT FUNCTIONS**
  - F-IF.7.c: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
  - F-IF.7.e: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic

functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

## Unit 16: Systems of Equations

### • SOLVING THREE-VARIABLE SYSTEMS OF LINEAR EQUATIONS

- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

### • SYSTEMS OF NONLINEAR EQUATIONS

- A-REI.11: Algebra Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or

inequalities, and interpret solutions as viable or non-viable options in a modeling context.

## Unit 17: Statistics and Probability

### • ANALYZING STATISTICAL SAMPLES

- S-IC.1: Statistics and Probability Making Inferences and Justifying Conclusions Understand and evaluate random processes underlying statistical experiments Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
- S-IC.4: Statistics and Probability Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- S-IC.2: Statistics and Probability Making Inferences and Justifying Conclusions Understand and evaluate random processes underlying statistical experiments Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

### • EXPERIMENTAL AND OBSERVATIONAL DESIGN

- S-IC.3: Statistics and Probability Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

### • CONCLUSIONS IN DATA

- S-IC.5: Statistics and Probability Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

### • NORMAL DISTRIBUTION

- S-ID.4: Statistics and Probability Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

### • ANALYZING DECISIONS IN PROBABILITY

- S-MD.6: Statistics and Probability Using Probability to Make Decisions Use probability to evaluate outcomes of decisions Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
- S-MD.7: Statistics and Probability Using Probability to Make Decisions Use probability to evaluate outcomes of decisions Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).