

New Jersey Tutorials are designed specifically for the New Jersey Core Curriculum Content Standards to prepare students for the PARCC assessments, the New Jersey Biology Competency Test (NJBCT).

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

Unit 1: Introduction to Algebraic Concepts

• MONITORING PRECISION AND ACCURACY

- N-Q.A.1: Quantities Reason quantitatively and use units to solve problems. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- N-Q.A.2: Quantities Reason quantitatively and use units to solve problems. Define appropriate quantities for the purpose of descriptive modeling.
- N-Q.A.3: Quantities Reason quantitatively and use units to solve problems. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

• LAWS OF EXPONENTS

- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x + y)(x - y)$, thus recognizing it as a difference of squares that can be factored as $(x + y)(x - y)$.
- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x + y)(x - y)$, thus recognizing it as a difference of squares that can be factored as $(x + y)(x - y)$.

- A-APR.D.6: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- A-REI.A.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- N-RN.A.1: The Real Number System Extend the properties of exponents to rational exponents. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
- N-RN.A.2: The Real Number System Extend the properties of exponents to rational exponents. Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- **AXIOMS OF EQUALITY**
 - A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x + y)(x - y)$, thus recognizing it as a difference of squares that can be factored as $(x + y)(x - y)$.
 - A-REI.A.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Unit 2: Sequences

- **SEQUENCES**
 - F-IF.A.3: Interpreting Functions Understand the concept of a function and use function notation Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
 - F-BF.A.1a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
 - F-BF.A.2: Building Functions Build a function that models a relationship between two quantities Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
 - F-LE.A.2: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Construct linear and exponential functions, including arithmetic and

geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- **ARITHMETIC AND GEOMETRIC SEQUENCES**

- F-BF.A.2: Building Functions Build a function that models a relationship between two quantities Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- F-IF.A.3: Interpreting Functions Understand the concept of a function and use function notation Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- F-BF.A.1a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- F-LE.A.2: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- **SUMS OF GEOMETRIC SEQUENCES**

- A-SSE.B.4: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

Unit 3: Linear Functions and Equations

- **SLOPE**

- F-IF.B.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- F-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- G-GPE.B.5: Expressing Geometric Properties with Equations Use coordinates to prove simple geometric theorems algebraically Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

- **GRAPHING AND MANIPULATING $Y = MX + B$**

- A-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- F-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- F-IF.B.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.C.7a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- F-LE.A.2: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- S-ID.C.7: Interpreting Categorical and Quantitative Data Interpret linear models Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- F-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-LE.A.1b: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- F-LE.B.5: Linear and Exponential Models Interpret expressions for functions in terms of the situation they model Interpret the parameters in a linear or exponential function in terms of a context.

Unit 4: Solving Two-Variable Systems of Equations

- **SOLVING SYSTEMS OF LINEAR EQUATIONS: GRAPHING**

- A-CED.A.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

- A-REI.C.6: Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- A-REI.D.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- A-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **SOLVING SYSTEMS OF LINEAR EQUATIONS: SUBSTITUTION**
 - A-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
 - A-REI.C.6: Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
 - A-CED.A.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
 - A-REI.C.5: Reasoning with Equations and Inequalities Solve systems of equations Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- **SOLVING SYSTEMS OF LINEAR EQUATIONS: ELIMINATION**
 - A-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
 - A-REI.C.6: Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
 - A-CED.A.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
 - A-REI.C.5: Reasoning with Equations and Inequalities Solve systems of equations Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

Unit 5: Exponential Functions and Equations**• EXPONENTIAL FUNCTIONS**

- A-SSE.A.1a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.A.1b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- F-IF.C.8b: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
- F-LE.A.1a: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- F-IF.B.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-LE.A.2: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- F-LE.A.3: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- F-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- F-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.B.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

- A-SSE.B.3c: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.
- A-CED.A.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
- F-BF.A.1a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- F-LE.A.1c: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- F-LE.B.5: Linear and Exponential Models Interpret expressions for functions in terms of the situation they model Interpret the parameters in a linear or exponential function in terms of a context.
- **EXPONENTIAL GROWTH AND DECAY**
 - A-SSE.A.1a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
 - A-SSE.A.1b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
 - F-IF.C.8b: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
 - F-LE.A.1a: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
 - F-LE.A.1c: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
 - F-LE.A.2: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- F-LE.B.5: Linear and Exponential Models Interpret expressions for functions in terms of the situation they model Interpret the parameters in a linear or exponential function in terms of a context.
- A-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- F-LE.A.1b: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- F-LE.A.3: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- **SOLVING EXPONENTIAL EQUATIONS**
 - A-SSE.B.3c: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.
 - F-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
 - F-IF.C.8b: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
 - F-BF.B.4a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
 - F-LE.A.4: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^ct = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.
 - F-LE.A.2: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- F-IF.C.7e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Unit 6: Relating Exponential and Logarithmic Functions

• INVERSE FUNCTIONS

- F-BF.B.4a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
- F-BF.B.4c: Building Functions Build new functions from existing functions Find inverse functions. Read values of an inverse function from a graph or a table, given that the function has an inverse.
- F-BF.B.4d: Building Functions Build new functions from existing functions Find inverse functions. Produce an invertible function from a non-invertible function by restricting the domain.

• LOGARITHMIC FUNCTIONS

- F-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.C.7e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- F-BF.B.4a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
- F-BF.B.4c: Building Functions Build new functions from existing functions Find inverse functions. Read values of an inverse function from a graph or a table, given that the function has an inverse.
- F-LE.A.4: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $abct = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.
- F-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and

x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.

Unit 7: Logarithmic Expressions and Equations

• EVALUATING LOGARITHMIC EXPRESSIONS

- A-SSE.A.1a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.A.1b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- F-LE.A.4: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^ct = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.

• SOLVING LOGARITHMIC EQUATIONS

- F-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- F-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-BF.B.4a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
- F-LE.A.4: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^ct = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.
- A-REI.A.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Unit 8: Operations with Polynomials

• ADDITION AND SUBTRACTION OF POLYNOMIALS

- A-APR.A.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x - y)(x + y)$, thus recognizing it as a difference of squares that can be factored as $(x - y)(x + y)$.

• MULTIPLICATION OF POLYNOMIALS

- A-APR.A.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x - y)(x + y)$, thus recognizing it as a difference of squares that can be factored as $(x - y)(x + y)$.

• DIVISION OF POLYNOMIALS

- A-SSE.A.1a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.A.1b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x - y)(x + y)$, thus recognizing it as a difference of squares that can be factored as $(x - y)(x + y)$.
- A-APR.D.6: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

Unit 9: Graphs and Representations of Quadratic Functions

• ANALYZING GRAPHS OF QUADRATIC FUNCTIONS

- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x - y)(x + y)$, thus recognizing it as a difference of squares that can be factored as $(x - y)(x + y)$.
- F-IF.C.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

- F-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- F-IF.B.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.C.7a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- A-APR.B.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- F-IF.C.8a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- **REPRESENTATIONS OF QUADRATIC FUNCTIONS**
 - A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x + y)(x - y)$, thus recognizing it as a difference of squares that can be factored as $(x + y)(x - y)$.

- A-REI.B.4a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p) = q$ that has the same solutions. Derive the quadratic formula from this form.
- A-SSE.B.3a: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
- A-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- F-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- F-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.C.8a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- F-IF.C.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- F-BF.A.1a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **PARABOLAS**
 - G-GMD.B.4: Geometric Measurement and Dimension Visualize relationships between two-dimensional and three-dimensional objects Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
 - G-GPE.A.2: Expressing Geometric Properties with Equations Translate between the geometric description and the equation for a conic section Derive the equation of a parabola given a focus and directrix.

- A-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Unit 10: Working with Equations and Functions

• SYSTEMS OF NONLINEAR EQUATIONS

- A-REI.C.6: Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- A-REI.C.7: Reasoning with Equations and Inequalities Solve systems of equations Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
- A-REI.C.5: Reasoning with Equations and Inequalities Solve systems of equations Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- A-REI.D.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- A-CED.A.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
- F-LE.A.2: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

• ARITHMETIC OPERATIONS ON FUNCTIONS

- F-BF.A.1b: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations.

Unit 11: Comparing Functions

• MULTIPLE REPRESENTATIONS OF FUNCTIONS

- A-CED.A.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- F-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key

features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- F-IF.C.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- F-LE.A.2: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- F-LE.A.1a: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **LINEAR VERSUS NONLINEAR FUNCTIONS**
 - F-IF.B.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
 - F-LE.A.1a: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
 - F-LE.A.2: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
 - F-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
 - F-IF.C.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
 - F-LE.A.1b: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
 - F-LE.A.1c: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Distinguish between situations that can be modeled with linear

functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Unit 12: Solving Quadratic Equations

• SOLVING QUADRATIC EQUATIONS BY FACTORING

- A-SSE.B.3a: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- F-IF.C.8a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- A-APR.B.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- A-APR.C.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .

- F-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- F-IF.C.7a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- F-BF.A.1a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **COMPLETING THE SQUARE**
 - A-SSE.B.3b: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
 - A-REI.B.4a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p) = q$ that has the same solutions. Derive the quadratic formula from this form.
 - A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
 - F-IF.C.8a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
 - A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x + y)(x - y)$, thus recognizing it as a difference of squares that can be factored as $(x + y)(x - y)$.
 - A-REI.B.4a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p) = q$ that has the same solutions. Derive the quadratic formula from this form.

- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- A-REI.B.4a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p) = q$ that has the same solutions. Derive the quadratic formula from this form.
- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- F-IF.C.7a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Unit 13: Quadratic Formula and Complex Numbers

• QUADRATIC FORMULA

- A-SSE.A.1a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.A.1b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- A-REI.B.4a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p) = q$ that has the same solutions. Derive the quadratic formula from this form.
- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .

- N-CN.C.7: The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- F-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- F-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-BF.A.1a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- A-CED.A.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- **COMPLEX NUMBERS AND QUADRATIC FUNCTIONS**
 - A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
 - N-CN.C.7: The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
 - A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and

factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .

- N-CN.A.1: The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- N-CN.A.1: The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- N-CN.A.2: The Complex Number System Perform arithmetic operations with complex numbers. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Unit 14: Factoring Special Cases and Cubic Polynomials

• FACTORING SPECIAL CASES

- A-SSE.A.1a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.A.1b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x + y)(x - y)$, thus recognizing it as a difference of squares that can be factored as $(x + y)(x - y)$.
- A-APR.C.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x + y)(x - y)$, thus recognizing it as a difference of squares that can be factored as $(x + y)(x - y)$.
- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x + y)(x - y)$, thus recognizing it as a difference of squares that can be factored as $(x + y)(x - y)$.
- A-APR.B.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable

factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x - y)(x + y)$, thus recognizing it as a difference of squares that can be factored as $(x - y)(x + y)$.

• **FACTORING CUBIC POLYNOMIALS**

- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x - y)(x + y)$, thus recognizing it as a difference of squares that can be factored as $(x - y)(x + y)$.
- A-APR.B.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- A-APR.C.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x - y)(x + y)$, thus recognizing it as a difference of squares that can be factored as $(x - y)(x + y)$.

Unit 15: Factoring Polynomials and the Factor Theorem

• **FACTORING HIGHER-ORDER POLYNOMIALS**

- A-SSE.A.1a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.A.1b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- A-APR.B.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x - y)(x + y)$, thus recognizing it as a difference of squares that can be factored as $(x - y)(x + y)$.
- A-APR.C.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.

• **FACTOR THEOREM AND REMAINDER THEOREM**

- A-APR.B.2: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Know and apply the Remainder Theorem: For a

polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

- F-IF.A.2: Interpreting Functions Understand the concept of a function and use function notation Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- A-APR.B.2: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

Unit 16: Polynomial Identities and Complex Numbers

• POLYNOMIAL IDENTITIES

- A-APR.C.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- A-REI.B.4a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x + y)(x - y)$, thus recognizing it as a difference of squares that can be factored as $(x + y)(x - y)$.
- A-REI.B.4a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x + y)(x - y)$, thus recognizing it as a difference of squares that can be factored as $(x + y)(x - y)$.
- A-APR.C.5: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers

of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

- **COMPLEX NUMBERS**

- N-CN.A.1: The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- N-CN.A.2: The Complex Number System Perform arithmetic operations with complex numbers. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

- **POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS**

- A-SSE.A.1a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.A.1b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x + y)(x - y)$, thus recognizing it as a difference of squares that can be factored as $(x + y)(x - y)$.
- A-APR.C.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- N-CN.C.8: The Complex Number System Use complex numbers in polynomial identities and equations. Extend polynomial identities to the complex numbers.
- A-REI.B.4b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- N-CN.A.1: The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- N-CN.C.7: The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
- N-CN.C.9: The Complex Number System Use complex numbers in polynomial identities and equations. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Unit 17: Radical Functions and Equations

- **ANALYZING GRAPHS OF SQUARE ROOT FUNCTIONS**

- F-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- G-CO.B.6: Congruence Understand congruence in terms of rigid motions Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- F-BF.B.4a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
- F-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.C.7b: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- F-BF.B.4c: Building Functions Build new functions from existing functions Find inverse functions. Read values of an inverse function from a graph or a table, given that the function has an inverse.
- F-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- **SOLVING SQUARE ROOT EQUATIONS**
 - A-REI.A.2: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
 - A-REI.A.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
 - F-BF.A.1a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Unit 18: Rational Expressions and Equations

• OPERATIONS WITH RATIONAL EXPRESSIONS

- A-SSE.A.1a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.A.1b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- A-APR.D.7: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
- A-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x + y)(x - y)$, thus recognizing it as a difference of squares that can be factored as $(x + y)(x - y)$.
- A-APR.D.6: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

• SOLVING RATIONAL EQUATIONS

- A-REI.A.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- F-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- A-CED.A.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
- A-REI.A.2: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Unit 19: Rational Functions

• ANALYZING GRAPHS OF RATIONAL FUNCTIONS

- F-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- F-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- F-IF.B.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **MODELING SITUATIONS WITH RATIONAL FUNCTIONS**
 - A-SSE.A.1a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
 - A-SSE.A.1b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
 - A-REI.A.2: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
 - F-BF.A.1a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Unit 20: Trigonometry and Trigonometric Functions

- **RADIANS AND THE UNIT CIRCLE**
 - F-TF.A.1: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
 - F-TF.A.2: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

- G-C.B.5: Circles Find arc lengths and areas of sectors of circles Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
- F-TF.A.3: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x , $+x$, and $2x$ in terms of their values for x , where x is any real number.
- F-TF.A.4: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- G-SRT.C.8: Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

• TRIGONOMETRIC FUNCTIONS

- F-IF.C.7e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- F-TF.B.5: Trigonometric Functions Model periodic phenomena with trigonometric functions Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- F-TF.C.8: Trigonometric Functions Prove and apply trigonometric identities Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
- F-TF.A.2: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- F-TF.C.8: Trigonometric Functions Prove and apply trigonometric identities Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

Unit 21: Parent Functions and Transformations

• PARENT FUNCTIONS

- F-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- F-IF.C.7b: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- F-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- F-IF.B.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-LE.A.2: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- F-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- F-IF.C.7c: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- F-IF.C.7e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- **TRANSFORMATIONS OF PARENT FUNCTIONS**
 - F-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
 - G-CO.A.2: Congruence Experiment with transformations in the plane Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

- G-CO.B.6: Congruence Understand congruence in terms of rigid motions Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- F-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- F-IF.C.7e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- **MULTIPLE TRANSFORMATIONS OF PARENT FUNCTIONS**
 - F-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
 - G-CO.A.2: Congruence Experiment with transformations in the plane Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
 - G-CO.B.6: Congruence Understand congruence in terms of rigid motions Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
 - G-CO.A.5: Congruence Experiment with transformations in the plane Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
 - F-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
 - F-IF.C.7c: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

- F-IF.C.7e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Unit 22: Statistical Design and Analysis

• ANALYZING STATISTICAL SAMPLES

- S-IC.A.1: Making Inferences and Justifying Conclusions Understand and evaluate random processes underlying statistical experiments Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
- S-IC.A.2: Making Inferences and Justifying Conclusions Understand and evaluate random processes underlying statistical experiments Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
- S-IC.B.4: Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

• EXPERIMENTAL AND OBSERVATIONAL DESIGN

- S-IC.B.3: Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

• CONCLUSIONS IN DATA

- S-IC.B.5: Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
- S-IC.B.6: Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Evaluate reports based on data.

Unit 23: Statistics

• SCATTERPLOTS AND MODELING

- S-ID.B.6a: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data.
- S-ID.B.6b: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative

variables on a scatter plot, and describe how the variables are related. Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.

- S-ID.B.6c: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a linear function for a scatter plot that suggests a linear association.
- S-ID.C.8: Interpreting Categorical and Quantitative Data Interpret linear models Compute (using technology) and interpret the correlation coefficient of a linear fit.
- F-LE.A.1a: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- F-LE.A.1c: Linear and Exponential Models Construct and compare linear and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- S-ID.C.7: Interpreting Categorical and Quantitative Data Interpret linear models Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

• **NORMAL DISTRIBUTION**

- S-ID.A.3: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
- S-ID.A.4: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
- S-IC.B.4: Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

Unit 24: Probability

• **INTRODUCTION TO PROBABILITY**

- S-CP.A.2: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

- S-CP.B.8: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.
 - S-CP.A.5: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
 - S-CP.B.8: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.
 - S-CP.B.7: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
- **CONDITIONAL PROBABILITY**
- S-CP.A.3: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
 - S-CP.A.5: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
 - S-CP.B.6: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of A given B as the fraction of Bs outcomes that also belong to A, and interpret the answer in terms of the model.
 - S-CP.A.2: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
 - S-CP.B.6: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of A given B as the fraction of Bs outcomes that also belong to A, and interpret the answer in terms of the model.
 - S-ID.B.5: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the

data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

- S-CP.A.4: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
- S-CP.A.1: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
- S-CP.B.6: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of A given B as the fraction of Bs outcomes that also belong to A, and interpret the answer in terms of the model.
- S-CP.B.6: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of A given B as the fraction of Bs outcomes that also belong to A, and interpret the answer in terms of the model.
- S-CP.B.6: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Find the conditional probability of A given B as the fraction of Bs outcomes that also belong to A, and interpret the answer in terms of the model.
- **GEOMETRIC PROBABILITIES**
 - G-MG.A.3: Modeling with Geometry Apply geometric concepts in modeling situations Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
 - S-MD.B.7: Using Probability to Make Decisions Use probability to evaluate outcomes of decisions Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
 - S-CP.A.1: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
 - S-CP.B.7: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
 - S-CP.A.1: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Describe events as subsets of a sample

space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).

- S-CP.B.7: Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.