

California Tutorials are designed specifically for the California Common Core State Standards and the California Next Generation Science Standards to prepare students for the Smarter Balanced Assessment Consortium (SBAC) exams and the California Science Tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

Unit 1: Rational and Irrational Numbers

- **OPERATIONS ON RATIONAL AND IRRATIONAL NUMBERS**

- N-RN.3: Number and Quantity The Real Number System Use properties of rational and irrational numbers. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

- **LAWS OF EXPONENTS**

- N-RN.1: Number and Quantity The Real Number System Extend the properties of exponents to rational exponents. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
- N-RN.2: Number and Quantity The Real Number System Extend the properties of exponents to rational exponents. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Unit 2: Expressions and Equations

- **FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS**

- F-BF.1.a: Functions Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- A-SSE.1.a: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- **ONE-STEP EQUATIONS AND INEQUALITIES**
 - A-CED.1: Algebra Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.
 - F-BF.1.a: Functions Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **MULTI-STEP EQUATIONS AND INEQUALITIES**
 - A-CED.1: Algebra Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.
- **LITERAL EQUATIONS**
 - A-CED.1: Algebra Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.
 - A-CED.4: Algebra Creating Equations Create equations that describe numbers or relationships Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Unit 3: Functions

- **FUNCTIONS AND RELATIONS**
 - F-IF.7.b: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **DOMAIN AND RANGE**
 - F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**

- F-IF.9: Functions Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- A-CED.1: Algebra Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.
- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Unit 4: Points, Lines, and Angles

- **PARALLEL LINES AND ANGLE RELATIONSHIPS**

- G-CO.9: Geometry Congruence Prove geometric theorems Prove theorems about lines and angles.

- **PERPENDICULAR BISECTOR AND ANGLE BISECTOR THEOREMS**

- G-CO.9: Geometry Congruence Prove geometric theorems Prove theorems about lines and angles.
- G-CO.10: Geometry Congruence Prove geometric theorems Prove theorems about triangles.
- G-SRT.4: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Prove theorems about triangles.

Unit 5: The Coordinate Plane

- **LENGTH AND THE DISTANCE FORMULA**

- G-GPE.6: Geometry Expressing Geometric Properties with Equations Use coordinates to prove simple geometric theorems algebraically Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

- **MIDPOINT FORMULA ON THE COORDINATE PLANE**

- G-GPE.6: Geometry Expressing Geometric Properties with Equations Use coordinates to prove simple geometric theorems algebraically Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

- **CONJECTURES IN COORDINATE GEOMETRY**

- G-GPE.4: Geometry Expressing Geometric Properties with Equations Use coordinates to prove simple geometric theorems algebraically Use coordinates to prove simple geometric theorems algebraically.

Unit 6: Conic Sections

- **CIRCLES**

- G-GPE.1: Geometry Expressing Geometric Properties with Equations Translate between the geometric description and the equation for a conic section Derive the equation of a circle of

given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

- **PARABOLAS**

- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- G-GPE.2: Geometry Expressing Geometric Properties with Equations Translate between the geometric description and the equation for a conic section Derive the equation of a parabola given a focus and directrix.

Unit 7: Geometric Transformations

- **TRANSFORMATIONS ON THE COORDINATE PLANE**

- G-SRT.1.a: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Verify experimentally the properties of dilations given by a center and a scale factor: A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- G-SRT.2: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- G-SRT.1.b: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Verify experimentally the properties of dilations given by a center and a scale factor: The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

- **DILATIONS, TRANSLATIONS, ROTATIONS, AND REFLECTIONS**

- G-SRT.1.a: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Verify experimentally the properties of dilations given by a center and a scale factor: A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- G-SRT.1.b: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Verify experimentally the properties of dilations given by a center and a scale factor: The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
- G-SRT.2: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Unit 8: Congruence and Similarity

• TRIANGLES AND CONGRUENCE TRANSFORMATIONS

- G-CO.10: Geometry Congruence Prove geometric theorems Prove theorems about triangles.
- G-SRT.4: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Prove theorems about triangles.
- G-SRT.5: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

• TRIANGLES AND SIMILARITY TRANSFORMATIONS

- G-SRT.2: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- G-SRT.5: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- G-CO.10: Geometry Congruence Prove geometric theorems Prove theorems about triangles.
- G-SRT.3: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
- G-SRT.4: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Prove theorems about triangles.

• SIMILARITY OF OTHER POLYGONS

- G-SRT.2: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Unit 9: Triangles and Quadrilaterals

• CLASSIFYING TRIANGLES

- G-CO.10: Geometry Congruence Prove geometric theorems Prove theorems about triangles.
- G-GMD.6: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Verify experimentally that in a triangle, angles opposite longer sides are larger, sides opposite larger angles are longer, and the sum of any two side lengths is greater than the remaining side length; apply these relationships to solve real-world and mathematical problems.

- G-SRT.4: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Prove theorems about triangles.

- **TRIANGLE ANGLE THEOREMS**

- G-CO.10: Geometry Congruence Prove geometric theorems Prove theorems about triangles.
- G-SRT.4: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Prove theorems about triangles.

- **TRIANGLE BISECTORS**

- G-CO.9: Geometry Congruence Prove geometric theorems Prove theorems about lines and angles.
- G-CO.10: Geometry Congruence Prove geometric theorems Prove theorems about triangles.
- G-SRT.4: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Prove theorems about triangles.
- G-SRT.5: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- G-C.3: Geometry Circles Understand and apply theorems about circles Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

- **MEDIANS AND ALTITUDES OF TRIANGLES**

- G-CO.10: Geometry Congruence Prove geometric theorems Prove theorems about triangles.
- G-SRT.4: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Prove theorems about triangles.

Unit 10: Quadrilaterals and Constructions

- **PARALLELOGRAMS AND RECTANGLES**

- G-CO.11: Geometry Congruence Prove geometric theorems Prove theorems about parallelograms.

- **SQUARES AND RHOMBI**

- G-CO.11: Geometry Congruence Prove geometric theorems Prove theorems about parallelograms.

- **CONSTRUCTIONS**

- G-C.4: Geometry Circles Understand and apply theorems about circles Construct a tangent line from a point outside a given circle to the circle.

Unit 11: Triangles and Trigonometry

- **THE PYTHAGOREAN THEOREM**

- G-CO.10: Geometry Congruence Prove geometric theorems Prove theorems about triangles.

- G-SRT.8: Geometry Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- G-SRT.4: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Prove theorems about triangles.
- G-SRT.5: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- **TRIGONOMETRIC RATIOS**
 - G-SRT.6: Geometry Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
 - G-SRT.8: Geometry Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
 - G-SRT.8.1: Geometry Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Derive and use the trigonometric ratios for special right triangles (30,60,90 and 45,45,90).
 - G-SRT.8.1: Geometry Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Derive and use the trigonometric ratios for special right triangles (30,60,90 and 45,45,90).
 - G-SRT.8.1: Geometry Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Derive and use the trigonometric ratios for special right triangles (30,60,90 and 45,45,90).
 - G-SRT.7: Geometry Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Explain and use the relationship between the sine and cosine of complementary angles.
- **RADIANS AND THE UNIT CIRCLE**
 - G-C.5: Geometry Circles Find arc lengths and areas of sectors of circles Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert between degrees and radians.
 - G-SRT.8: Geometry Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
 - G-SRT.8.1: Geometry Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Derive and use the trigonometric ratios for special

right triangles (30,60,90 and 45,45,90).

- **TRIGONOMETRIC FUNCTIONS**

- F-TF.8: Functions Trigonometric Functions Prove and apply trigonometric identities Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

Unit 12: Circles

- **CIRCLE BASICS**

- G-C.2: Geometry Circles Understand and apply theorems about circles Identify and describe relationships among inscribed angles, radii, and chords.

- **CENTRAL ANGLES, INSCRIBED ANGLES, AND CHORDS**

- G-C.2: Geometry Circles Understand and apply theorems about circles Identify and describe relationships among inscribed angles, radii, and chords.
- G-CO.9: Geometry Congruence Prove geometric theorems Prove theorems about lines and angles.
- G-C.3: Geometry Circles Understand and apply theorems about circles Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

- **SECANTS, ANGLES, AND INTERCEPTED ARCS**

- G-CO.9: Geometry Congruence Prove geometric theorems Prove theorems about lines and angles.
- G-C.2: Geometry Circles Understand and apply theorems about circles Identify and describe relationships among inscribed angles, radii, and chords.

- **TANGENTS, ANGLES, AND INTERCEPTED ARCS**

- G-CO.9: Geometry Congruence Prove geometric theorems Prove theorems about lines and angles.
- G-C.2: Geometry Circles Understand and apply theorems about circles Identify and describe relationships among inscribed angles, radii, and chords.

Unit 13: Advanced Circle Properties

- **CONGRUENT AND SIMILAR CIRCLES**

- G-C.1: Geometry Circles Understand and apply theorems about circles Prove that all circles are similar.
- G-SRT.2: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

- **CIRCUMFERENCE AND ARC LENGTH**

- G-GMD.1: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

- **AREA OF CIRCLES AND SECTORS**

- G-GMD.1: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- G-C.5: Geometry Circles Find arc lengths and areas of sectors of circles Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert between degrees and radians.

Unit 14: Exponential Functions, Equations, and Inequalities

- **EXPONENTIAL FUNCTIONS**

- A-SSE.3.c: Algebra Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.
- F-IF.8.b: Functions Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
- A-SSE.1.b: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- A-CED.1: Algebra Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.

- **EXPONENTIAL GROWTH AND DECAY**

- F-IF.8.b: Functions Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
- A-SSE.1.b: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- F-LE.3: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Observe using graphs and tables that a

quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- **SOLVING EXPONENTIAL INEQUALITIES**

- A-SSE.1.b: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Unit 15: Polynomials

- **POLYNOMIAL BASICS**

- A-SSE.1.a: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.

- **ADDITION AND SUBTRACTION OF POLYNOMIALS**

- A-APR.1: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

- **MULTIPLICATION OF POLYNOMIALS**

- A-APR.1: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

- **ARITHMETIC OPERATIONS ON FUNCTIONS**

- F-BF.1.b: Functions Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations.

Unit 16: Factoring

- **FACTORING QUADRATIC TRINOMIALS**

- A-SSE.3.a: Algebra Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.

- A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .

- **FACTORING SPECIAL CASES**

- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- A-SSE.1.b: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Unit 17: Complex Numbers

- **COMPLEX NUMBERS**

- N-CN.1: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- N-CN.2: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

- **POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS**

- N-CN.8: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Extend polynomial identities to the complex numbers.
- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- N-CN.7: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
- A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- N-CN.9: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Unit 18: Representations of Quadratic Functions

- **QUADRATIC FUNCTIONS**

- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.7.a: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- F-LE.6: Functions Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.
- **ANALYZING GRAPHS OF QUADRATIC FUNCTIONS**
 - F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
 - F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
 - F-IF.7.a: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
 - F-IF.9: Functions Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- **REPRESENTATIONS OF QUADRATIC FUNCTIONS**
 - A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
 - A-REI.4.a: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p) = q$ that has the same solutions. Derive the quadratic formula from this form.
 - F-IF.8.a: Functions Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-SSE.3.a: Algebra Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-LE.6: Functions Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.
- **QUADRATIC PARENT FUNCTION**
 - F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **TRANSFORMATIONS OF THE QUADRATIC PARENT FUNCTION**
 - F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
 - F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
 - F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Unit 19: Solving Quadratic Equations

- **SOLVING QUADRATIC FUNCTIONS WITH FACTORING**
 - A-SSE.3.a: Algebra Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
 - A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by

inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .

- F-IF.8.a: Functions Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- F-LE.6: Functions Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.
- A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- F-IF.7.a: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.

• COMPLETING THE SQUARE

- A-REI.4.a: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p) = q$ that has the same solutions. Derive the quadratic formula from this form.
- A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- F-IF.8.a: Functions Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- A-SSE.3.b: Algebra Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

- A-REI.4.a: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p) = q$ that has the same solutions. Derive the quadratic formula from this form.
 - A-REI.4.a: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p) = q$ that has the same solutions. Derive the quadratic formula from this form.
 - A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
 - F-IF.7.a: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **QUADRATIC FORMULA**
- A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
 - F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
 - A-REI.4.a: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p) = q$ that has the same solutions. Derive the quadratic formula from this form.
 - A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
 - F-BF.1.a: Functions Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

- **COMPLEX NUMBERS AND QUADRATIC FUNCTIONS**

- A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- N-CN.7: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
- A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- N-CN.1: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .

Unit 20: Nonlinear Functions

- **INVERSE FUNCTIONS**

- F-BF.4.a: Functions Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.

- **ABSOLUTE VALUE FUNCTIONS**

- F-IF.7.b: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

- **SYSTEMS OF NONLINEAR EQUATIONS**

- A-REI.7: Algebra Reasoning with Equations and Inequalities Solve systems of equations Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Unit 21: Volume

• VOLUME OF PRISMS AND PYRAMIDS

- G-GMD.1: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- G-GMD.3: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

• VOLUME OF CYLINDERS AND CONES

- G-GMD.1: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- G-GMD.3: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

• VOLUME OF COMPOSITE SOLIDS

- G-GMD.3: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

• VOLUME OF SIMILAR SOLIDS

- G-GMD.3: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- G-GMD.5: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Know that the effect of a scale factor k greater than zero on length, area, and volume is to multiply each by k , k , and k , respectively; determine length, area and volume measures using scale factors.
- G-GMD.5: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Know that the effect of a scale factor k greater than zero on length, area, and volume is to multiply each by k , k , and k , respectively; determine length, area and volume measures using scale factors.
- G-GMD.5: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Know that the effect of a scale factor k greater than zero on length, area, and volume is to multiply each by k , k , and k , respectively; determine length, area and volume measures using scale factors.

Unit 22: Basic Probability Concepts

• INTRODUCTION TO PROBABILITY

- S-CP.1: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
- S-CP.2: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand that two events and are independent if the probability of and occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- S-CP.8: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the general Multiplication Rule in a uniform probability model, $(\text{and}) = ()() = ()()$, and interpret the answer in terms of the model.
- S-CP.2: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand that two events and are independent if the probability of and occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- S-CP.8: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the general Multiplication Rule in a uniform probability model, $(\text{and}) = ()() = ()()$, and interpret the answer in terms of the model.
- S-CP.7: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the Addition Rule, $(\text{or}) = () + ()$ (and), and interpret the answer in terms of the model.

• COMBINATIONS AND PERMUTATIONS

- S-CP.9: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Use permutations and combinations to compute probabilities of compound events and solve problems.

Unit 23: Advanced Probability Concepts

• CONDITIONAL PROBABILITY

- S-CP.3: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand the conditional probability of given as $(\text{and})/()$, and interpret independence of

and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .

- S-CP.5: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
- S-CP.6: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.
- S-CP.2: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand that two events and are independent if the probability of and occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- S-CP.3: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand the conditional probability of given as $(\text{and})/()$, and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
- S-CP.6: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.
- S-CP.4: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
- S-CP.3: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand the conditional probability of given as $(\text{and})/()$, and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
- S-CP.6: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.
- S-CP.3: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data

Understand the conditional probability of given as $(\text{and})/()$, and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .

- S-CP.6: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.
- S-CP.3: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand the conditional probability of given as $(\text{and})/()$, and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
- S-CP.6: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.

• GEOMETRIC PROBABILITIES

- S-CP.1: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
- S-MD.7: Statistics and Probability Using Probability to Make Decisions Use probability to evaluate outcomes of decisions Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
- S-CP.1: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
- S-CP.7: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the Addition Rule, $(\text{or}) = () + ()$ (and) , and interpret the answer in terms of the model.
- S-CP.1: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
- S-CP.7: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability

model Apply the Addition Rule, $(\text{or}) = () + ()$ (and), and interpret the answer in terms of the model.

• **ANALYZING DECISIONS IN PROBABILITY**

- S-MD.6: Statistics and Probability Using Probability to Make Decisions Use probability to evaluate outcomes of decisions Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
- S-MD.7: Statistics and Probability Using Probability to Make Decisions Use probability to evaluate outcomes of decisions Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).