

California Tutorials are designed specifically for the California Common Core State Standards and the California Next Generation Science Standards to prepare students for the Smarter Balanced Assessment Consortium (SBAC) exams and the California Science Tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

## Unit 1: Real Number System

### • LAWS OF EXPONENTS

- N-RN.1: Number and Quantity The Real Number System Extend the properties of exponents to rational exponents. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- N-RN.2: Number and Quantity The Real Number System Extend the properties of exponents to rational exponents. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

### • OPERATIONS ON RATIONAL AND IRRATIONAL NUMBERS

- N-RN.3: Number and Quantity The Real Number System Use properties of rational and irrational numbers. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

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- **MONITORING PRECISION AND ACCURACY**

- N-Q.1: Number and Quantity Quantities Reason quantitatively and use units to solve problems. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- N-Q.2: Number and Quantity Quantities Reason quantitatively and use units to solve problems. Define appropriate quantities for the purpose of descriptive modeling.
- N-Q.3: Number and Quantity Quantities Reason quantitatively and use units to solve problems. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

## Unit 2: Equations and Inequalities

- **ONE-STEP EQUATIONS AND INEQUALITIES**

- A-CED.1: Algebra Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.
- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- A-REI.3: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- F-BF.1.a: Functions Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

- **MULTI-STEP EQUATIONS AND INEQUALITIES**

- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- A-CED.1: Algebra Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.
- A-REI.3: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- A-REI.1: Algebra Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the

assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

- **AXIOMS OF EQUALITY**

- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- A-REI.1: Algebra Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

- **LITERAL EQUATIONS**

- A-REI.3: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- A-CED.1: Algebra Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.
- A-CED.4: Algebra Creating Equations Create equations that describe numbers or relationships Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

### Unit 3: Writing Expressions and Equations

- **FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS**

- F-BF.1.a: Functions Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- A-SSE.1.a: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.

- **FORMULATING AND SOLVING EQUATIONS FROM WORD PROBLEMS**

- F-LE.2: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- F-LE.1.b: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can

be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

- A-CED.1: Algebra Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.

- **FORMULATING AND SOLVING INEQUALITIES FROM WORD PROBLEMS**

- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- A-CED.1: Algebra Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.
- A-REI.3: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

#### Unit 4: Functions

- **FUNCTIONS AND RELATIONS**

- F-IF.2: Functions Interpreting Functions Understand the concept of a function and use function notation Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- F-IF.7.b: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

- **DOMAIN AND RANGE**

- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

- **EVALUATING FUNCTIONS**

- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a

function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- F-IF.2: Functions Interpreting Functions Understand the concept of a function and use function notation Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

## Unit 5: Graphs of Linear Equations and Inequalities

### • SLOPE

- F-IF.6: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

### • GRAPHING AND ANALYZING LINEAR FUNCTIONS

- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- F-LE.2: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.6: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **GRAPHING AND MANIPULATING  $Y = MX + B$** 
  - F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - F-LE.2: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
  - F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - F-IF.6: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
  - F-LE.1.b: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
  - F-LE.5: Functions Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Interpret the parameters in a linear or exponential function in terms of a context.

- F-IF.7.a: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.

- **GRAPHS OF LINEAR INEQUALITIES**

- A-REI.12: Algebra Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

## Unit 6: Linear Equations

- **SLOPE-INTERCEPT FORM OF A LINEAR EQUATION**

- A-REI.10: Algebra Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- F-IF.7.a: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- F-LE.2: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- F-IF.6: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented

symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

- **POINT-SLOPE FORM OF A LINEAR EQUATION**

- A-REI.10: Algebra Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- F-LE.2: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- F-IF.7.a: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

## Unit 7: Linear Systems

- **SOLVING SYSTEMS OF LINEAR EQUATIONS: GUESS AND CHECK**

- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- F-BF.1.a: Functions Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- A-REI.6: Algebra Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of



linear equations in two variables.

- **SOLVING SYSTEMS OF LINEAR EQUATIONS: GRAPHING**

- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- A-REI.6: Algebra Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- A-REI.11: Algebra Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = (x)$  intersect are the solutions of the equation  $f(x) = (x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- A-REI.11: Algebra Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = (x)$  intersect are the solutions of the equation  $f(x) = (x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

- **SOLVING SYSTEMS OF LINEAR EQUATIONS: SUBSTITUTION**

- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- A-REI.6: Algebra Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

- **SOLVING SYSTEMS OF LINEAR EQUATIONS: ELIMINATION**

- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- A-REI.5: Algebra Reasoning with Equations and Inequalities Solve systems of equations Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- A-REI.6: Algebra Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

#### • SOLVING SYSTEMS OF LINEAR INEQUALITIES

- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- A-REI.12: Algebra Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

#### • SOLVING THREE-VARIABLE SYSTEMS OF LINEAR EQUATIONS

- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- A-REI.5: Algebra Reasoning with Equations and Inequalities Solve systems of equations Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

### Unit 8: Exponential Functions, Equations, and Inequalities

#### • EXPONENTIAL FUNCTIONS

- A-SSE.1.b: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- A-SSE.3.c: Algebra Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.

- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.8.b: Functions Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
- F-LE.1.a: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- F-IF.6: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-IF.7.e: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- F-LE.1.c: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- F-LE.2: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- F-LE.5: Functions Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Interpret the parameters in a linear or exponential function in terms of a context.

- A-CED.1: Algebra Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.
- **EXPONENTIAL GROWTH AND DECAY**
  - F-IF.8.b: Functions Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
  - F-LE.1.a: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
  - F-LE.1.c: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
  - F-LE.5: Functions Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Interpret the parameters in a linear or exponential function in terms of a context.
  - A-SSE.1.b: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - F-LE.2: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
  - F-LE.3: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
  - F-LE.1.b: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
  - A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- **SOLVING EXPONENTIAL INEQUALITIES**

- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- A-SSE.1.b: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- F-LE.1.c: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

## Unit 9: Sequences

### • SEQUENCES

- F-BF.1.a: Functions Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- F-BF.2: Functions Building Functions Build a function that models a relationship between two quantities Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- F-IF.3: Functions Interpreting Functions Understand the concept of a function and use function notation Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- F-LE.2: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

### • ARITHMETIC AND GEOMETRIC SEQUENCES

- F-BF.2: Functions Building Functions Build a function that models a relationship between two quantities Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- F-IF.3: Functions Interpreting Functions Understand the concept of a function and use function notation Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- F-LE.2: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential

functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- F-BF.1.a: Functions Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

## Unit 10: Polynomials

### • POLYNOMIAL BASICS

- A-SSE.1.a: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.

### • ADDITION AND SUBTRACTION OF POLYNOMIALS

- A-APR.1: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

### • MULTIPLICATION OF POLYNOMIALS

- A-APR.1: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

## Unit 11: Factoring

### • FACTORING QUADRATIC TRINOMIALS

- A-SSE.3.a: Algebra Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
- A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .

### • FACTORING SPECIAL CASES

- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- A-SSE.1.b: Algebra Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.

- **FACTORING HIGHER-ORDER POLYNOMIALS**

- A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.

## Unit 12: Graphs of Quadratic Functions

- **QUADRATIC FUNCTIONS**

- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-LE.6: Functions Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.

- **ANALYZING GRAPHS OF QUADRATIC FUNCTIONS**

- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.7.a: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .

- F-IF.9: Functions Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- **REPRESENTATIONS OF QUADRATIC FUNCTIONS**
  - A-SSE.2: Algebra Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - A-REI.4.a: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p) = q$  that has the same solutions. Derive the quadratic formula from this form.
  - F-IF.8.a: Functions Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  - A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - A-SSE.3.a: Algebra Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
  - F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - F-LE.6: Functions Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.

### Unit 13: Solving Quadratic Functions

- **SOLVING QUADRATIC FUNCTIONS WITH FACTORING**

- A-SSE.3.a: Algebra Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain



properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.

- A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- F-IF.8.a: Functions Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- F-LE.6: Functions Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.
- A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- F-IF.7.a: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **COMPLETING THE SQUARE**
  - A-REI.4.a: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p) = q$  that has the same solutions. Derive the quadratic formula from this form.
  - A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .

- F-IF.8.a: Functions Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- A-SSE.3.b: Algebra Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- A-REI.4.a: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p) = q$  that has the same solutions. Derive the quadratic formula from this form.
- A-REI.4.a: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p) = q$  that has the same solutions. Derive the quadratic formula from this form.
- A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- F-IF.7.a: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **QUADRATIC FORMULA**
  - F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
  - F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a

function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- A-REI.4.a: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p) = q$  that has the same solutions. Derive the quadratic formula from this form.
- A-REI.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- F-BF.1.a: Functions Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

## Unit 14: Parent Functions and Transformations

### • LINEAR AND EXPONENTIAL PARENT FUNCTIONS

- A-REI.10: Algebra Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.7.e: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic

functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

- F-LE.1.c: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

- **QUADRATIC PARENT FUNCTION**

- A-REI.10: Algebra Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

- **TRANSFORMATIONS OF THE LINEAR AND EXPONENTIAL PARENT FUNCTIONS**

- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

- **TRANSFORMATIONS OF THE QUADRATIC PARENT FUNCTION**

- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

## Unit 15: Nonlinear Functions

- **LINEAR VERSUS NONLINEAR FUNCTIONS**

- F-LE.1.a: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- F-LE.1.b: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can

be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

- F-IF.9: Functions Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- F-LE.2: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- F-IF.6: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-LE.1.c: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- **ABSOLUTE VALUE FUNCTIONS**
  - F-IF.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
  - F-IF.1: Functions Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - F-IF.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - F-IF.7.b: Functions Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
  - F-BF.3: Functions Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of

$k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

#### • SYSTEMS OF NONLINEAR EQUATIONS

- A-REI.6: Algebra Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- A-REI.7: Algebra Reasoning with Equations and Inequalities Solve systems of equations Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
- A-REI.11: Algebra Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = (x)$  intersect are the solutions of the equation  $f(x) = (x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- A-REI.5: Algebra Reasoning with Equations and Inequalities Solve systems of equations Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- A-REI.11: Algebra Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = (x)$  intersect are the solutions of the equation  $f(x) = (x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- A-CED.3: Algebra Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- A-REI.11: Algebra Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = (x)$  intersect are the solutions of the equation  $f(x) = (x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- A-REI.11: Algebra Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = (x)$  intersect are the solutions of the equation  $f(x) = (x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

## Unit 16: Working with Functions

### • ARITHMETIC OPERATIONS ON FUNCTIONS

- F-BF.1.b: Functions Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations.

### • MULTIPLE REPRESENTATIONS OF FUNCTIONS

- A-CED.2: Algebra Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- F-IF.9: Functions Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- F-LE.1.a: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

### • INVERSE FUNCTIONS

- F-BF.4.a: Functions Building Functions Build new functions from existing functions Find inverse functions. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.

## Unit 17: Statistics

### • DATA ANALYSIS

- S-ID.1: Statistics and Probability Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable Represent data with plots on the real number line (dot plots, histograms, and box plots).
- S-ID.2: Statistics and Probability Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- S-ID.3: Statistics and Probability Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

### • FREQUENCY TABLES

- S-ID.5: Statistics and Probability Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies

in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

- S-ID.3: Statistics and Probability Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

## • SCATTERPLOTS

- S-ID.6.a: Statistics and Probability Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
- S-ID.6.b: Statistics and Probability Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Informally assess the fit of a function by plotting and analyzing residuals.
- S-ID.6.c: Statistics and Probability Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a linear function for a scatter plot that suggests a linear association.
- S-ID.9: Statistics and Probability Interpreting Categorical and Quantitative Data Interpret linear models Distinguish between correlation and causation.
- S-ID.7: Statistics and Probability Interpreting Categorical and Quantitative Data Interpret linear models Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

## • SCATTERPLOTS AND MODELING

- S-ID.6.a: Statistics and Probability Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
- S-ID.6.b: Statistics and Probability Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Informally assess the fit of a function by plotting and analyzing residuals.
- S-ID.6.c: Statistics and Probability Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a linear function for a scatter plot that suggests a linear association.
- S-ID.8: Statistics and Probability Interpreting Categorical and Quantitative Data Interpret linear models Compute (using technology) and interpret the correlation coefficient of a linear fit.



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- S-ID.7: Statistics and Probability Interpreting Categorical and Quantitative Data Interpret linear models Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
  - F-LE.1.a: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
  - F-LE.1.c: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
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