

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

This Tutorial is built to state standards.

Unit 1: Expressions, Equations, and Inequalities

• FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS

- CCSS.Math.Content.HSF-BF.A.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- CCSS.Math.Content.HSA-SSE.A.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- CCSS.Math.Content.HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.

• LITERAL EQUATIONS

- CCSS.Math.Content.HSA-CED.A.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
- CCSS.Math.Content.HSA-REI.B.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- CCSS.Math.Content.HSA-CED.A.4: Creating Equations Create equations that describe numbers or relationships Rearrange formulas to highlight a quantity of interest, using the same

reasoning as in solving equations.

- **SUMS OF GEOMETRIC SEQUENCES**

- CCSS.Math.Content.HSA-SSE.B.4: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

Unit 2: Logarithmic Expressions and Functions

- **LOGARITHMIC FUNCTIONS**

- CCSS.Math.Content.HSF-BF.B.4.c: Building Functions Build new functions from existing functions Find inverse functions. Read values of an inverse function from a graph or a table, given that the function has an inverse.
- CCSS.Math.Content.HSF-BF.B.5: Building Functions Build new functions from existing functions Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
- CCSS.Math.Content.HSF-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- CCSS.Math.Content.HSF-LE.A.4: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems For exponential models, express as a logarithm the solution to $ab^t = c$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.
- CCSS.Math.Content.HSF-IF.C.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

- **EVALUATING LOGARITHMIC EXPRESSIONS**

- CCSS.Math.Content.HSA-SSE.A.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- CCSS.Math.Content.HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.

Unit 3: Exponential and Logarithmic Equations

- **SOLVING EXPONENTIAL EQUATIONS**

- CCSS.Math.Content.HSA-SSE.B.3.c: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.

- CCSS.Math.Content.HSF-IF.C.8.b: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
- CCSS.Math.Content.HSF-IF.C.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

- **SOLVING LOGARITHMIC EQUATIONS**

- CCSS.Math.Content.HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- CCSS.Math.Content.HSF-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Unit 4: Arithmetic with Polynomials

- **POLYNOMIAL BASICS**

- CCSS.Math.Content.HSA-SSE.A.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- CCSS.Math.Content.HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.

- **ADDITION AND SUBTRACTION OF POLYNOMIALS**

- CCSS.Math.Content.HSA-APR.A.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

- **MULTIPLICATION OF POLYNOMIALS**

- CCSS.Math.Content.HSA-APR.A.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

- **DIVISION OF POLYNOMIALS**

- CCSS.Math.Content.HSA-APR.B.2: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
- CCSS.Math.Content.HSA-APR.D.6: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- CCSS.Math.Content.HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- CCSS.Math.Content.HSA-APR.B.2: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
- CCSS.Math.Content.HSA-APR.D.6: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

Unit 5: Factoring and Graphing with Polynomials

• GRAPHS OF POLYNOMIAL FUNCTIONS

- CCSS.Math.Content.HSA-APR.B.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- CCSS.Math.Content.HSF-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- CCSS.Math.Content.HSF-IF.C.7.c: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

• FACTORING CUBIC POLYNOMIALS

- CCSS.Math.Content.HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.

- CCSS.Math.Content.HSA-APR.C.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- **FACTORING HIGHER-ORDER POLYNOMIALS**
 - CCSS.Math.Content.HSA-SSE.A.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
 - CCSS.Math.Content.HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
 - CCSS.Math.Content.HSA-APR.C.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- **FACTOR THEOREM AND REMAINDER THEOREM**
 - CCSS.Math.Content.HSF-IF.A.2: Interpreting Functions Understand the concept of a function and use function notation Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
 - CCSS.Math.Content.HSA-APR.B.2: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

Unit 6: Polynomial Identities

- **POLYNOMIAL IDENTITIES**
 - CCSS.Math.Content.HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
 - CCSS.Math.Content.HSA-APR.C.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
 - CCSS.Math.Content.HSA-REI.B.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p) = q$ that has the same solutions. Derive the quadratic formula from this form.
 - CCSS.Math.Content.HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .

- CCSS.Math.Content.HSA-APR.C.5: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascals Triangle.
- **POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS**
- CCSS.Math.Content.HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- CCSS.Math.Content.HSA-APR.C.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- CCSS.Math.Content.HSN-CN.C.8: The Complex Number System Use complex numbers in polynomial identities and equations. Extend polynomial identities to the complex numbers.
- CCSS.Math.Content.HSA-REI.B.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b .
- CCSS.Math.Content.HSN-CN.C.7: The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
- CCSS.Math.Content.HSN-CN.C.9: The Complex Number System Use complex numbers in polynomial identities and equations. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Unit 7: Radical Expressions, Equations, and Functions

- **ANALYZING GRAPHS OF SQUARE ROOT FUNCTIONS**
- CCSS.Math.Content.HSF-IF.C.7.b: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- CCSS.Math.Content.HSF-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- CCSS.Math.Content.HSG-CO.B.6: Congruence Understand congruence in terms of rigid motions Use geometric descriptions of rigid motions to transform figures and to predict the effect of a

given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

- CCSS.Math.Content.HSF-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- CCSS.Math.Content.HSF-BF.B.4.c: Building Functions Build new functions from existing functions Find inverse functions. Read values of an inverse function from a graph or a table, given that the function has an inverse.

• SOLVING SQUARE ROOT EQUATIONS

- CCSS.Math.Content.HSA-REI.A.2: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Unit 8: Rational Expressions, Equations, and Functions

• OPERATIONS WITH RATIONAL EXPRESSIONS

- CCSS.Math.Content.HSA-APR.D.7: Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
- CCSS.Math.Content.HSA-SSE.A.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.

• ANALYZING GRAPHS OF RATIONAL FUNCTIONS

- CCSS.Math.Content.HSF-IF.C.7.d: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
- CCSS.Math.Content.HSF-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- CCSS.Math.Content.HSF-IF.B.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

• SOLVING RATIONAL EQUATIONS

- CCSS.Math.Content.HSA-REI.A.2: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- CCSS.Math.Content.HSF-IF.C.7.d: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

- **MODELING SITUATIONS WITH RATIONAL FUNCTIONS**

Unit 9: Trigonometry

- **RADIANS AND THE UNIT CIRCLE**

- CCSS.Math.Content.HSF-TF.A.1: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- CCSS.Math.Content.HSF-TF.A.2: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- CCSS.Math.Content.HSG-C.B.5: Circles Find arc lengths and areas of sectors of circles Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
- CCSS.Math.Content.HSF-TF.A.3: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x , $+\pi$, and $2x$ in terms of their values for x , where x is any real number.
- CCSS.Math.Content.HSF-TF.A.4: Trigonometric Functions Extend the domain of trigonometric functions using the unit circle Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

- **TRIGONOMETRIC FUNCTIONS**

- CCSS.Math.Content.HSF-IF.C.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- CCSS.Math.Content.HSF-TF.B.5: Trigonometric Functions Model periodic phenomena with trigonometric functions Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

- CCSS.Math.Content.HSF-TF.C.8: Trigonometric Functions Prove and apply trigonometric identities Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

- **LAWS OF SINE AND COSINE**

- CCSS.Math.Content.HSG-SRT.D.9: Similarity, Right Triangles, and Trigonometry Apply trigonometry to general triangles Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- CCSS.Math.Content.HSG-SRT.D.10: Similarity, Right Triangles, and Trigonometry Apply trigonometry to general triangles Prove the Laws of Sines and Cosines and use them to solve problems.
- CCSS.Math.Content.HSG-SRT.D.11: Similarity, Right Triangles, and Trigonometry Apply trigonometry to general triangles Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
- CCSS.Math.Content.HSG-SRT.C.8: Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Unit 10: Transforming Functions

- **DOMAIN AND RANGE**

- CCSS.Math.Content.HSF-IF.B.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

- **PARENT FUNCTIONS**

- CCSS.Math.Content.HSF-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- CCSS.Math.Content.HSF-IF.B.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- CCSS.Math.Content.HSF-IF.C.7.b: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- CCSS.Math.Content.HSF-IF.C.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential

and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

• TRANSFORMATIONS OF PARENT FUNCTIONS

- CCSS.Math.Content.HSF-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- CCSS.Math.Content.HSF-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- CCSS.Math.Content.HSG-CO.B.6: Congruence Understand congruence in terms of rigid motions Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

• MULTIPLE TRANSFORMATIONS OF PARENT FUNCTIONS

- CCSS.Math.Content.HSF-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- CCSS.Math.Content.HSG-CO.B.6: Congruence Understand congruence in terms of rigid motions Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- CCSS.Math.Content.HSF-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- CCSS.Math.Content.HSF-BF.B.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- CCSS.Math.Content.HSG-CO.A.2: Congruence Experiment with transformations in the plane Represent transformations in the plane using, e.g., transparencies and geometry software;

describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

Unit 11: Working with Functions

• ARITHMETIC OPERATIONS ON FUNCTIONS

- CCSS.Math.Content.HSF-BF.A.1.b: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations.
- CCSS.Math.Content.HSA-APR.A.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

• REPRESENTATIONS OF QUADRATIC FUNCTIONS

- CCSS.Math.Content.HSA-REI.B.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p) = q$ that has the same solutions. Derive the quadratic formula from this form.
- CCSS.Math.Content.HSF-IF.C.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- CCSS.Math.Content.HSF-IF.A.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- CCSS.Math.Content.HSA-SSE.B.3.a: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
- CCSS.Math.Content.HSA-APR.B.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- CCSS.Math.Content.HSF-IF.B.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- CCSS.Math.Content.HSF-IF.C.7.c: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**

- CCSS.Math.Content.HSF-IF.C.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- CCSS.Math.Content.HSF-LE.A.1.a: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

- **INVERSE FUNCTIONS**

- CCSS.Math.Content.HSF-BF.B.4.a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
- CCSS.Math.Content.HSF-BF.B.4.c: Building Functions Build new functions from existing functions Find inverse functions. Read values of an inverse function from a graph or a table, given that the function has an inverse.
- CCSS.Math.Content.HSF-BF.B.4.a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
- CCSS.Math.Content.HSF-BF.B.4.b: Building Functions Build new functions from existing functions Find inverse functions. Verify by composition that one function is the inverse of another.
- CCSS.Math.Content.HSF-BF.B.4.a: Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
- CCSS.Math.Content.HSF-BF.B.4.d: Building Functions Build new functions from existing functions Find inverse functions. Produce an invertible function from a non-invertible function by restricting the domain.

Unit 12: Systems of Equations

- **SOLVING THREE-VARIABLE SYSTEMS OF LINEAR EQUATIONS**

- CCSS.Math.Content.HSA-CED.A.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of

equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

- CCSS.Math.Content.HSA-REI.C.5: Reasoning with Equations and Inequalities Solve systems of equations Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

• SYSTEMS OF NONLINEAR EQUATIONS

- CCSS.Math.Content.HSA-REI.C.6: Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- CCSS.Math.Content.HSA-REI.C.7: Reasoning with Equations and Inequalities Solve systems of equations Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
- CCSS.Math.Content.HSA-REI.D.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = (x)$ intersect are the solutions of the equation $f(x) = (x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- CCSS.Math.Content.HSA-REI.C.5: Reasoning with Equations and Inequalities Solve systems of equations Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- CCSS.Math.Content.HSA-REI.D.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = (x)$ intersect are the solutions of the equation $f(x) = (x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- CCSS.Math.Content.HSA-REI.D.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = (x)$ intersect are the solutions of the equation $f(x) = (x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
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tables of values, or find successive approximations. Include cases where $f(x)$ and/or (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

- CCSS.Math.Content.HSA-REI.D.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = (x)$ intersect are the solutions of the equation $f(x) = (x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Unit 13: Three-Dimensional Geometry I

• RELATING TWO-DIMENSIONAL FIGURES TO THREE-DIMENSIONAL SOLIDS

- CCSS.Math.Content.HSG-GMD.B.4: Geometric Measurement and Dimension Visualize relationships between two-dimensional and three-dimensional objects Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

• SURFACE AREA AND VOLUME OF SPHERES

- CCSS.Math.Content.HSG-GMD.A.2: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Give an informal argument using Cavalieris principle for the formulas for the volume of a sphere and other solid figures.
- CCSS.Math.Content.HSG-GMD.B.4: Geometric Measurement and Dimension Visualize relationships between two-dimensional and three-dimensional objects Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
- CCSS.Math.Content.HSG-MG.A.1: Modeling with Geometry Apply geometric concepts in modeling situations Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- CCSS.Math.Content.HSG-GMD.A.3: Geometric Measurement and Dimension Explain volume formulas and use them to solve problems Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

• SURFACE AREA OF COMPOSITE SOLIDS

- CCSS.Math.Content.HSG-MG.A.1: Modeling with Geometry Apply geometric concepts in modeling situations Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Unit 14: Three-Dimensional Geometry II

• SURFACE AREA OF SIMILAR SOLIDS

- CCSS.Math.Content.HSG-MG.A.1: Modeling with Geometry Apply geometric concepts in modeling situations Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

• MODELING SITUATIONS WITH GEOMETRY

- CCSS.Math.Content.HSG-MG.A.2: Modeling with Geometry Apply geometric concepts in modeling situations Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
- CCSS.Math.Content.HSG-MG.A.3: Modeling with Geometry Apply geometric concepts in modeling situations Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Unit 15: Statistical Design and Analysis

• ANALYZING STATISTICAL SAMPLES

- CCSS.Math.Content.HSS-IC.A.1: Making Inferences and Justifying Conclusions Understand and evaluate random processes underlying statistical experiments Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
- CCSS.Math.Content.HSS-IC.B.4: Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- CCSS.Math.Content.HSS-IC.A.2: Making Inferences and Justifying Conclusions Understand and evaluate random processes underlying statistical experiments Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

• EXPERIMENTAL AND OBSERVATIONAL DESIGN

- CCSS.Math.Content.HSS-IC.B.3: Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

• CONCLUSIONS IN DATA

- CCSS.Math.Content.HSS-IC.B.5: Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
- CCSS.Math.Content.HSS-IC.B.6: Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Evaluate reports based on data.

Unit 16: Statistics and Probability

• NORMAL DISTRIBUTION

- CCSS.Math.Content.HSS-ID.A.3: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable Interpret differences

in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

- CCSS.Math.Content.HSS-ID.A.4: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
- CCSS.Math.Content.HSS-IC.B.4: Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- **ANALYZING DECISIONS IN PROBABILITY**
 - CCSS.Math.Content.HSS-MD.B.6: Using Probability to Make Decisions Use probability to evaluate outcomes of decisions Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
 - CCSS.Math.Content.HSS-MD.B.7: Using Probability to Make Decisions Use probability to evaluate outcomes of decisions Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).