Regents Tutorials are designed specifically to prepare students for the New York Regents Exams.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

Unit 1: Expressions and Equations

- **AXIOMS OF EQUALITY**
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - NY.CCLS.Math.9-12.A-REI.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- **LITERAL EQUATIONS**
  - NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
  - NY.CCLS.Math.9-12.A-CED.4: Creating Equations Create equations that describe numbers or relationships Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

- NY.CCLS.Math.9-12.A-REI.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Unit 2: Equations, and Inequalities

- **FORMULATING AND SOLVING EQUATIONS FROM WORD PROBLEMS**
  - NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
  - NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
  - NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
  - NY.CCLS.Math.9-12.F-LE.1.b: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
  - NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - NY.CCLS.Math.9-12.A-REI.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
  - NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- **FORMULATING AND SOLVING INEQUALITIES FROM WORD PROBLEMS**

- NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
- NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- NY.CCLS.Math.9-12.A-REI.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- NY.CCLS.Math.9-12.A-REI.12: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Unit 3: Series and Sequences

- **SEQUENCES**
  - NY.CCLS.Math.9-12.F-IF.3: Interpreting Functions Understand the concept of a function and use function notation Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
  - NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
  - NY.CCLS.Math.9-12.F-BF.2: Building Functions Build a function that models a relationship between two quantities Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
  - NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **ARITHMETIC AND GEOMETRIC SEQUENCES**
  - NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

- NY.CCLS.Math.9-12.F-BF.2: Building Functions Build a function that models a relationship between two quantities Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- NY.CCLS.Math.9-12.F-IF.3: Interpreting Functions Understand the concept of a function and use function notation Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- **SUMS OF GEOMETRIC SEQUENCES**
- NY.CCLS.Math.9-12.A-SSE.4: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

Unit 4: Linear Functions, Equations, and Inequalities

- **SLOPE**
- NY.CCLS.Math.9-12.F-IF.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.S-ID.7: Interpreting Categorical and Quantitative Data Interpret linear models Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- NY.CCLS.Math.9-12.G-GPE.5: Expressing Geometric Properties with Equations Use coordinates to prove simple geometric theorems algebraically Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

- **GRAPHING AND ANALYZING LINEAR FUNCTIONS**

- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- NY.CCLS.Math.9-12.F-IF.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- NY.CCLS.Math.9-12.F-IF.2: Interpreting Functions Understand the concept of a function and use function notation Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- NY.CCLS.Math.9-12.F-LE.5: Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Interpret the parameters in a linear or exponential function in terms of a context.
- NY.CCLS.Math.9-12.F-IF.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.

- NY.CCLS.Math.9-12.S-ID.7: Interpreting Categorical and Quantitative Data Interpret linear models Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

- **GRAPHING AND MANIPULATING Y = MX + B**
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
  - NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - NY.CCLS.Math.9-12.F-IF.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
  - NY.CCLS.Math.9-12.F-IF.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
  - NY.CCLS.Math.9-12.F-IF.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
  - NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f

is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- NY.CCLS.Math.9-12.S-ID.7: Interpreting Categorical and Quantitative Data Interpret linear models Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- NY.CCLS.Math.9-12.F-LE.1.b: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- NY.CCLS.Math.9-12.F-LE.5: Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Interpret the parameters in a linear or exponential function in terms of a context.

Unit 5: Exponential Functions

- **LAWS OF EXPONENTS**
  - NY.CCLS.Math.9-12.N-RN.1: The Real Number System Extend the properties of exponents to rational exponents. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
  - NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - NY.CCLS.Math.9-12.A-APR.6: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
  - NY.CCLS.Math.9-12.A-REI.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

- NY.CCLS.Math.9-12.F-IF.8.b: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
- NY.CCLS.Math.9-12.N-RN.2: The Real Number System Extend the properties of exponents to rational exponents. Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.

- **EXPONENTIAL FUNCTIONS**
  - NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - NY.CCLS.Math.9-12.F-IF.8.b: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
  - NY.CCLS.Math.9-12.F-LE.1.a: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
  - NY.CCLS.Math.9-12.F-IF.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
  - NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential

functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- NY.CCLS.Math.9-12.F-LE.3: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- NY.CCLS.Math.9-12.F-IF.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- NY.CCLS.Math.9-12.A-SSE.3.c: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.
- NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.A-REI.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities.

Determine an explicit expression, a recursive process, or steps for calculation from a context.

- NY.CCLS.Math.9-12.F-LE.1.c: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- NY.CCLS.Math.9-12.F-LE.5: Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Interpret the parameters in a linear or exponential function in terms of a context.

Unit 6: Solving Exponential Equations and Inequalities

- **EXPONENTIAL GROWTH AND DECAY**
  - NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - NY.CCLS.Math.9-12.A-SSE.3.c: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.
  - NY.CCLS.Math.9-12.F-IF.8.b: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
  - NY.CCLS.Math.9-12.F-LE.1.a: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
  - NY.CCLS.Math.9-12.F-LE.1.b: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
  - NY.CCLS.Math.9-12.F-LE.1.c: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- NY.CCLS.Math.9-12.F-LE.3: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- NY.CCLS.Math.9-12.F-LE.5: Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Interpret the parameters in a linear or exponential function in terms of a context.
- NY.CCLS.Math.9-12.F-IF.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.F-IF.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

- **SOLVING EXPONENTIAL EQUATIONS**
  - NY.CCLS.Math.9-12.A-SSE.3.c: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.
  - NY.CCLS.Math.9-12.F-IF.8.b: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
  - NY.CCLS.Math.9-12.F-BF.4.a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an

inverse and write an expression for the inverse.

- NY.CCLS.Math.9-12.F-LE.4: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems For exponential models, express as a logarithm the solution to ab to the ct power = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.F-IF.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- **SOLVING EXPONENTIAL INEQUALITIES**
  - NY.CCLS.Math.9-12.F-IF.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
  - NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
  - NY.CCLS.Math.9-12.F-IF.8.b: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
  - NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.

- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- NY.CCLS.Math.9-12.F-LE.1.c: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Unit 7: Logarithmic Expressions, Equations, and Functions

- **LOGARITHMIC FUNCTIONS**
  - NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - NY.CCLS.Math.9-12.F-BF.4.a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.
  - NY.CCLS.Math.9-12.F-BF.4.c: Building Functions Build new functions from existing functions Find inverse functions. Read values of an inverse function from a graph or a table, given that the function has an inverse.
  - NY.CCLS.Math.9-12.F-BF.5: Building Functions Build new functions from existing functions Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
  - NY.CCLS.Math.9-12.F-LE.4: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems For exponential models, express as a logarithm the solution to ab to the ct power = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
  - NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

- NY.CCLS.Math.9-12.F-IF.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

- **EVALUATING LOGARITHMIC EXPRESSIONS**
  - NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - NY.CCLS.Math.9-12.F-LE.4: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems For exponential models, express as a logarithm the solution to ab to the ct power = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
- **SOLVING LOGARITHMIC EQUATIONS**
  - NY.CCLS.Math.9-12.F-BF.4.a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
  - NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - NY.CCLS.Math.9-12.F-LE.4: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems For exponential models, express as a logarithm the solution to ab to the ct power = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
  - NY.CCLS.Math.9-12.A-REI.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

- NY.CCLS.Math.9-12.F-LE.4: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems For exponential models, express as a logarithm the solution to ab to the ct power = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Unit 8: Polynomials

- **DIVISION OF POLYNOMIALS**
  - NY.CCLS.Math.9-12.A-APR.6: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
  - NY.CCLS.Math.9-12.A-APR.2: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x a is p(a), so p(a) = 0 if and only if (x a) is a factor of p(x).
  - NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - NY.CCLS.Math.9-12.A-APR.6: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
  - NY.CCLS.Math.9-12.A-APR.6: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
  - NY.CCLS.Math.9-12.A-APR.2: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x a is p(a), so p(a) = 0 if and only if (x a) is a factor of p(x).
- **ARITHMETIC OPERATIONS ON FUNCTIONS**

- NY.CCLS.Math.9-12.F-BF.1.b: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations.
- NY.CCLS.Math.9-12.A-APR.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Unit 9: Graphs and Representations of Quadratic Functions

- **ANALYZING GRAPHS OF QUADRATIC FUNCTIONS**
  - NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - NY.CCLS.Math.9-12.F-IF.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
  - NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
  - NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - NY.CCLS.Math.9-12.F-IF.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
  - NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.

Experiment with cases and illustrate an explanation of the effects on the graph using technology.

- NY.CCLS.Math.9-12.A-REI.10: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- NY.CCLS.Math.9-12.A-APR.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- NY.CCLS.Math.9-12.F-IF.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

- **REPRESENTATIONS OF QUADRATIC FUNCTIONS**
  - NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - NY.CCLS.Math.9-12.A-SSE.3.a: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
  - NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
  - NY.CCLS.Math.9-12.A-REI.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of

completing the square to transform any quadratic equation in x into an equation of the form (x p) = q that has the same solutions. Derive the quadratic formula from this form.

- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- NY.CCLS.Math.9-12.F-IF.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.F-IF.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- NY.CCLS.Math.9-12.A-APR.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- NY.CCLS.Math.9-12.F-IF.7.c: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**

- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.F-IF.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.F-LE.1.a: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

Unit 10: Solving Quadratic Equations 1

- **SOLVING QUADRATIC EQUATIONS BY FACTORING**

- NY.CCLS.Math.9-12.A-SSE.3.a: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
- NY.CCLS.Math.9-12.A-APR.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- NY.CCLS.Math.9-12.F-IF.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- NY.CCLS.Math.9-12.A-APR.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f

is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

- NY.CCLS.Math.9-12.F-IF.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- NY.CCLS.Math.9-12.A-APR.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.

- **QUADRATIC FORMULA**
  - NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize

when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.

- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- NY.CCLS.Math.9-12.N-CN.7: The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- NY.CCLS.Math.9-12.A-REI.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x p) = q that has the same solutions. Derive the quadratic formula from this form.

- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.

Unit 11: Solving Quadratic Equations 2

- **COMPLETING THE SQUARE**
  - NY.CCLS.Math.9-12.A-SSE.3.b: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
  - NY.CCLS.Math.9-12.A-REI.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x p) = q that has the same solutions. Derive the quadratic formula from this form.
  - NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
  - NY.CCLS.Math.9-12.F-IF.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
  - NY.CCLS.Math.9-12.F-IF.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  - NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - NY.CCLS.Math.9-12.A-REI.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x p) = q that has the same solutions. Derive the quadratic formula from this form.
  - NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the

quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.

- NY.CCLS.Math.9-12.A-REI.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x p) = q that has the same solutions. Derive the quadratic formula from this form.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.

- **COMPLEX NUMBERS AND QUADRATIC FUNCTIONS**
  - NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
  - NY.CCLS.Math.9-12.N-CN.7: The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
  - NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
  - NY.CCLS.Math.9-12.N-CN.1: The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number i such that i = 1, and every complex number has the form a + bi with a and b real.
  - NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic

equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.

- NY.CCLS.Math.9-12.N-CN.1: The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number i such that i = 1, and every complex number has the form a + bi with a and b real.
- NY.CCLS.Math.9-12.N-CN.2: The Complex Number System Perform arithmetic operations with complex numbers. Use the relation i = 1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Unit 12: Factoring Polynomials

- **FACTORING SPECIAL CASES**
  - NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - NY.CCLS.Math.9-12.A-APR.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
  - NY.CCLS.Math.9-12.A-APR.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
  - NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
  - NY.CCLS.Math.9-12.F-IF.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.

- **FACTORING CUBIC POLYNOMIALS**

- NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- NY.CCLS.Math.9-12.A-APR.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- NY.CCLS.Math.9-12.A-APR.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Unit 13: Factoring Higher-Order Polynomials

- **FACTORING HIGHER-ORDER POLYNOMIALS**

- NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- NY.CCLS.Math.9-12.A-APR.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- NY.CCLS.Math.9-12.A-APR.6: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
- NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.

- NY.CCLS.Math.9-12.A-APR.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.

- **FACTOR THEOREM AND REMAINDER THEOREM**
  - NY.CCLS.Math.9-12.A-APR.2: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x a is p(a), so p(a) = 0 if and only if (x a) is a factor of p(x).
  - NY.CCLS.Math.9-12.A-APR.6: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
  - NY.CCLS.Math.9-12.F-IF.2: Interpreting Functions Understand the concept of a function and use function notation Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
  - NY.CCLS.Math.9-12.A-APR.2: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x a is p(a), so p(a) = 0 if and only if (x a) is a factor of p(x).

Unit 14: Polynomial Functions and Polynomial Identities

- **GRAPHS OF POLYNOMIAL FUNCTIONS**
  - NY.CCLS.Math.9-12.A-APR.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
  - NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
  - NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.

Experiment with cases and illustrate an explanation of the effects on the graph using technology.

- NY.CCLS.Math.9-12.F-IF.7.c: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

- **COMPLEX NUMBERS**
  - NY.CCLS.Math.9-12.N-CN.1: The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number i such that i = 1, and every complex number has the form a + bi with a and b real.
  - NY.CCLS.Math.9-12.N-CN.2: The Complex Number System Perform arithmetic operations with complex numbers. Use the relation i = 1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- **POLYNOMIAL IDENTITIES**
  - NY.CCLS.Math.9-12.A-APR.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
  - NY.CCLS.Math.9-12.A-REI.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x p) = q that has the same solutions. Derive the quadratic formula from this form.
  - NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
  - NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - NY.CCLS.Math.9-12.A-REI.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x p) = q that has the same solutions. Derive the quadratic formula from this form.
  - NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize

when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.

- NY.CCLS.Math.9-12.A-APR.5: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Know and apply the Binomial Theorem for the expansion of (x + y) in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascals Triangle.

Unit 15: Radical Equations and Functions

- **ANALYZING GRAPHS OF SQUARE ROOT FUNCTIONS**
  - NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
  - NY.CCLS.Math.9-12.G-CO.6: Congruence Understand congruence in terms of rigid motions Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
  - NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - NY.CCLS.Math.9-12.F-IF.7.b: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
  - NY.CCLS.Math.9-12.F-BF.4.a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.
  - NY.CCLS.Math.9-12.F-BF.4.c: Building Functions Build new functions from existing functions Find inverse functions. Read values of an inverse function from a graph or a table, given that the function has an inverse.
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
  - NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the

quantitative relationship it describes.

- **SOLVING SQUARE ROOT EQUATIONS**
  - NY.CCLS.Math.9-12.A-REI.2: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
  - NY.CCLS.Math.9-12.A-REI.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
  - NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Unit 16: Rational Expressions and Functions

- **OPERATIONS WITH RATIONAL EXPRESSIONS**
  - NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - NY.CCLS.Math.9-12.A-APR.7: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
  - NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - NY.CCLS.Math.9-12.A-APR.6: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
- **ANALYZING GRAPHS OF RATIONAL FUNCTIONS**
  - NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.

Experiment with cases and illustrate an explanation of the effects on the graph using technology.

- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

Unit 17: Solving Rational Equations

- **SOLVING RATIONAL EQUATIONS**
  - NY.CCLS.Math.9-12.A-REI.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
  - NY.CCLS.Math.9-12.A-REI.2: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- **MODELING SITUATIONS WITH RATIONAL FUNCTIONS**
  - NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between

quantities; graph equations on coordinate axes with labels and scales.

- NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- NY.CCLS.Math.9-12.A-REI.2: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Unit 18: Nonlinear Functions

- **LINEAR VERSUS NONLINEAR FUNCTIONS**
  - NY.CCLS.Math.9-12.F-IF.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
  - NY.CCLS.Math.9-12.F-LE.1.a: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
  - NY.CCLS.Math.9-12.F-LE.1.b: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
  - NY.CCLS.Math.9-12.F-LE.1.c: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
  - NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
  - NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities,

interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- NY.CCLS.Math.9-12.F-IF.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- NY.CCLS.Math.9-12.F-IF.7.b: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

- **PARABOLAS**
  - NY.CCLS.Math.9-12.G-GMD.4: Geometric Measurement and Dimension Visualize relationships between two-dimensional and three-dimensional objects Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
  - NY.CCLS.Math.9-12.G-GPE.2: Expressing Geometric Properties with Equations Translate between the geometric description and the equation for a conic section Derive the equation of a parabola given a focus and directrix.
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **INVERSE FUNCTIONS**
  - NY.CCLS.Math.9-12.F-BF.4.c: Building Functions Build new functions from existing functions Find inverse functions. Read values of an inverse function from a graph or a table, given that the function has an inverse.
  - NY.CCLS.Math.9-12.F-BF.4.a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.
  - NY.CCLS.Math.9-12.F-BF.4.a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.
  - NY.CCLS.Math.9-12.F-BF.4.d: Building Functions Build new functions from existing functions Find inverse functions. Produce an invertible function from a non-invertible function by restricting the domain.
  - NY.CCLS.Math.9-12.F-BF.4.a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.

Unit 19: Trigonometry

- **RADIANS AND THE UNIT CIRCLE**
  - NY.CCLS.Math.9-12.F-TF.1: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
  - NY.CCLS.Math.9-12.F-TF.2: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
  - NY.CCLS.Math.9-12.G-C.5: Circles Find arc lengths and areas of sectors of circles Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
  - NY.CCLS.Math.9-12.F-TF.3: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Use special triangles to determine geometrically the values of sine, cosine, tangent for /3, /4 and /6, and use the unit circle to express the values of sine, cosine, and tangent for x, +x, and 2x in terms of their values for x, where x is any real number.
  - NY.CCLS.Math.9-12.F-TF.4: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
  - NY.CCLS.Math.9-12.G-SRT.8: Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- **TRIGONOMETRIC FUNCTIONS**
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - NY.CCLS.Math.9-12.F-IF.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
  - NY.CCLS.Math.9-12.F-TF.2: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
  - NY.CCLS.Math.9-12.F-TF.5: Trigonometric Functions Model periodic phenomena with trigonometric functions Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

- NY.CCLS.Math.9-12.F-TF.8: Trigonometric Functions Prove and apply trigonometric identities Prove the Pythagorean identity sin() + cos() = 1 and use it to find sin(), cos(), or tan() given sin(), cos(), or tan() and the quadrant of the angle.
- NY.CCLS.Math.9-12.F-TF.8: Trigonometric Functions Prove and apply trigonometric identities Prove the Pythagorean identity sin() + cos() = 1 and use it to find sin(), cos(), or tan() given sin(), cos(), or tan() and the quadrant of the angle.

Unit 20: Parent Functions and Transformations

- **PARENT FUNCTIONS**
  - NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - NY.CCLS.Math.9-12.F-IF.7.b: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
  - NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
  - NY.CCLS.Math.9-12.F-IF.7.c: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
  - NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
  - NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- NY.CCLS.Math.9-12.F-IF.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

- **TRANSFORMATIONS OF PARENT FUNCTIONS**
  - NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
  - NY.CCLS.Math.9-12.G-CO.2: Congruence Experiment with transformations in the plane Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
  - NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
  - NY.CCLS.Math.9-12.G-CO.6: Congruence Understand congruence in terms of rigid motions Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

- NY.CCLS.Math.9-12.F-IF.7.c: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- NY.CCLS.Math.9-12.F-IF.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- NY.CCLS.Math.9-12.F-IF.7.b: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

- **MULTIPLE TRANSFORMATIONS OF PARENT FUNCTIONS**

- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.F-IF.7.b: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- NY.CCLS.Math.9-12.F-IF.7.c: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- NY.CCLS.Math.9-12.F-IF.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- NY.CCLS.Math.9-12.G-CO.2: Congruence Experiment with transformations in the plane Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- NY.CCLS.Math.9-12.G-CO.6: Congruence Understand congruence in terms of rigid motions Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- NY.CCLS.Math.9-12.G-CO.5: Congruence Experiment with transformations in the plane Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.

Experiment with cases and illustrate an explanation of the effects on the graph using technology.

- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Unit 21: Systems of Equations 1

- **SOLVING SYSTEMS OF LINEAR EQUATIONS: GRAPHING**
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - NY.CCLS.Math.9-12.A-REI.6: Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
  - NY.CCLS.Math.9-12.A-REI.10: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
  - NY.CCLS.Math.9-12.A-REI.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = (x) intersect are the solutions of the equation f(x) = (x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

- NY.CCLS.Math.9-12.A-REI.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = (x) intersect are the solutions of the equation f(x) = (x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- NY.CCLS.Math.9-12.A-REI.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = (x) intersect are the solutions of the equation f(x) = (x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

- **SOLVING SYSTEMS OF LINEAR EQUATIONS: SUBSTITUTION**
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
  - NY.CCLS.Math.9-12.A-REI.6: Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
  - NY.CCLS.Math.9-12.A-REI.5: Reasoning with Equations and Inequalities Solve systems of equations Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

Unit 22: Systems of Equations 2

- **SOLVING SYSTEMS OF LINEAR EQUATIONS: ELIMINATION**
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations

and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

- NY.CCLS.Math.9-12.A-REI.6: Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- NY.CCLS.Math.9-12.A-REI.5: Reasoning with Equations and Inequalities Solve systems of equations Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

- **SYSTEMS OF NONLINEAR EQUATIONS**
  - NY.CCLS.Math.9-12.A-REI.7: Reasoning with Equations and Inequalities Solve systems of equations Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
  - NY.CCLS.Math.9-12.A-REI.5: Reasoning with Equations and Inequalities Solve systems of equations Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
  - NY.CCLS.Math.9-12.A-REI.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = (x) intersect are the solutions of the equation f(x) = (x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
  - NY.CCLS.Math.9-12.A-REI.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = (x) intersect are the solutions of the equation f(x) = (x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
  - NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
  - NY.CCLS.Math.9-12.A-REI.6: Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
  - NY.CCLS.Math.9-12.A-REI.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = (x) intersect are the solutions of the equation f(x) =

(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

- NY.CCLS.Math.9-12.A-REI.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = (x) intersect are the solutions of the equation f(x) = (x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- NY.CCLS.Math.9-12.A-REI.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = (x) intersect are the solutions of the equation f(x) = (x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Unit 23: Statistical Design and Analysis 1

- **ANALYZING STATISTICAL SAMPLES**
  - NY.CCLS.Math.9-12.S-IC.1: Making Inferences and Justifying Conclusions Understand and evaluate random processes underlying statistical experiments Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
  - NY.CCLS.Math.9-12.S-IC.4: Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
  - NY.CCLS.Math.9-12.S-IC.2: Making Inferences and Justifying Conclusions Understand and evaluate random processes underlying statistical experiments Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
- **EXPERIMENTAL AND OBSERVATIONAL DESIGN**
  - NY.CCLS.Math.9-12.S-IC.3: Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Unit 24: Statistical Design and Analysis 2

- **CONCLUSIONS IN DATA**
  - NY.CCLS.Math.9-12.S-IC.5: Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

- NY.CCLS.Math.9-12.S-IC.6: Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Evaluate reports based on data.

- **SCATTERPLOTS AND MODELING**
  - NY.CCLS.Math.9-12.S-ID.6.a: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
  - NY.CCLS.Math.9-12.S-ID.6.b: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Informally assess the fit of a function by plotting and analyzing residuals.
  - NY.CCLS.Math.9-12.S-ID.7: Interpreting Categorical and Quantitative Data Interpret linear models Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
  - NY.CCLS.Math.9-12.S-ID.8: Interpreting Categorical and Quantitative Data Interpret linear models Compute (using technology) and interpret the correlation coefficient of a linear fit.
  - NY.CCLS.Math.9-12.S-ID.6.c: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a linear function for a scatter plot that suggests a linear association.
  - NY.CCLS.Math.9-12.F-LE.1.a: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
  - NY.CCLS.Math.9-12.F-LE.1.c: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Unit 25: Probability

- **INTRODUCTION TO PROBABILITY**
  - NY.CCLS.Math.9-12.S-CP.2: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand that two events and are independent if the probability of and occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
  - NY.CCLS.Math.9-12.S-CP.8: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply

the general Multiplication Rule in a uniform probability model, (and) = ()() = ()(), and interpret the answer in terms of the model.

- NY.CCLS.Math.9-12.S-CP.1: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
- NY.CCLS.Math.9-12.S-CP.2: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand that two events and are independent if the probability of and occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- NY.CCLS.Math.9-12.S-CP.5: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
- NY.CCLS.Math.9-12.S-CP.8: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the general Multiplication Rule in a uniform probability model, (and) = ()() = ()(), and interpret the answer in terms of the model.
- NY.CCLS.Math.9-12.S-CP.7: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the Addition Rule, (or) = () + () (and), and interpret the answer in terms of the model.

- **CONDITIONAL PROBABILITY**
  - NY.CCLS.Math.9-12.S-CP.3: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand the conditional probability of given as (and)/(), and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
  - NY.CCLS.Math.9-12.S-CP.5: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
  - NY.CCLS.Math.9-12.S-CP.6: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.
  - NY.CCLS.Math.9-12.S-CP.2: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand that two events and are independent if the probability of and occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

- NY.CCLS.Math.9-12.S-CP.3: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand the conditional probability of given as (and)/(), and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
- NY.CCLS.Math.9-12.S-CP.6: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.
- NY.CCLS.Math.9-12.S-ID.5: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
- NY.CCLS.Math.9-12.S-CP.4: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
- NY.CCLS.Math.9-12.S-CP.1: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
- NY.CCLS.Math.9-12.S-CP.3: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand the conditional probability of given as (and)/(), and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
- NY.CCLS.Math.9-12.S-CP.6: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.
- NY.CCLS.Math.9-12.S-CP.3: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand the conditional probability of given as (and)/(), and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
- NY.CCLS.Math.9-12.S-CP.6: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find

the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.

- NY.CCLS.Math.9-12.S-CP.3: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand the conditional probability of given as (and)/(), and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
- NY.CCLS.Math.9-12.S-CP.6: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.

Unit 26: Advanced Probability Concepts

- **GEOMETRIC PROBABILITIES**
  - NY.CCLS.Math.9-12.G-MG.3: Modeling with Geometry Apply geometric concepts in modeling situations Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
  - NY.CCLS.Math.9-12.S-MD.7: Using Probability to Make Decisions Use probability to evaluate outcomes of decisions Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
  - NY.CCLS.Math.9-12.S-CP.1: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
  - NY.CCLS.Math.9-12.S-CP.7: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the Addition Rule, (or) = () + () (and), and interpret the answer in terms of the model.
  - NY.CCLS.Math.9-12.S-CP.1: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
  - NY.CCLS.Math.9-12.S-CP.7: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the Addition Rule, (or) = () + () (and), and interpret the answer in terms of the model.
- **NORMAL DISTRIBUTION**
  - NY.CCLS.Math.9-12.S-ID.3: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

- NY.CCLS.Math.9-12.S-ID.4: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
- NY.CCLS.Math.9-12.S-IC.4: Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.