

Massachusetts Tutorials are designed specifically for the Learning Standards found in the Massachusetts Curriculum Frameworks to prepare students for the MCAS tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

### Unit 1: Writing and Solving Equations

- **LITERAL EQUATIONS**

- AII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from simple root and rational functions and exponential functions.

- **FORMULATING AND SOLVING EQUATIONS FROM WORD PROBLEMS**

- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

- AII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.

## Unit 2: Solving Inequalities and Geometric Sequences

### • FORMULATING AND SOLVING INEQUALITIES FROM WORD PROBLEMS

- AII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from simple root and rational functions and exponential functions.
- AII.A-CED.A.3: Algebra Creating Equations Create equations that describe numbers or relationships. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

### • SUMS OF GEOMETRIC SEQUENCES

- AII.A-SSE.B.4: Algebra Seeing Structure in Expressions Write expressions in equivalent forms to solve problems. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

## Unit 3: Functions

### • FUNCTIONS AND RELATIONS

- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- AII.F-IF.C.7.b: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph square root and cube root functions.

### • DOMAIN AND RANGE

- AII.F-IF.B.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

### • INVERSE FUNCTIONS

- AII.F-BF.4.a: Functions Building Functions Build new functions from existing functions. Find inverse functions algebraically and graphically. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.

#### Unit 4: Linear Functions, Equations, and Inequalities

- **SLOPE**

- AII.F-IF.B.6: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

- **GRAPHING AND ANALYZING LINEAR FUNCTIONS**

- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- AII.F-IF.B.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

- **GRAPHING AND MANIPULATING  $Y = MX + B$**

- AII.A-CED.A.2: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph

equations on coordinate axes with labels and scales.

- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- AII.F-IF.B.6: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

## Unit 5: Exponential Functions

### • EXPONENTIAL FUNCTIONS

- AII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- AII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- AII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from simple root and rational functions and exponential functions.
- AII.F-IF.C.10: Functions Interpreting Functions Analyze functions using different representations. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.
- AII.F-IF.B.6: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- AII.F-IF.B.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- AII.A-CED.A.2: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- AII.F-IF.C.7.e: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.
- **EXPONENTIAL GROWTH AND DECAY**
  - AII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - AII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - AII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from simple root and rational functions and exponential functions.
  - AII.F-IF.C.10: Functions Interpreting Functions Analyze functions using different representations. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.
  - AII.A-CED.A.2: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

## Unit 6: Exponential Equations and Inequalities

- **SOLVING EXPONENTIAL EQUATIONS**
  - AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic

functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

- AII.F-IF.C.7.e: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.
- AII.F-BF.B.4.a: Functions Building Functions Build new functions from existing functions. Find inverse functions algebraically and graphically. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
- AII.F-LE.A.4: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems. For exponential models, express as a logarithm the solution to  $ab$  to the  $ct$  power =  $d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.
- AII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from simple root and rational functions and exponential functions.
- AII.F-IF.C.10: Functions Interpreting Functions Analyze functions using different representations. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.
- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **SOLVING EXPONENTIAL INEQUALITIES**
  - AII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from simple root and rational functions and exponential functions.
  - AII.A-CED.A.3: Algebra Creating Equations Create equations that describe numbers or relationships. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
  - AII.A-CED.A.2: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

## Unit 7: Logarithmic Expressions, Equations, and Functions

- **LOGARITHMIC FUNCTIONS**

- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- AII.F-IF.C.10: Functions Interpreting Functions Analyze functions using different representations. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.
- AII.F-LE.A.4: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems. For exponential models, express as a logarithm the solution to  $ab^{ct} = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.
- AII.F-IF.B.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- AII.F-IF.C.7.c: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- AII.F-BF.B.4.a: Functions Building Functions Build new functions from existing functions. Find inverse functions algebraically and graphically. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
- **EVALUATING LOGARITHMIC EXPRESSIONS**
  - AII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - AII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - AII.F-LE.A.4: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems. For exponential models, express as a

logarithm the solution to  $ab$  to the  $ct$  power =  $d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.

- **SOLVING LOGARITHMIC EQUATIONS**

- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- AII.F-IF.C.10: Functions Interpreting Functions Analyze functions using different representations. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.
- AII.F-BF.B.4.a: Functions Building Functions Build new functions from existing functions. Find inverse functions algebraically and graphically. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
- AII.F-LE.A.4: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems. For exponential models, express as a logarithm the solution to  $ab$  to the  $ct$  power =  $d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.
- AII.F-LE.A.4: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems. For exponential models, express as a logarithm the solution to  $ab$  to the  $ct$  power =  $d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.

## Unit 8: Simplifying, Adding, and Subtracting Polynomial Expressions

- **POLYNOMIAL BASICS**

- AII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- AII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- AII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Use the structure of an expression to identify ways to rewrite it.

- **ADDITION AND SUBTRACTION OF POLYNOMIALS**

- AII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Use the structure of an expression to identify ways to rewrite it.
- AII.A-APR.A.1.a: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. Perform operations on polynomial expressions (addition, subtraction, multiplication, and division), and compare the system of polynomials to the system of integers when performing operations.

## Unit 9: Multiplying and Dividing Polynomial Expressions and Composing Functions

### • MULTIPLICATION OF POLYNOMIALS

- AII.A-APR.A.1.a: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. Perform operations on polynomial expressions (addition, subtraction, multiplication, and division), and compare the system of polynomials to the system of integers when performing operations.

### • DIVISION OF POLYNOMIALS

- AII.A-APR.A.1.a: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. Perform operations on polynomial expressions (addition, subtraction, multiplication, and division), and compare the system of polynomials to the system of integers when performing operations.
- AII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Use the structure of an expression to identify ways to rewrite it.
- AII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- AII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- AII.A-APR.D.6: Algebra Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions. Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

### • ARITHMETIC OPERATIONS ON FUNCTIONS

- AII.F-BF.A.1.b: Functions Building Functions Build a function that models a relationship between two quantities. Write a function (simple rational, radical, logarithmic, and trigonometric functions) that describes a relationship between two quantities. Combine standard function types using arithmetic operations.

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**Unit 10: Graphs and Representations of Quadratic Functions****• ANALYZING GRAPHS OF QUADRATIC FUNCTIONS**

- AII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Use the structure of an expression to identify ways to rewrite it.
- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- AII.F-IF.B.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- AII.A-APR.B.3: Algebra Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

**• REPRESENTATIONS OF QUADRATIC FUNCTIONS**

- AII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Use the structure of an expression to identify ways to rewrite it.
- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- AII.A-CED.A.2: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph

equations on coordinate axes with labels and scales.

- AII.A-CED.A.3: Algebra Creating Equations Create equations that describe numbers or relationships. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**

- AII.A-CED.A.2: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- AII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from simple root and rational functions and exponential functions.
- AII.F-IF.C.10: Functions Interpreting Functions Analyze functions using different representations. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.

## Unit 11: Solving Quadratic Equations

- **SOLVING QUADRATIC EQUATIONS BY FACTORING**

- AII.F-IF.C.8.a: Functions Interpreting Functions Analyze functions using different representations. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring in a polynomial function to show zeros, extreme values, and symmetry of the graph and interpret these in terms of a context.
- AII.A-APR.B.3: Algebra Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- AII.A-APR.C.4: Algebra Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems. Prove polynomial identities and use them to describe numerical relationships.
- AII.A-CED.A.2: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph

equations on coordinate axes with labels and scales.

- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

- **QUADRATIC FORMULA**

- AII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- AII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- AII.N-CN.C.7: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- AII.A-CED.A.3: Algebra Creating Equations Create equations that describe numbers or relationships. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

- **COMPLEX NUMBERS AND QUADRATIC FUNCTIONS**

- AII.N-CN.C.7: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
- AII.N-CN.A.1: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $x=a$  and  $b$  real.
- AII.N-CN.A.1: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $x=a$  and  $b$  real.
- AII.N-CN.A.2: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Use the relation  $i^2 = -1$  and the Commutative, Associative, and Distributive properties to add, subtract, and multiply complex numbers.

- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

## Unit 12: Factoring Special Cases and Cubic Polynomials

### • FACTORING SPECIAL CASES

- AII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Use the structure of an expression to identify ways to rewrite it.
- AII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- AII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- AII.A-APR.C.4: Algebra Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems. Prove polynomial identities and use them to describe numerical relationships.
- AII.A-APR.B.3: Algebra Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

### • FACTORING CUBIC POLYNOMIALS

- AII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Use the structure of an expression to identify ways to rewrite it.
- AII.A-APR.C.4: Algebra Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems. Prove polynomial identities and use them to describe numerical relationships.
- AII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- AII.A-APR.B.3: Algebra Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

## Unit 13: Factoring Polynomials and the Factor Theorem

### • FACTORING HIGHER-ORDER POLYNOMIALS

- AII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.

- AII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- AII.A-APR.B.3: Algebra Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- AII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Use the structure of an expression to identify ways to rewrite it.
- AII.A-APR.C.4: Algebra Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems. Prove polynomial identities and use them to describe numerical relationships.
- **FACTOR THEOREM AND REMAINDER THEOREM**
  - AII.A-APR.B.2: Algebra Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials. Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .
  - AII.A-APR.D.6: Algebra Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions. Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

## Unit 14: Polynomial Functions and Polynomial Identities

- **GRAPHS OF POLYNOMIAL FUNCTIONS**
  - AII.A-APR.B.3: Algebra Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
  - AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
  - AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

- AII.F-IF.C.10: Functions Interpreting Functions Analyze functions using different representations. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.
- AII.F-IF.C.7.c: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- AII.F-IF.C.8.a: Functions Interpreting Functions Analyze functions using different representations. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring in a polynomial function to show zeros, extreme values, and symmetry of the graph and interpret these in terms of a context.
- **COMPLEX NUMBERS**
  - AII.N-CN.A.1: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
  - AII.N-CN.A.2: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Use the relation  $i^2 = -1$  and the Commutative, Associative, and Distributive properties to add, subtract, and multiply complex numbers.

## Unit 15: Polynomial Identities and Complex Numbers

- **POLYNOMIAL IDENTITIES**
  - AII.A-APR.C.4: Algebra Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems. Prove polynomial identities and use them to describe numerical relationships.
  - AII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Use the structure of an expression to identify ways to rewrite it.
  - AII.A-APR.C.5: Algebra Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems. Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle.
- **POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS**
  - AII.N-CN.C.8: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Extend polynomial identities to the complex numbers.
  - AII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Use the structure of an expression to identify ways to rewrite it.
  - AII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - AII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its

context. Interpret complicated expressions by viewing one or more of their parts as a single entity.

- AII.A-APR.C.4: Algebra Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems. Prove polynomial identities and use them to describe numerical relationships.
- AII.N-CN.A.1: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $x=a$  and  $b$  real.
- AII.N-CN.C.7: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
- AII.N-CN.C.9: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

## Unit 16: Radical Functions and Equations

### • ANALYZING GRAPHS OF SQUARE ROOT FUNCTIONS

- AII.F-BF.B.3: Functions Building Functions Build new functions from existing functions. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Include simple rational, radical, logarithmic, and trigonometric functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- AII.F-IF.B.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- AII.F-IF.C.7.b: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph square root and cube root functions.
- AII.F-BF.B.3: Functions Building Functions Build new functions from existing functions. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Include simple rational, radical, logarithmic, and trigonometric functions. Utilize technology to experiment with cases and illustrate an

explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

- AII.F-BF.B.4.a: Functions Building Functions Build new functions from existing functions. Find inverse functions algebraically and graphically. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
- **SOLVING SQUARE ROOT EQUATIONS**
  - AII.A-REI.A.2: Algebra Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
  - AII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from simple root and rational functions and exponential functions.
  - AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

## Unit 17: Rational Expressions and Equations

- **OPERATIONS WITH RATIONAL EXPRESSIONS**
  - AII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - AII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - AII.A-APR.D.7: Algebra Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
  - AII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Use the structure of an expression to identify ways to rewrite it.
  - AII.A-APR.D.6: Algebra Arithmetic with Polynomials and Rational Expressions Rewrite rational expressions. Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
- **SOLVING RATIONAL EQUATIONS**
  - AII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations

arising from simple root and rational functions and exponential functions.

- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- AII.F-IF.C.10: Functions Interpreting Functions Analyze functions using different representations. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.
- AII.A-CED.A.3: Algebra Creating Equations Create equations that describe numbers or relationships. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- AII.A-REI.A.2: Algebra Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

## Unit 18: Rational Functions

### • ANALYZING GRAPHS OF RATIONAL FUNCTIONS

- AII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from simple root and rational functions and exponential functions.
- AII.F-IF.C.10: Functions Interpreting Functions Analyze functions using different representations. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.
- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- AII.F-IF.B.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

### • MODELING SITUATIONS WITH RATIONAL FUNCTIONS

- AII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- AII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of exponential, polynomial, and rational expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- AII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from simple root and rational functions and exponential functions.
- AII.A-CED.A.2: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- AII.A-CED.A.3: Algebra Creating Equations Create equations that describe numbers or relationships. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- AII.A-REI.A.2: Algebra Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

## Unit 19: Nonlinear Functions I

### • LINEAR VERSUS NONLINEAR FUNCTIONS

- AII.F-IF.B.6: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

- **ABSOLUTE VALUE FUNCTIONS**

- AII.F-IF.B.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- AII.F-IF.C.7.b: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph square root and cube root functions.

## Unit 20: Nonlinear Functions II

- **RADIANS AND THE UNIT CIRCLE**

- AII.F-TF.A.1: Functions Trigonometric Functions Extend the domain of trigonometric functions using the unit circle. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- AII.F-TF.A.2: Functions Trigonometric Functions Extend the domain of trigonometric functions using the unit circle. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

- **TRIGONOMETRIC FUNCTIONS**

- AII.F-IF.C.7.e: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.
- AII.F-TF.B.5: Functions Trigonometric Functions Model periodic phenomena with trigonometric functions. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- AII.F-TF.C.8: Functions Trigonometric Functions Prove and apply trigonometric identities. Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant.
- AII.F-TF.A.2: Functions Trigonometric Functions Extend the domain of trigonometric functions using the unit circle. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- AII.F-TF.C.8: Functions Trigonometric Functions Prove and apply trigonometric identities. Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant.

## Unit 21: Parent Functions and Transformations

- **PARENT FUNCTIONS**

- AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- AII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- AII.F-IF.B.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- AII.F-IF.C.10: Functions Interpreting Functions Analyze functions using different representations. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.
- AII.F-IF.C.7.b: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph square root and cube root functions.
- AII.F-IF.C.7.c: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- AII.F-IF.C.7.e: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.
- AII.F-BF.B.3: Functions Building Functions Build new functions from existing functions. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Include simple rational, radical, logarithmic, and trigonometric functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- **TRANSFORMATIONS OF PARENT FUNCTIONS**
  - AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of

graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

- AII.F-IF.B.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- AII.F-IF.C.7.b: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph square root and cube root functions.
- AII.F-IF.C.7.c: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- AII.F-IF.C.7.e: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.
- AII.F-BF.B.3: Functions Building Functions Build new functions from existing functions. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Include simple rational, radical, logarithmic, and trigonometric functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- **MULTIPLE TRANSFORMATIONS OF PARENT FUNCTIONS**
  - AII.F-BF.B.3: Functions Building Functions Build new functions from existing functions. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Include simple rational, radical, logarithmic, and trigonometric functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
  - AII.F-IF.B.4: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

- AII.F-IF.B.5: Functions Interpreting Functions Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- AII.F-IF.C.7.b: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph square root and cube root functions.
- AII.F-IF.C.7.c: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- AII.F-IF.C.7.e: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.
- AII.F-BF.B.3: Functions Building Functions Build new functions from existing functions. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Include simple rational, radical, logarithmic, and trigonometric functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

## Unit 22: Systems of Equations

### • SOLVING THREE-VARIABLE SYSTEMS OF LINEAR EQUATIONS

- AII.A-CED.A.2: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- AII.A-CED.A.3: Algebra Creating Equations Create equations that describe numbers or relationships. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

### • SYSTEMS OF NONLINEAR EQUATIONS

- AII.A-REI.D.11: Algebra Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically. Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are polynomial, rational, and logarithmic functions.
- AII.A-CED.A.3: Algebra Creating Equations Create equations that describe numbers or relationships. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities,

and interpret solutions as viable or non-viable options in a modeling context.

### Unit 23: Statistical Design and Analysis

#### • ANALYZING STATISTICAL SAMPLES

- AII.S-IC.A.1: Statistics and Probability Making Inferences and Justifying Conclusions Understand and evaluate random processes underlying statistical experiments. Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population.
- AII.S-IC.A.2: Statistics and Probability Making Inferences and Justifying Conclusions Understand and evaluate random processes underlying statistical experiments. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
- AII.S-IC.B.4: Statistics and Probability Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

#### • EXPERIMENTAL AND OBSERVATIONAL DESIGN

- AII.S-IC.B.3: Statistics and Probability Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

#### • CONCLUSIONS IN DATA

- AII.S-IC.B.5: Statistics and Probability Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

### Unit 24: Statistics and Probability

#### • NORMAL DISTRIBUTION

- AII.S-ID.A.4: Statistics and Probability Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
- AII.S-IC.B.4: Statistics and Probability Making Inferences and Justifying Conclusions Make inferences and justify conclusions from sample surveys, experiments, and observational studies. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

#### • ANALYZING DECISIONS IN PROBABILITY

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- AII.S-MD.B.6: Statistics and Probability Using Probability to Make Decisions Use probability to evaluate outcomes of decisions. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
  - AII.S-MD.B.7: Statistics and Probability Using Probability to Make Decisions Use probability to evaluate outcomes of decisions. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game and replacing the goalie with an extra skater).
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