

Massachusetts Tutorials are designed specifically for the Learning Standards found in the Massachusetts Curriculum Frameworks to prepare students for the MCAS tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

Unit 1: The Number System

- **RATIONAL AND IRRATIONAL NUMBERS**

- 8.NS.A.1: The Number System Know that there are numbers that are not rational, and approximate them by rational numbers. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion. For rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

- **APPROXIMATING IRRATIONAL NUMBERS**

- 8.NS.A.2: The Number System Know that there are numbers that are not rational, and approximate them by rational numbers. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,).
- 8.EE.A.2: Expressions and Equations Work with radicals and integer exponents. Use square root and cube root symbols to represent solutions to equations of the form $x = p$ and $x = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that 2 is irrational.

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Unit 2: Exponents

- **PROPERTIES OF EXPONENTS**

- 8.EE.A.1: Expressions and Equations Work with radicals and integer exponents. Know and apply the properties of integer exponents to generate equivalent numerical expressions.

- **POWERS OF 10**

- 8.EE.A.3: Expressions and Equations Work with radicals and integer exponents. Use numbers expressed in the form of a single digit multiplied by an integer power of 10 to estimate very large or very small quantities, and express how many times as much one is than the other.
- 8.EE.A.4: Expressions and Equations Work with radicals and integer exponents. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

- **SCIENTIFIC NOTATION**

- 8.EE.A.3: Expressions and Equations Work with radicals and integer exponents. Use numbers expressed in the form of a single digit multiplied by an integer power of 10 to estimate very large or very small quantities, and express how many times as much one is than the other.
- 8.EE.A.4: Expressions and Equations Work with radicals and integer exponents. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Unit 3: Proportional Reasoning and Slope

- **SLOPE**

- 8.EE.B.6: Expressions and Equations Understand the connections between proportional relationships, lines, and linear equations. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .
- 8.F.B.4: Functions Use functions to model relationships between quantities. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- 8.F.B.5: Functions Use functions to model relationships between quantities. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
- 8.EE.B.5: Expressions and Equations Understand the connections between proportional relationships, lines, and linear equations. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
- 8.EE.B.6: Expressions and Equations Understand the connections between proportional relationships, lines, and linear equations. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .

- **MULTIPLE REPRESENTATIONS OF PROPORTIONS**

- 8.EE.B.5: Expressions and Equations Understand the connections between proportional relationships, lines, and linear equations. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
- 8.F.B.4: Functions Use functions to model relationships between quantities. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- 8.F.B.5: Functions Use functions to model relationships between quantities. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Unit 4: Functions

- **RELATIONS AND FUNCTIONS**

- 8.F.A.1: Functions Define, evaluate, and compare functions. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting

of an input and the corresponding output.

- **COMPARING FUNCTIONS**

- 8.F.A.2: Functions Define, evaluate, and compare functions. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- 8.F.B.4: Functions Use functions to model relationships between quantities. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
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- **GRAPHS OF FUNCTIONS**

- 8.F.A.3: Functions Define, evaluate, and compare functions. Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear.
- 8.F.B.4: Functions Use functions to model relationships between quantities. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- 8.F.B.5: Functions Use functions to model relationships between quantities. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
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Unit 5: Linear Functions

- **SLOPE-INTERCEPT FORM**

- 8.EE.B.6: Expressions and Equations Understand the connections between proportional relationships, lines, and linear equations. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .
- 8.F.A.3: Functions Define, evaluate, and compare functions. Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear.
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- 8.F.B.4: Functions Use functions to model relationships between quantities. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- **WRITING LINEAR FUNCTIONS**
 - 8.F.B.4: Functions Use functions to model relationships between quantities. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Unit 6: Solving Equations

- **SOLVING LINEAR EQUATIONS**
 - 8.EE.C.7.a: Expressions and Equations Analyze and solve linear equations and pairs of simultaneous linear equations. Solve linear equations in one variable. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities

is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

- 8.EE.C.7.b: Expressions and Equations Analyze and solve linear equations and pairs of simultaneous linear equations. Solve linear equations in one variable. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
- 8.EE.C.7.a: Expressions and Equations Analyze and solve linear equations and pairs of simultaneous linear equations. Solve linear equations in one variable. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
- 8.EE.C.7.a: Expressions and Equations Analyze and solve linear equations and pairs of simultaneous linear equations. Solve linear equations in one variable. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
- 8.EE.C.8.b: Expressions and Equations Analyze and solve linear equations and pairs of simultaneous linear equations. Analyze and solve pairs of simultaneous linear equations. Solve systems of two linear equations in two variables algebraically (using substitution and elimination strategies), and estimate solutions by graphing the equations. Solve simple cases by inspection.
- 8.EE.C.8.c: Expressions and Equations Analyze and solve linear equations and pairs of simultaneous linear equations. Analyze and solve pairs of simultaneous linear equations. Solve real-world and mathematical problems leading to two linear equations in two variables.
- **SOLVING SYSTEMS OF LINEAR EQUATIONS**
 - 8.EE.C.8.a: Expressions and Equations Analyze and solve linear equations and pairs of simultaneous linear equations. Analyze and solve pairs of simultaneous linear equations. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
 - 8.EE.C.8.b: Expressions and Equations Analyze and solve linear equations and pairs of simultaneous linear equations. Analyze and solve pairs of simultaneous linear equations. Solve systems of two linear equations in two variables algebraically (using substitution and elimination strategies), and estimate solutions by graphing the equations. Solve simple cases by inspection.
 - 8.EE.C.8.c: Expressions and Equations Analyze and solve linear equations and pairs of simultaneous linear equations. Analyze and solve pairs of simultaneous linear equations. Solve real-world and mathematical problems leading to two linear equations in two variables.
- **SOLVING EQUATIONS USING ROOTS**
 - 8.EE.A.2: Expressions and Equations Work with radicals and integer exponents. Use square root and cube root symbols to represent solutions to equations of the form $x = p$ and $x = p$, where p is a

positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Unit 7: The Pythagorean Theorem and Distance Formula

- **PYTHAGOREAN THEOREM**

- 8.G.B.6.b: Geometry Understand and apply the Pythagorean Theorem. Analyze and justify the Pythagorean Theorem and its converse using pictures, diagrams, narratives, or models.
- 8.G.B.7: Geometry Understand and apply the Pythagorean Theorem. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
- 8.G.B.6.a: Geometry Understand and apply the Pythagorean Theorem. Understand the relationship among the sides of a right triangle.

- **THE CONVERSE OF THE PYTHAGOREAN THEOREM**

- 8.G.B.6.b: Geometry Understand and apply the Pythagorean Theorem. Analyze and justify the Pythagorean Theorem and its converse using pictures, diagrams, narratives, or models.
- 8.G.B.7: Geometry Understand and apply the Pythagorean Theorem. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

- **DISTANCE ON THE COORDINATE PLANE**

- 8.G.B.7: Geometry Understand and apply the Pythagorean Theorem. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
- 8.G.B.8: Geometry Understand and apply the Pythagorean Theorem. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Unit 8: Three-Dimensional Geometry

- **VOLUME OF CYLINDERS AND CONES**

- 8.G.C.9: Geometry Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. Know the formulas for the volumes of cones, cylinders, and spheres, and use them to solve real-world and mathematical problems.

- **SPHERES**

- 8.G.C.9: Geometry Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. Know the formulas for the volumes of cones, cylinders, and spheres, and use them to solve real-world and mathematical problems.

Unit 9: Rotations, Reflections, and Translations

- **BASICS OF TRANSFORMATIONS**

- 8.G.A.3: Geometry Understand congruence and similarity using physical models, transparencies, or geometry software. Describe the effects of dilations, translations, rotations, and reflections on two-

dimensional figures using coordinates.

- 8.G.A.4: Geometry Understand congruence and similarity using physical models, transparencies, or geometry software. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- 8.G.A.1.b: Geometry Understand congruence and similarity using physical models, transparencies, or geometry software. Verify experimentally the properties of rotations, reflections, and translations: Angles are transformed to angles of the same measure.
- 8.G.A.1.a: Geometry Understand congruence and similarity using physical models, transparencies, or geometry software. Verify experimentally the properties of rotations, reflections, and translations: Lines are transformed to lines, and line segments to line segments of the same length.
- 8.G.A.1.c: Geometry Understand congruence and similarity using physical models, transparencies, or geometry software. Verify experimentally the properties of rotations, reflections, and translations: Parallel lines are transformed to parallel lines.
- **TRANSFORMATIONS AND CONGRUENCE**
 - 8.G.A.2: Geometry Understand congruence and similarity using physical models, transparencies, or geometry software. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. Given two congruent figures, describe a sequence that exhibits the congruence between them.
 - 8.G.A.3: Geometry Understand congruence and similarity using physical models, transparencies, or geometry software. Describe the effects of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Unit 10: Congruence and Similarity Transformations

- **TRANSFORMATIONS ON THE COORDINATE PLANE**
 - 8.G.A.3: Geometry Understand congruence and similarity using physical models, transparencies, or geometry software. Describe the effects of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
 - 8.G.A.4: Geometry Understand congruence and similarity using physical models, transparencies, or geometry software. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- **SIMILARITY AND DILATIONS**
 - 8.G.A.3: Geometry Understand congruence and similarity using physical models, transparencies, or geometry software. Describe the effects of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
 - 8.G.A.4: Geometry Understand congruence and similarity using physical models, transparencies, or geometry software. Understand that a two-dimensional figure is similar to another if the second can

be obtained from the first by a sequence of rotations, reflections, translations, and dilations. Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

- 8.G.B.7: Geometry Understand and apply the Pythagorean Theorem. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Unit 11: Angles and Angle Relationships

• PARALLEL LINES AND ANGLE RELATIONSHIPS

- 8.G.A.5: Geometry Understand congruence and similarity using physical models, transparencies, or geometry software. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

• ANGLE RELATIONSHIPS IN TRIANGLES

- 8.G.A.5: Geometry Understand congruence and similarity using physical models, transparencies, or geometry software. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Unit 12: Probability and Statistics

• SCATTERPLOTS

- 8.SPA.1: Statistics and Probability Investigate patterns of association in bivariate data. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- 8.SPA.2: Statistics and Probability Investigate patterns of association in bivariate data. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line.

• LINEAR MODELS IN DATA

- 8.SPA.1: Statistics and Probability Investigate patterns of association in bivariate data. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- 8.SPA.2: Statistics and Probability Investigate patterns of association in bivariate data. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line.
- 8.SPA.3: Statistics and Probability Investigate patterns of association in bivariate data. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

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- 8.F.B.4: Functions Use functions to model relationships between quantities. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
 - **FREQUENCY TABLES**
 - 8.SP.A.4: Statistics and Probability Investigate patterns of association in bivariate data. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.
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