

Kentucky Tutorials are designed specifically for the Kentucky Academic Standards to prepare students for the K-PREP, EOC exams, ACT, and ACT Plan.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

## Unit 1: Rational and Irrational Numbers

### • OPERATIONS ON RATIONAL AND IRRATIONAL NUMBERS

- HSN-RN.B.3: The Real Number System Use properties of rational and irrational numbers. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

### • LAWS OF EXPONENTS

- HSN-RN.A.1: The Real Number System Extend the properties of exponents to rational exponents. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
- HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- HSA-APR.D.6: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

- HSA-REI.A.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- HSN-RN.A.2: The Real Number System Extend the properties of exponents to rational exponents. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

## Unit 2: Expressions and Equations 1

### • FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS

- HSF-BF.A.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- HSA-SSE.A.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- HSA-SSE.A.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.

### • ONE-STEP EQUATIONS AND INEQUALITIES

- HSA-CED.A.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
- HSA-CED.A.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- HSA-REI.A.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- HSA-REI.B.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- HSF-BF.A.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

### Unit 3: Expressions and Equations 2

#### • MULTI-STEP EQUATIONS AND INEQUALITIES

- HSA-CED.A.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
- HSA-CED.A.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- HSA-REI.A.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- HSA-REI.B.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

#### • LITERAL EQUATIONS

- HSA-CED.A.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
- HSA-CED.A.4: Creating Equations Create equations that describe numbers or relationships Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
- HSA-REI.B.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

### Unit 4: Functions

#### • FUNCTIONS AND RELATIONS

- HSF-IF.A.2: Interpreting Functions Understand the concept of a function and use function notation Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- HSF-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- HSF-LE.A.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- HSF-IF.C.7.b: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

- **DOMAIN AND RANGE**

- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- HSF-IF.B.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
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#### • MULTIPLE REPRESENTATIONS OF FUNCTIONS

- HSA-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- HSF-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- HSF-IF.C.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- HSF-LE.A.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- HSF-LE.A.1.a: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

### Unit 5: Points, Lines, and Angles

#### • PARALLEL LINES AND ANGLE RELATIONSHIPS

- HSG-CO.A.1: Congruence Experiment with transformations in the plane Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- HSG-CO.C.9: Congruence Prove geometric theorems Prove theorems about lines and angles.

- **PERPENDICULAR BISECTOR AND ANGLE BISECTOR THEOREMS**

- HSG-CO.A.1: Congruence Experiment with transformations in the plane Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- HSG-CO.C.10: Congruence Prove geometric theorems Prove theorems about triangles.
- HSG-CO.C.9: Congruence Prove geometric theorems Prove theorems about lines and angles.

- **CONJECTURES IN COORDINATE GEOMETRY**

- HSG-CO.A.2: Congruence Experiment with transformations in the plane Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- HSG-CO.C.10: Congruence Prove geometric theorems Prove theorems about triangles.
- HSG-GPE.B.4: Expressing Geometric Properties with Equations Use coordinates to prove simple geometric theorems algebraically Use coordinates to prove simple geometric theorems algebraically.

## Unit 6: Conic Sections

- **CIRCLES**

- HSG-CO.A.1: Congruence Experiment with transformations in the plane Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- HSG-GPE.A.1: Expressing Geometric Properties with Equations Translate between the geometric description and the equation for a conic section Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

- **PARABOLAS**

- HSG-GMD.B.4: Geometric Measurement and Dimension Visualize relationships between two-dimensional and three-dimensional objects Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
- HSG-GPE.A.2: Expressing Geometric Properties with Equations Translate between the geometric description and the equation for a conic section Derive the equation of a parabola given a focus and directrix.
- HSA-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

## Unit 7: Geometric Transformations

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**• TRANSFORMATIONS ON THE COORDINATE PLANE**

- HSG-CO.A.2: Congruence Experiment with transformations in the plane Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- HSG-CO.B.6: Congruence Understand congruence in terms of rigid motions Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- HSG-CO.A.5: Congruence Experiment with transformations in the plane Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
- HSG-CO.A.3: Congruence Experiment with transformations in the plane Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- HSG-CO.A.4: Congruence Experiment with transformations in the plane Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- HSG-SRT.A.1.b: Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Verify experimentally the properties of dilations given by a center and a scale factor: The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
- HSG-SRT.A.2: Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- HSG-SRT.A.1.a: Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Verify experimentally the properties of dilations given by a center and a scale factor: A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

**• DILATIONS, TRANSLATIONS, ROTATIONS, AND REFLECTIONS**

- HSG-CO.B.6: Congruence Understand congruence in terms of rigid motions Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

- HSG-CO.A.5: Congruence Experiment with transformations in the plane Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
- HSG-CO.A.3: Congruence Experiment with transformations in the plane Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- HSG-CO.A.2: Congruence Experiment with transformations in the plane Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- HSG-CO.A.4: Congruence Experiment with transformations in the plane Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- HSG-SRT.A.2: Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- HSG-SRT.A.1.a: Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Verify experimentally the properties of dilations given by a center and a scale factor: A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- HSG-SRT.A.1.b: Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Verify experimentally the properties of dilations given by a center and a scale factor: The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

## Unit 8: Congruence and Similarity

### • TRIANGLES AND CONGRUENCE TRANSFORMATIONS

- HSG-CO.B.6: Congruence Understand congruence in terms of rigid motions Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- HSG-CO.B.7: Congruence Understand congruence in terms of rigid motions Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- HSG-CO.B.8: Congruence Understand congruence in terms of rigid motions Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in



terms of rigid motions.

- HSG-CO.C.10: Congruence Prove geometric theorems Prove theorems about triangles.
- HSG-SRT.B.5: Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

#### • **TRIANGLES AND SIMILARITY TRANSFORMATIONS**

- HSG-CO.C.10: Congruence Prove geometric theorems Prove theorems about triangles.
- HSG-SRT.A.2: Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- HSG-SRT.B.5: Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- HSG-SRT.B.4: Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Prove theorems about triangles.
- HSG-SRT.A.3: Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

#### • **SIMILARITY OF OTHER POLYGONS**

- HSG-SRT.A.2: Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

### Unit 9: Triangles and Quadrilaterals

#### • **TRIANGLE ANGLE THEOREMS**

- HSG-CO.C.10: Congruence Prove geometric theorems Prove theorems about triangles.

#### • **TRIANGLE BISECTORS**

- HSG-CO.C.10: Congruence Prove geometric theorems Prove theorems about triangles.
- HSG-CO.C.9: Congruence Prove geometric theorems Prove theorems about lines and angles.
- HSG-SRT.B.5: Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

- HSG-SRT.B.4: Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Prove theorems about triangles.
- HSG-CO.D.12: Congruence Make geometric constructions Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
- HSG-C.A.3: Circles Understand and apply theorems about circles Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
- **MEDIANS AND ALTITUDES OF TRIANGLES**
  - HSG-CO.C.10: Congruence Prove geometric theorems Prove theorems about triangles.

### Unit 10: Quadrilaterals and Constructions

- **PARALLELOGRAMS AND RECTANGLES**

- HSG-CO.C.11: Congruence Prove geometric theorems Prove theorems about parallelograms.
- HSG-MG.A.1: Modeling with Geometry Apply geometric concepts in modeling situations Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

- **SQUARES AND RHOMBI**

- HSG-CO.C.11: Congruence Prove geometric theorems Prove theorems about parallelograms.
- HSG-MG.A.1: Modeling with Geometry Apply geometric concepts in modeling situations Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

- **CONSTRUCTIONS**

- HSG-CO.D.12: Congruence Make geometric constructions Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
- HSG-CO.D.13: Congruence Make geometric constructions Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
- HSG-C.A.4: Circles Understand and apply theorems about circles Construct a tangent line from a point outside a given circle to the circle.

### Unit 11: Right Triangles and Trigonometric Ratios

- **THE PYTHAGOREAN THEOREM**

- HSG-SRT.C.8: Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- HSG-MG.A.1: Modeling with Geometry Apply geometric concepts in modeling situations Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a

tree trunk or a human torso as a cylinder).

- HSG-SRT.B.5: Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- HSG-SRT.B.4: Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Prove theorems about triangles.
- HSG-CO.C.10: Congruence Prove geometric theorems Prove theorems about triangles.

#### • TRIGONOMETRIC RATIOS

- HSF-TF.A.3: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\frac{1}{3}$ ,  $\frac{1}{4}$  and  $\frac{1}{6}$ , and use the unit circle to express the values of sine, cosine, and tangent for  $x$ ,  $+x$ , and  $2x$  in terms of their values for  $x$ , where  $x$  is any real number.
- HSG-SRT.C.8: Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- HSG-SRT.C.6: Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- HSF-TF.A.3: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\frac{1}{3}$ ,  $\frac{1}{4}$  and  $\frac{1}{6}$ , and use the unit circle to express the values of sine, cosine, and tangent for  $x$ ,  $+x$ , and  $2x$  in terms of their values for  $x$ , where  $x$  is any real number.
- HSF-TF.A.3: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\frac{1}{3}$ ,  $\frac{1}{4}$  and  $\frac{1}{6}$ , and use the unit circle to express the values of sine, cosine, and tangent for  $x$ ,  $+x$ , and  $2x$  in terms of their values for  $x$ , where  $x$  is any real number.
- HSG-SRT.C.7: Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Explain and use the relationship between the sine and cosine of complementary angles.
- HSG-SRT.B.5: Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- HSG-MG.A.1: Modeling with Geometry Apply geometric concepts in modeling situations Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

### Unit 12: Trigonometry

#### • RADIANS AND THE UNIT CIRCLE

- HSF-TF.A.1: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- HSF-TF.A.2: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- HSG-C.B.5: Circles Find arc lengths and areas of sectors of circles Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
- HSF-TF.A.3: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $x$ ,  $+\pi$ , and  $2x$  in terms of their values for  $x$ , where  $x$  is any real number.
- HSF-TF.A.4: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- HSG-SRT.C.8: Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- **TRIGONOMETRIC FUNCTIONS**
  - HSF-IF.C.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
  - HSF-TF.A.2: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
  - HSF-TF.B.5: Trigonometric Functions Model periodic phenomena with trigonometric functions Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
  - HSF-TF.C.8: Trigonometric Functions Prove and apply trigonometric identities Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.

## Unit 13: Circles 1

- **CIRCLE BASICS**

- HSG-CO.A.1: Congruence Experiment with transformations in the plane Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- HSG-C.A.2: Circles Understand and apply theorems about circles Identify and describe relationships among inscribed angles, radii, and chords.
- **CENTRAL ANGLES, INSCRIBED ANGLES, AND CHORDS**
  - HSG-C.A.2: Circles Understand and apply theorems about circles Identify and describe relationships among inscribed angles, radii, and chords.
  - HSG-CO.A.1: Congruence Experiment with transformations in the plane Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
  - HSG-CO.C.9: Congruence Prove geometric theorems Prove theorems about lines and angles.
  - HSG-C.A.3: Circles Understand and apply theorems about circles Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
  - HSG-C.B.5: Circles Find arc lengths and areas of sectors of circles Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

#### Unit 14: Circles 2

- **SECANTS, ANGLES, AND INTERCEPTED ARCS**
  - HSG-CO.A.1: Congruence Experiment with transformations in the plane Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
  - HSG-CO.C.9: Congruence Prove geometric theorems Prove theorems about lines and angles.
  - HSG-C.A.2: Circles Understand and apply theorems about circles Identify and describe relationships among inscribed angles, radii, and chords.
- **TANGENTS, ANGLES, AND INTERCEPTED ARCS**
  - HSG-CO.A.1: Congruence Experiment with transformations in the plane Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
  - HSG-CO.C.9: Congruence Prove geometric theorems Prove theorems about lines and angles.
  - HSG-C.A.2: Circles Understand and apply theorems about circles Identify and describe relationships among inscribed angles, radii, and chords.

#### Unit 15: Advanced Circle Properties

- **CONGRUENT AND SIMILAR CIRCLES**

- HSG-CO.B.6: Congruence Understand congruence in terms of rigid motions Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
  - HSG-CO.A.1: Congruence Experiment with transformations in the plane Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
  - HSG-C.A.1: Circles Understand and apply theorems about circles Prove that all circles are similar.
  - HSG-CO.A.4: Congruence Experiment with transformations in the plane Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
  - HSG-SRT.A.2: Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- **CIRCUMFERENCE AND ARC LENGTH**
    - HSG-CO.A.1: Congruence Experiment with transformations in the plane Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
    - HSG-GMD.A.1: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
    - HSG-MG.A.1: Modeling with Geometry Apply geometric concepts in modeling situations Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
  - **AREA OF CIRCLES AND SECTORS**
    - HSG-GMD.A.1: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
    - HSG-MG.A.1: Modeling with Geometry Apply geometric concepts in modeling situations Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
    - HSG-CO.A.1: Congruence Experiment with transformations in the plane Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

- HSG-C.B.5: Circles Find arc lengths and areas of sectors of circles Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

## Unit 16: Exponential Functions, Equations, and Inequalities

### • EXPONENTIAL FUNCTIONS

- HSA-SSE.A.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- HSA-SSE.A.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- HSF-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- HSF-IF.C.8.b: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
- HSF-LE.A.1.a: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- HSF-IF.B.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- HSF-LE.A.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- HSF-LE.A.3: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a

function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- HSA-SSE.B.3.c: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.
- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- HSF-IF.B.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- HSF-IF.C.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- HSA-CED.A.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
- HSA-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- HSA-REI.B.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- HSF-LE.B.5: Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Interpret the parameters in a linear or exponential function in terms of a context.
- HSF-BF.A.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- HSF-LE.A.1.c: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

- **EXPONENTIAL GROWTH AND DECAY**



- HSA-SSE.A.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- HSA-SSE.A.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- HSF-LE.A.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- HSF-LE.B.5: Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Interpret the parameters in a linear or exponential function in terms of a context.
- HSF-IF.C.8.b: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
- HSF-LE.A.1.a: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- HSF-LE.A.1.c: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- HSF-LE.A.3: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- HSF-LE.A.1.b: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- HSA-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- **SOLVING EXPONENTIAL INEQUALITIES**

- HSF-LE.A.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- HSA-CED.A.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- HSA-CED.A.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
- HSA-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- HSA-SSE.A.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- HSF-LE.A.1.c: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

## Unit 17: Polynomials 1

### • POLYNOMIAL BASICS

- HSA-SSE.A.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- HSA-SSE.A.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- HSA-APR.A.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

### • ADDITION AND SUBTRACTION OF POLYNOMIALS

- HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- HSA-APR.A.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the

integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

## Unit 18: Polynomials 2

### • MULTIPLICATION OF POLYNOMIALS

- HSA-APR.A.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.

### • ARITHMETIC OPERATIONS ON FUNCTIONS

- HSF-BF.A.1.b: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations.

## Unit 19: Factoring

### • FACTORING QUADRATIC TRINOMIALS

- HSA-SSE.B.3.a: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
- HSA-SSE.A.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- HSA-SSE.A.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .

### • FACTORING SPECIAL CASES

- HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- HSA-APR.C.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.

- HSA-SSE.A.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- HSA-SSE.A.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- HSA-APR.B.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

## Unit 20: Complex Numbers

### • COMPLEX NUMBERS

- HSN-CN.A.1: The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
- HSN-CN.A.2: The Complex Number System Perform arithmetic operations with complex numbers. Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

### • POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS

- HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- HSA-APR.C.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- HSA-SSE.A.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- HSA-SSE.A.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- HSN-CN.C.8: The Complex Number System Use complex numbers in polynomial identities and equations. Extend polynomial identities to the complex numbers.
- HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .

- HSN-CN.A.1: The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
- HSN-CN.C.7: The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
- HSN-CN.C.9: The Complex Number System Use complex numbers in polynomial identities and equations. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

## Unit 21: Representations of Quadratic Functions

### • QUADRATIC FUNCTIONS

- HSF-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- HSF-IF.C.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- HSA-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- HSA-SSE.A.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- HSA-SSE.A.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- HSA-SSE.B.3.b: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- HSF-IF.C.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- HSF-BF.A.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

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**• ANALYZING GRAPHS OF QUADRATIC FUNCTIONS**

- HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- HSF-IF.C.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- HSF-IF.B.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- HSF-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- HSF-IF.C.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- HSF-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- HSA-APR.B.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when

suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

- HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- HSF-IF.C.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

### • REPRESENTATIONS OF QUADRATIC FUNCTIONS

- HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- HSA-REI.B.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p) = q$  that has the same solutions. Derive the quadratic formula from this form.
- HSF-IF.C.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- HSF-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- HSF-IF.C.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- HSA-APR.B.2: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Know and apply the Remainder

Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .

- HSA-APR.B.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- HSA-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- HSF-BF.A.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- HSA-CED.A.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- HSA-SSE.B.3.a: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
- HSF-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- HSF-IF.C.7.c: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

## Unit 22: Quadratic Parent Functions and Transformations

### • QUADRATIC PARENT FUNCTION

- HSF-IF.C.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- HSA-REI.D.10: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a



function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- HSF-IF.B.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

### • TRANSFORMATIONS OF THE QUADRATIC PARENT FUNCTION

- HSF-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- HSG-CO.A.2: Congruence Experiment with transformations in the plane Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- HSF-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- HSF-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- HSF-IF.B.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- HSF-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

## Unit 23: Solving Quadratic Equations

### • SOLVING QUADRATIC EQUATIONS BY FACTORING

- HSA-SSE.B.3.a: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain

properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.

- HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- HSF-IF.C.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- HSA-APR.B.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- HSA-APR.C.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- HSF-IF.C.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- HSA-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- HSF-BF.A.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **COMPLETING THE SQUARE**
  - HSA-SSE.B.3.b: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
  - HSA-REI.B.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p) = q$  that has the same solutions. Derive the quadratic formula from this form.
  - HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
  - HSF-IF.C.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  - HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - HSA-REI.B.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p) = q$  that has the same solutions. Derive the quadratic formula from this form.
  - HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula

and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .

- HSA-REI.B.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.
- HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- HSF-IF.C.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.

## Unit 24: Quadratic Formula and Complex Numbers

### • QUADRATIC FORMULA

- HSA-SSE.A.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- HSA-SSE.A.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- HSF-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- HSA-CED.A.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- HSA-REI.B.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x + p) = q$  that has the same solutions. Derive the quadratic formula from this form.
- HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- HSF-BF.A.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **COMPLEX NUMBERS AND QUADRATIC FUNCTIONS**
  - HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
  - HSN-CN.C.7: The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
  - HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
  - HSN-CN.A.1: The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has

the form  $a + bi$  with  $a$  and  $b$  real.

- HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- HSN-CN.A.1: The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
- HSN-CN.A.2: The Complex Number System Perform arithmetic operations with complex numbers. Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

## Unit 25: Nonlinear Functions

### • INVERSE FUNCTIONS

- HSF-BF.B.4.c: Building Functions Build new functions from existing functions Find inverse functions. Read values of an inverse function from a graph or a table, given that the function has an inverse.
- HSF-BF.B.4.a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
- HSF-BF.B.4.d: Building Functions Build new functions from existing functions Find inverse functions. Produce an invertible function from a non-invertible function by restricting the domain.

### • ABSOLUTE VALUE FUNCTIONS

- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- HSF-IF.B.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- HSF-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- HSF-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- HSF-IF.C.7.b: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **SYSTEMS OF NONLINEAR EQUATIONS**
  - HSA-REI.C.6: Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
  - HSA-REI.C.7: Reasoning with Equations and Inequalities Solve systems of equations Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
  - HSA-REI.C.5: Reasoning with Equations and Inequalities Solve systems of equations Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
  - HSA-REI.D.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = (x)$  intersect are the solutions of the equation  $f(x) = (x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
  - HSA-CED.A.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
  - HSA-REI.D.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = (x)$  intersect are the solutions of the equation  $f(x) = (x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
  - HSF-LE.A.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential

functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

## Unit 26: Volume

### • VOLUME OF PRISMS AND PYRAMIDS

- HSG-MG.A.1: Modeling with Geometry Apply geometric concepts in modeling situations Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- HSG-GMD.A.2: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument using Cavalieris principle for the formulas for the volume of a sphere and other solid figures.
- HSG-GMD.A.1: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- HSG-GMD.A.2: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument using Cavalieris principle for the formulas for the volume of a sphere and other solid figures.
- HSG-GMD.A.3: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- HSG-GMD.A.2: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument using Cavalieris principle for the formulas for the volume of a sphere and other solid figures.
- HSG-GMD.B.4: Geometric Measurement and Dimension Visualize relationships between two-dimensional and three-dimensional objects Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

### • VOLUME OF CYLINDERS AND CONES

- HSG-GMD.A.1: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- HSG-GMD.A.2: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument using Cavalieris principle for the formulas for the volume of a sphere and other solid figures.
- HSG-GMD.A.3: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- HSG-GMD.A.2: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument using Cavalieris principle for the formulas for the



volume of a sphere and other solid figures.

- HSG-MG.A.1: Modeling with Geometry Apply geometric concepts in modeling situations Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- HSG-GMD.B.4: Geometric Measurement and Dimension Visualize relationships between two-dimensional and three-dimensional objects Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
- HSG-GMD.A.2: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument using Cavalieris principle for the formulas for the volume of a sphere and other solid figures.
- **VOLUME OF COMPOSITE SOLIDS**
  - HSG-MG.A.1: Modeling with Geometry Apply geometric concepts in modeling situations Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
  - HSG-GMD.A.3: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
  - HSG-GMD.A.2: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument using Cavalieris principle for the formulas for the volume of a sphere and other solid figures.

## Unit 27: Basic Probability Concepts

- **INTRODUCTION TO PROBABILITY**
  - HSS-CP.A.2: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand that two events and are independent if the probability of and occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
  - HSS-CP.B.8: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the general Multiplication Rule in a uniform probability model, ( and ) = ()() = ()(), and interpret the answer in terms of the model.
  - HSS-CP.A.2: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand that two events and are independent if the probability of and occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
  - HSS-CP.A.5: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

- HSS-CP.B.8: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the general Multiplication Rule in a uniform probability model,  $(A \text{ and } B) = (A)(B) = (A)(B)$ , and interpret the answer in terms of the model.
- HSS-CP.A.1: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
- HSS-CP.B.7: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the Addition Rule,  $(A \text{ or } B) = (A) + (B) - (A \text{ and } B)$ , and interpret the answer in terms of the model.
- **COMBINATIONS AND PERMUTATIONS**
  - HSS-CP.B.9: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Use permutations and combinations to compute probabilities of compound events and solve problems.

## Unit 28: Advanced Probability Concepts

### • **CONDITIONAL PROBABILITY**

- HSS-CP.A.3: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand the conditional probability of given as  $(A \text{ and } B)/(A)$ , and interpret independence of  $A$  and  $B$  as saying that the conditional probability of given is the same as the probability of  $B$ , and the conditional probability of given is the same as the probability of  $A$ .
- HSS-CP.A.5: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
- HSS-CP.B.6: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of given as the fraction of  $S$  outcomes that also belong to  $A$ , and interpret the answer in terms of the model.
- HSS-CP.A.2: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand that two events  $A$  and  $B$  are independent if the probability of  $A$  and  $B$  occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- HSS-CP.A.3: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand the conditional probability of given as  $(A \text{ and } B)/(A)$ , and interpret independence of  $A$  and  $B$  as saying that the conditional probability of given is the same as the probability of  $B$ , and the conditional probability of given is the same as the probability of  $A$ .

- HSS-CP.B.6: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.
- HSS-ID.B.5: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
- HSS-CP.A.4: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
- HSS-CP.A.3: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand the conditional probability of given as ( and )/( ), and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
- HSS-CP.B.6: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.
- HSS-CP.A.3: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand the conditional probability of given as ( and )/( ), and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
- HSS-CP.B.6: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.
- HSS-CP.A.3: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand the conditional probability of given as ( and )/( ), and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
- HSS-CP.B.6: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional

probability of given as the fraction of  $s$  outcomes that also belong to  $t$ , and interpret the answer in terms of the model.

- **GEOMETRIC PROBABILITIES**

- HSG-MG.A.3: Modeling with Geometry Apply geometric concepts in modeling situations Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
- HSS-MD.B.7: Using Probability to Make Decisions Use probability to evaluate outcomes of decisions Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
- HSS-CP.A.1: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
- HSS-CP.B.7: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the Addition Rule,  $(\text{or}) = () + ()$  ( and ), and interpret the answer in terms of the model.
- HSS-CP.A.1: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
- HSS-CP.B.7: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the Addition Rule,  $(\text{or}) = () + ()$  ( and ), and interpret the answer in terms of the model.

- **ANALYZING DECISIONS IN PROBABILITY**

- HSS-MD.B.6: Using Probability to Make Decisions Use probability to evaluate outcomes of decisions Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
- HSS-MD.B.7: Using Probability to Make Decisions Use probability to evaluate outcomes of decisions Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).