

Regents Tutorials are designed specifically to prepare students for the New York Regents Exams.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

### Unit 1: Real Number System

- **OPERATIONS ON RATIONAL AND IRRATIONAL NUMBERS**

- NY.CCLS.Math.9-12.N-RN.3: The Real Number System Use properties of rational and irrational numbers. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

- **MONITORING PRECISION AND ACCURACY**

- NY.CCLS.Math.9-12.N-Q.1: Quantities Reason quantitatively and use units to solve problems. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- NY.CCLS.Math.9-12.N-Q.2: Quantities Reason quantitatively and use units to solve problems. Define appropriate quantities for the purpose of descriptive modeling.
- NY.CCLS.Math.9-12.N-Q.3: Quantities Reason quantitatively and use units to solve problems. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

### Unit 2: Equations and Inequalities

- **ONE-STEP EQUATIONS AND INEQUALITIES**

- NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- NY.CCLS.Math.9-12.A-REI.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- NY.CCLS.Math.9-12.A-REI.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- NY.CCLS.Math.9-12.A-REI.2: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- **MULTI-STEP EQUATIONS AND INEQUALITIES**
  - NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
  - NY.CCLS.Math.9-12.A-REI.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
  - NY.CCLS.Math.9-12.A-REI.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

### Unit 3: Applying Properties of Equations and Inequalities

- **AXIOMS OF EQUALITY**
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated

expressions by viewing one or more of their parts as a single entity.

- NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- NY.CCLS.Math.9-12.A-REI.1: Reasoning with Equations and Inequalities Understand solving equations as a process of reasoning and explain the reasoning Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- **LITERAL EQUATIONS**
  - NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
  - NY.CCLS.Math.9-12.A-CED.4: Creating Equations Create equations that describe numbers or relationships Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
  - NY.CCLS.Math.9-12.A-REI.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

#### Unit 4: Writing Expressions and Equations

- **FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS**
  - NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
  - NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - NY.CCLS.Math.9-12.A-APR.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
  - NY.CCLS.Math.9-12.A-APR.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.

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- **FORMULATING AND SOLVING EQUATIONS FROM WORD PROBLEMS**

- NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
- NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- NY.CCLS.Math.9-12.F-LE.1.b: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.A-REI.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.

- **FORMULATING AND SOLVING INEQUALITIES FROM WORD PROBLEMS**

- NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
- NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- NY.CCLS.Math.9-12.A-REI.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations

with coefficients represented by letters.

- NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- NY.CCLS.Math.9-12.A-REI.12: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

## Unit 5: Functions

### • FUNCTIONS AND RELATIONS

- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-IF.2: Interpreting Functions Understand the concept of a function and use function notation Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called

the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- NY.CCLS.Math.9-12.F-IF.7.b: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **DOMAIN AND RANGE**
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
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- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **EVALUATING FUNCTIONS**
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - NY.CCLS.Math.9-12.F-IF.2: Interpreting Functions Understand the concept of a function and use function notation Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
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- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

## Unit 6: Graphs of Linear Equations and Inequalities

### • SLOPE

- NY.CCLS.Math.9-12.F-IF.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.S-ID.7: Interpreting Categorical and Quantitative Data Interpret linear models Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- NY.CCLS.Math.9-12.G-GPE.5: Expressing Geometric Properties with Equations Use coordinates to prove simple geometric theorems algebraically Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

### • GRAPHING AND ANALYZING LINEAR FUNCTIONS

- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions,



including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- NY.CCLS.Math.9-12.F-IF.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- NY.CCLS.Math.9-12.F-IF.2: Interpreting Functions Understand the concept of a function and use function notation Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-IF.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- NY.CCLS.Math.9-12.S-ID.7: Interpreting Categorical and Quantitative Data Interpret linear models Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- **GRAPHING AND MANIPULATING  $Y = MX + B$** 
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function

and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.F-IF.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- NY.CCLS.Math.9-12.F-IF.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- NY.CCLS.Math.9-12.F-IF.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.S-ID.7: Interpreting Categorical and Quantitative Data Interpret linear models Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

- NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- NY.CCLS.Math.9-12.F-LE.1.b: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- NY.CCLS.Math.9-12.F-LE.5: Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Interpret the parameters in a linear or exponential function in terms of a context.
- **GRAPHS OF LINEAR INEQUALITIES**
  - NY.CCLS.Math.9-12.A-REI.12: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
  - NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
  - NY.CCLS.Math.9-12.A-REI.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

## Unit 7: Linear Equations

- **SLOPE-INTERCEPT FORM OF A LINEAR EQUATION**
  - NY.CCLS.Math.9-12.S-ID.7: Interpreting Categorical and Quantitative Data Interpret linear models Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - NY.CCLS.Math.9-12.A-REI.10: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- NY.CCLS.Math.9-12.F-IF.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.A-CED.4: Creating Equations Create equations that describe numbers or relationships Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
- NY.CCLS.Math.9-12.G-GPE.5: Expressing Geometric Properties with Equations Use coordinates to prove simple geometric theorems algebraically Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
- NY.CCLS.Math.9-12.F-IF.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **POINT-SLOPE FORM OF A LINEAR EQUATION**
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function

and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- NY.CCLS.Math.9-12.F-IF.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- NY.CCLS.Math.9-12.A-REI.10: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.G-GPE.5: Expressing Geometric Properties with Equations Use coordinates to prove simple geometric theorems algebraically Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

## Unit 8: Two-Variable Linear Systems

### • SOLVING SYSTEMS OF LINEAR EQUATIONS: GUESS AND CHECK

- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- NY.CCLS.Math.9-12.A-REI.6: Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- NY.CCLS.Math.9-12.A-REI.7: Reasoning with Equations and Inequalities Solve systems of equations Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

### • SOLVING SYSTEMS OF LINEAR EQUATIONS: GRAPHING

- NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.A-REI.6: Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- NY.CCLS.Math.9-12.A-REI.10: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- NY.CCLS.Math.9-12.A-REI.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = (x)$  intersect are the solutions of the equation  $f(x) = (x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- NY.CCLS.Math.9-12.A-REI.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = (x)$  intersect are the solutions of the equation  $f(x) = (x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- NY.CCLS.Math.9-12.A-REI.11: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = (x)$  intersect are the solutions of the equation  $f(x) = (x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

## Unit 9: Solving Two-Variable Linear Systems

- **SOLVING SYSTEMS OF LINEAR EQUATIONS: SUBSTITUTION**
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

- NY.CCLS.Math.9-12.A-REI.6: Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- NY.CCLS.Math.9-12.A-REI.5: Reasoning with Equations and Inequalities Solve systems of equations Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- **SOLVING SYSTEMS OF LINEAR EQUATIONS: ELIMINATION**
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
  - NY.CCLS.Math.9-12.A-REI.6: Reasoning with Equations and Inequalities Solve systems of equations Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
  - NY.CCLS.Math.9-12.A-REI.5: Reasoning with Equations and Inequalities Solve systems of equations Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

## Unit 10: Linear Systems

- **SOLVING SYSTEMS OF LINEAR INEQUALITIES**
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
  - NY.CCLS.Math.9-12.A-REI.12: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
- **SOLVING THREE-VARIABLE SYSTEMS OF LINEAR EQUATIONS**
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

- NY.CCLS.Math.9-12.A-REI.5: Reasoning with Equations and Inequalities Solve systems of equations Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

## Unit 11: Exponential Functions, Equations, and Inequalities

### • EXPONENTIAL FUNCTIONS

- NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.F-IF.8.b: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
- NY.CCLS.Math.9-12.F-LE.1.a: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- NY.CCLS.Math.9-12.F-IF.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- NY.CCLS.Math.9-12.F-LE.3: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function



and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- NY.CCLS.Math.9-12.F-IF.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- NY.CCLS.Math.9-12.A-SSE.3.c: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.
- NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.A-REI.3: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- NY.CCLS.Math.9-12.F-LE.1.c: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- NY.CCLS.Math.9-12.F-LE.5: Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Interpret the parameters in a linear or exponential function in terms of a context.

- **EXPONENTIAL GROWTH AND DECAY**

- NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- NY.CCLS.Math.9-12.A-SSE.3.c: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.
- NY.CCLS.Math.9-12.F-IF.8.b: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
- NY.CCLS.Math.9-12.F-LE.1.a: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- NY.CCLS.Math.9-12.F-LE.1.b: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- NY.CCLS.Math.9-12.F-LE.1.c: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- NY.CCLS.Math.9-12.F-LE.3: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- NY.CCLS.Math.9-12.F-LE.5: Linear, Quadratic, and Exponential Models Interpret expressions for functions in terms of the situation they model Interpret the parameters in a linear or exponential function in terms of a context.

- NY.CCLS.Math.9-12.F-IF.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.F-IF.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **SOLVING EXPONENTIAL INEQUALITIES**
  - NY.CCLS.Math.9-12.F-IF.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
  - NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
  - NY.CCLS.Math.9-12.F-IF.8.b: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
  - NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- NY.CCLS.Math.9-12.F-LE.1.c: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

## Unit 12: Sequences

### • SEQUENCES

- NY.CCLS.Math.9-12.F-IF.3: Interpreting Functions Understand the concept of a function and use function notation Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- NY.CCLS.Math.9-12.F-BF.2: Building Functions Build a function that models a relationship between two quantities Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

### • ARITHMETIC AND GEOMETRIC SEQUENCES

- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- NY.CCLS.Math.9-12.F-BF.2: Building Functions Build a function that models a relationship between two quantities Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- NY.CCLS.Math.9-12.F-IF.3: Interpreting Functions Understand the concept of a function and use function notation Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

## Unit 13: Polynomials

### • POLYNOMIAL BASICS

- NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **ADDITION AND SUBTRACTION OF POLYNOMIALS**
  - NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - NY.CCLS.Math.9-12.A-APR.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- **MULTIPLICATION OF POLYNOMIALS**
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - NY.CCLS.Math.9-12.A-APR.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

## Unit 14: Factoring

- **FACTORING QUADRATIC TRINOMIALS**
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - NY.CCLS.Math.9-12.A-SSE.3.a: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
  - NY.CCLS.Math.9-12.A-APR.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
  - NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by

inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .

- NY.CCLS.Math.9-12.F-IF.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- NY.CCLS.Math.9-12.A-APR.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- **FACTORING SPECIAL CASES**
  - NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - NY.CCLS.Math.9-12.A-APR.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
  - NY.CCLS.Math.9-12.A-APR.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable

factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- NY.CCLS.Math.9-12.F-IF.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .

## Unit 15: Graphs of Quadratic Functions

### • QUADRATIC FUNCTIONS

- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- NY.CCLS.Math.9-12.F-IF.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- NY.CCLS.Math.9-12.F-IF.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

- NY.CCLS.Math.9-12.A-SSE.3.b: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- NY.CCLS.Math.9-12.F-IF.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **ANALYZING GRAPHS OF QUADRATIC FUNCTIONS**
  - NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - NY.CCLS.Math.9-12.F-IF.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - NY.CCLS.Math.9-12.F-IF.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.



- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- NY.CCLS.Math.9-12.A-REI.10: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.A-APR.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- NY.CCLS.Math.9-12.F-IF.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- **REPRESENTATIONS OF QUADRATIC FUNCTIONS**
  - NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
  - NY.CCLS.Math.9-12.A-SSE.3.a: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
  - NY.CCLS.Math.9-12.A-CED.1: Creating Equations Create equations that describe numbers or relationships Create equations and inequalities in one variable and use them to solve problems.
  - NY.CCLS.Math.9-12.A-REI.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p) = q$  that has the same solutions. Derive the quadratic formula from this form.

- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- NY.CCLS.Math.9-12.F-IF.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.F-IF.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- NY.CCLS.Math.9-12.A-APR.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- NY.CCLS.Math.9-12.F-IF.7.c: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

- NY.CCLS.Math.9-12.A-CED.3: Creating Equations Create equations that describe numbers or relationships Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

## Unit 16: Solving Quadratic Functions

### • SOLVING QUADRATIC EQUATIONS BY FACTORING

- NY.CCLS.Math.9-12.A-SSE.3.a: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
- NY.CCLS.Math.9-12.A-APR.3: Arithmetic with Polynomials and Rational Expressions Understand the relationship between zeros and factors of polynomials Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- NY.CCLS.Math.9-12.F-IF.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- NY.CCLS.Math.9-12.A-APR.4: Arithmetic with Polynomials and Rational Expressions Use polynomial identities to solve problems Prove polynomial identities and use them to describe numerical relationships.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and

factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .

- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-IF.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- NY.CCLS.Math.9-12.A-APR.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- **COMPLETING THE SQUARE**
  - NY.CCLS.Math.9-12.A-SSE.3.b: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain

properties of the quantity represented by the expression. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

- NY.CCLS.Math.9-12.A-REI.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p) = q$  that has the same solutions. Derive the quadratic formula from this form.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- NY.CCLS.Math.9-12.F-IF.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- NY.CCLS.Math.9-12.F-IF.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- NY.CCLS.Math.9-12.A-SSE.2: Seeing Structure in Expressions Interpret the structure of expressions Use the structure of an expression to identify ways to rewrite it.
- NY.CCLS.Math.9-12.A-REI.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p) = q$  that has the same solutions. Derive the quadratic formula from this form.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- NY.CCLS.Math.9-12.A-REI.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p) = q$  that has the same solutions. Derive the quadratic formula from this form.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .

- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- **QUADRATIC FORMULA**
- NY.CCLS.Math.9-12.A-SSE.1.a: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- NY.CCLS.Math.9-12.A-SSE.1.b: Seeing Structure in Expressions Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.A-REI.4.a: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x + p) = q$  that has the same solutions. Derive the quadratic formula from this form.
- NY.CCLS.Math.9-12.A-REI.4.b: Reasoning with Equations and Inequalities Solve equations and inequalities in one variable Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for  $x = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .

- NY.CCLS.Math.9-12.A-SSE.3.a: Seeing Structure in Expressions Write expressions in equivalent forms to solve problems Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
- NY.CCLS.Math.9-12.F-IF.8.a: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- NY.CCLS.Math.9-12.F-BF.1.a: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

## Unit 17: Parent Functions

### • LINEAR AND EXPONENTIAL PARENT FUNCTIONS

- NY.CCLS.Math.9-12.A-REI.10: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- NY.CCLS.Math.9-12.F-LE.1.a: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- NY.CCLS.Math.9-12.F-IF.7.e: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- NY.CCLS.Math.9-12.F-IF.8.b: Interpreting Functions Analyze functions using different representations Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-LE.1.c: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- **QUADRATIC PARENT FUNCTION**
  - NY.CCLS.Math.9-12.A-REI.10: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
  - NY.CCLS.Math.9-12.F-IF.7.a: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

## Unit 18: Transformations of Parent Functions

- **TRANSFORMATIONS OF THE LINEAR AND EXPONENTIAL PARENT FUNCTIONS**
  - NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both



positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

- NY.CCLS.Math.9-12.G-CO.2: Congruence Experiment with transformations in the plane Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- NY.CCLS.Math.9-12.G-CO.6: Congruence Understand congruence in terms of rigid motions Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- **TRANSFORMATIONS OF THE QUADRATIC PARENT FUNCTION**
  - NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
  - NY.CCLS.Math.9-12.G-CO.2: Congruence Experiment with transformations in the plane Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
  - NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
  - NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both

positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

- NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

## Unit 19: Nonlinear Functions

### • LINEAR VERSUS NONLINEAR FUNCTIONS

- NY.CCLS.Math.9-12.F-IF.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- NY.CCLS.Math.9-12.F-LE.1.a: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- NY.CCLS.Math.9-12.F-LE.1.b: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- NY.CCLS.Math.9-12.F-LE.1.c: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features

of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- NY.CCLS.Math.9-12.F-IF.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- NY.CCLS.Math.9-12.F-IF.7.b: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **ABSOLUTE VALUE FUNCTIONS**
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - NY.CCLS.Math.9-12.F-IF.5: Interpreting Functions Interpret functions that arise in applications in terms of the context Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
  - NY.CCLS.Math.9-12.F-IF.7.b: Interpreting Functions Analyze functions using different representations Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
  - NY.CCLS.Math.9-12.F-IF.1: Interpreting Functions Understand the concept of a function and use function notation Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - NY.CCLS.Math.9-12.F-BF.3: Building Functions Build new functions from existing functions Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

## Unit 20: Working with Functions

- **ARITHMETIC OPERATIONS ON FUNCTIONS**
  - NY.CCLS.Math.9-12.F-BF.1.b: Building Functions Build a function that models a relationship between two quantities Write a function that describes a relationship between two quantities. Combine

standard function types using arithmetic operations.

- NY.CCLS.Math.9-12.A-APR.1: Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**

- NY.CCLS.Math.9-12.F-IF.4: Interpreting Functions Interpret functions that arise in applications in terms of the context For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- NY.CCLS.Math.9-12.F-IF.9: Interpreting Functions Analyze functions using different representations Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- NY.CCLS.Math.9-12.F-LE.2: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- NY.CCLS.Math.9-12.A-CED.2: Creating Equations Create equations that describe numbers or relationships Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- NY.CCLS.Math.9-12.F-LE.1.a: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

## Unit 21: Statistics

- **DATA ANALYSIS**

- NY.CCLS.Math.9-12.S-ID.1: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable Represent data with plots on the real number line (dot plots, histograms, and box plots).
- NY.CCLS.Math.9-12.S-ID.2: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- NY.CCLS.Math.9-12.S-ID.3: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

- **FREQUENCY TABLES**

- NY.CCLS.Math.9-12.S-ID.5: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
- NY.CCLS.Math.9-12.S-CP.4: Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
- NY.CCLS.Math.9-12.S-ID.3: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on a single count or measurement variable Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

## Unit 22: Scatterplots and Regression

### • SCATTERPLOTS

- NY.CCLS.Math.9-12.S-ID.6.a: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
- NY.CCLS.Math.9-12.S-ID.6.b: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Informally assess the fit of a function by plotting and analyzing residuals.
- NY.CCLS.Math.9-12.S-ID.6.c: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a linear function for a scatter plot that suggests a linear association.
- NY.CCLS.Math.9-12.S-ID.9: Interpreting Categorical and Quantitative Data Interpret linear models Distinguish between correlation and causation.
- NY.CCLS.Math.9-12.F-IF.6: Interpreting Functions Interpret functions that arise in applications in terms of the context Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- NY.CCLS.Math.9-12.S-ID.7: Interpreting Categorical and Quantitative Data Interpret linear models Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

### • SCATTERPLOTS AND MODELING

- NY.CCLS.Math.9-12.S-ID.6.a: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative

variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.

- NY.CCLS.Math.9-12.S-ID.6.b: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Informally assess the fit of a function by plotting and analyzing residuals.
- NY.CCLS.Math.9-12.S-ID.7: Interpreting Categorical and Quantitative Data Interpret linear models Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- NY.CCLS.Math.9-12.S-ID.8: Interpreting Categorical and Quantitative Data Interpret linear models Compute (using technology) and interpret the correlation coefficient of a linear fit.
- NY.CCLS.Math.9-12.S-ID.6.c: Interpreting Categorical and Quantitative Data Summarize, represent, and interpret data on two categorical and quantitative variables Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a linear function for a scatter plot that suggests a linear association.
- NY.CCLS.Math.9-12.F-LE.1.a: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- NY.CCLS.Math.9-12.F-LE.1.c: Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.