

Ohio Tutorials are designed specifically for the Ohio Learning Standards to prepare students for the Ohio State Tests and end-of-course exams.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

## Unit 1: Real Number System

### • LAWS OF EXPONENTS

- OH.Math.HSA.SSE.2: Use the structure of an expression to identify ways to rewrite it.
- OH.Math.HSA.APR.6: Rewrite simple rational expressions in different forms; write  $\frac{p}{q}$  in the form  $\frac{a}{b} + \frac{c}{d}$ , where  $p$ ,  $q$ ,  $a$ ,  $b$ ,  $c$ , and  $d$  are polynomials with the degree of  $p$  less than the degree of  $q$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
- OH.Math.HSA.REI.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- OH.Math.HSA.SSE.3c: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.
- OH.Math.HSN.RN.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
- OH.Math.HSN.RN.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.

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- **OPERATIONS ON RATIONAL AND IRRATIONAL NUMBERS**

- OH.Math.HSN.RN.3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

## Unit 2: Equations and Inequalities

- **FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS**

- OH.Math.HSA.SSE.1a: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- OH.Math.HSA.SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- OH.Math.HSA.SSE.2: Use the structure of an expression to identify ways to rewrite it.
- OH.Math.HSA.APR.1b: Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic.

- **FORMULATING AND SOLVING EQUATIONS FROM WORD PROBLEMS**

- OH.Math.HSA.SSE.1a: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- OH.Math.HSF.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- OH.Math.HSF.LE.1b: Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- OH.Math.HSA.CED.1a: Create equations and inequalities in one variable and use them to solve problems. Focus on applying linear and simple exponential expressions.
- OH.Math.HSA.CED.1b: Create equations and inequalities in one variable and use them to solve problems. Focus on applying simple quadratic expressions.
- OH.Math.HSA.CED.2a: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying linear and simple exponential expressions.
- OH.Math.HSF.IF.4a: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear and exponential functions.
- OH.Math.HSF.IF.4b: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear, quadratic, and exponential functions.

- OH.Math.HSF.BF.1a.i: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from context. Focus on linear and exponential functions.
- OH.Math.HSA.CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- OH.Math.HSA.CED.3a: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
- OH.Math.HSA.REI.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- OH.Math.HSA.CED.1c: Create equations and inequalities in one variable and use them to solve problems. Extend to include more complicated function situations with the option to solve with technology.
- OH.Math.HSA.CED.1: Create equations and inequalities in one variable and use them to solve problems.
- **FORMULATING AND SOLVING INEQUALITIES FROM WORD PROBLEMS**
  - OH.Math.HSA.CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
  - OH.Math.HSA.REI.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
  - OH.Math.HSA.CED.1a: Create equations and inequalities in one variable and use them to solve problems. Focus on applying linear and simple exponential expressions.
  - OH.Math.HSA.CED.1b: Create equations and inequalities in one variable and use them to solve problems. Focus on applying simple quadratic expressions.
  - OH.Math.HSA.CED.1c: Create equations and inequalities in one variable and use them to solve problems. Extend to include more complicated function situations with the option to solve with technology.
  - OH.Math.HSA.CED.3a: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
  - OH.Math.HSA.SSE.1a: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.

### Unit 3: Applications of Equations and Inequalities

- **LITERAL EQUATIONS**
  - OH.Math.HSA.CED.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

- OH.Math.HSA.REI.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- OH.Math.HSA.CED.1a: Create equations and inequalities in one variable and use them to solve problems. Focus on applying linear and simple exponential expressions.
- OH.Math.HSA.CED.4b: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. Focus on formulas in which the variable of interest is linear.
- OH.Math.HSA.CED.4c: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. Focus on formulas in which the variable of interest is linear or square.
- OH.Math.HSA.CED.4d: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
- OH.Math.HSA.SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- OH.Math.HSA.CED.4a: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. Focus on formulas in which the variable of interest is linear or square.
- **SUMS OF GEOMETRIC SEQUENCES**
  - OH.Math.HSA.SSE.4: Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

#### Unit 4: Functions

- **FUNCTIONS AND RELATIONS**
  - OH.Math.HSF.IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
  - OH.Math.HSF.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - OH.Math.HSF.IF.4b: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear, quadratic, and exponential functions.
  - OH.Math.HSF.IF.5c: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Emphasize the selection of a type of function for a model based on behavior of data and context.
  - OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
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- OH.Math.HSF.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- OH.Math.HSF.IF.7c: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **DOMAIN AND RANGE**
  - OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - OH.Math.HSF.IF.5c: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Emphasize the selection of a type of function for a model based on behavior of data and context.
  - OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - OH.Math.HSF.IF.5b: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on linear, quadratic, and exponential functions.
  - OH.Math.HSF.IF.5a: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on linear and exponential functions.
  - OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function

and is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

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- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **INVERSE FUNCTIONS**
  - OH.Math.HSF.BF.4b: Find inverse functions. Read values of an inverse function from a graph or a table, given that the function has an inverse.
  - OH.Math.HSF.BF.4c: Find inverse functions. Verify by composition that one function is the inverse of another.
  - OH.Math.HSF.BF.4d: Find inverse functions. Find the inverse of a function algebraically, given that the function has an inverse.
  - OH.Math.HSF.BF.4a: Find inverse functions. Informally determine the input of a function when the output is known.
  - OH.Math.HSF.BF.4e: Find inverse functions. Produce an invertible function from a non-invertible function by restricting the domain.

## Unit 5: Linear Functions

- **SLOPE**
  - OH.Math.HSF.IF.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
  - OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - OH.Math.HSG.GPE.5: Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.
  - OH.Math.HSA.CED.2a: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying linear and

simple exponential expressions.

- OH.Math.HSF.IF.4b: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear, quadratic, and exponential functions.

- **GRAPHING AND ANALYZING LINEAR FUNCTIONS**

- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- OH.Math.HSF.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- OH.Math.HSF.IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- OH.Math.HSF.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- OH.Math.HSF.IF.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- OH.Math.HSF.LE.5: Interpret the parameters in a linear or exponential function in terms of a context.
- OH.Math.HSF.IF.7a: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph linear functions and indicate intercepts.
- OH.Math.HSA.CED.2a: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying linear and simple exponential expressions.
- OH.Math.HSF.BF.1a.i: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from context. Focus on linear and exponential functions.
- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- OH.Math.HSF.IF.4a: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear and exponential functions.



- OH.Math.HSF.IF.5b: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on linear, quadratic, and exponential functions.
- OH.Math.HSF.IF.5a: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on linear and exponential functions.
- OH.Math.HSA.REI.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- OH.Math.HSF.IF.4b: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear, quadratic, and exponential functions.
- **GRAPHING AND MANIPULATING  $Y = MX + B$** 
  - OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - OH.Math.HSF.IF.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
  - OH.Math.HSF.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
  - OH.Math.HSF.IF.7a: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph linear functions and indicate intercepts.
  - OH.Math.HSA.CED.2a: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying linear and simple exponential expressions.
  - OH.Math.HSF.IF.4b: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear, quadratic, and exponential functions.
  - OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function



and is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- OH.Math.HSF.LE.5: Interpret the parameters in a linear or exponential function in terms of a context.
- OH.Math.HSS.ID.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- OH.Math.HSA.REI.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- OH.Math.HSF.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- OH.Math.HSF.IF.4a: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear and exponential functions.
- OH.Math.HSF.IF.5a: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on linear and exponential functions.
- OH.Math.HSF.LE.1b: Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- OH.Math.HSF.BF.1a.i: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from context. Focus on linear and exponential functions.
- OH.Math.HSA.CED.2b: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying simple quadratic expressions.

## Unit 6: Exponential Functions

### • EXPONENTIAL FUNCTIONS

- OH.Math.HSA.SSE.1a: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- OH.Math.HSA.SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- OH.Math.HSA.SSE.3c: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.

- OH.Math.HSF.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- OH.Math.HSF.LE.5: Interpret the parameters in a linear or exponential function in terms of a context.
- OH.Math.HSF.IF.7e: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph simple exponential functions, indicating intercepts and end behavior.
- OH.Math.HSF.IF.7f: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- OH.Math.HSF.LE.1a: Distinguish between situations that can be modeled with linear functions and with exponential functions. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
- OH.Math.HSF.IF.4a: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear and exponential functions.
- OH.Math.HSF.IF.8b.i: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions. Focus on exponential functions evaluated at integer inputs.
- OH.Math.HSF.IF.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- OH.Math.HSF.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- OH.Math.HSF.LE.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.
- OH.Math.HSF.IF.8b: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
- OH.Math.HSF.LE.1c: Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- OH.Math.HSF.IF.4b: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear, quadratic, and exponential functions.

- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- OH.Math.HSF.IF.5b: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on linear, quadratic, and exponential functions.
- OH.Math.HSF.IF.5a: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on linear and exponential functions.
- OH.Math.HSA.CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- OH.Math.HSA.REI.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- OH.Math.HSA.REI.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- OH.Math.HSF.IF.7g: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior.
- OH.Math.HSA.CED.1a: Create equations and inequalities in one variable and use them to solve problems. Focus on applying linear and simple exponential expressions.
- OH.Math.HSA.CED.1c: Create equations and inequalities in one variable and use them to solve problems. Extend to include more complicated function situations with the option to solve with technology.
- OH.Math.HSA.CED.2a: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying linear and simple exponential expressions.
- OH.Math.HSA.CED.2c: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Extend to include more complicated function situations with the option to graph with technology.
- OH.Math.HSF.BF.1a.i: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from context. Focus on linear and exponential functions.

- OH.Math.HSF.BF.1a.ii: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from context. Focus on situations that exhibit quadratic or exponential relationships.
- **EXPONENTIAL GROWTH AND DECAY**
  - OH.Math.HSA.CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - OH.Math.HSA.SSE.1a: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - OH.Math.HSA.SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - OH.Math.HSF.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
  - OH.Math.HSF.LE.5: Interpret the parameters in a linear or exponential function in terms of a context.
  - OH.Math.HSF.IF.8b: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.
  - OH.Math.HSF.LE.1a: Distinguish between situations that can be modeled with linear functions and with exponential functions. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
  - OH.Math.HSF.LE.1c: Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
  - OH.Math.HSA.CED.2a: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying linear and simple exponential expressions.
  - OH.Math.HSF.IF.8b.i: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions. Focus on exponential functions evaluated at integer inputs.
  - OH.Math.HSA.SSE.3c: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.
  - OH.Math.HSA.REI.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
  - OH.Math.HSF.LE.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.
  - OH.Math.HSF.LE.1b: Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per

unit interval relative to another.

- OH.Math.HSF.IF.4a: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear and exponential functions.
- OH.Math.HSF.IF.5a: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on linear and exponential functions.
- OH.Math.HSA.CED.2c: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Extend to include more complicated function situations with the option to graph with technology.
- OH.Math.HSF.IF.7e: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph simple exponential functions, indicating intercepts and end behavior.
- OH.Math.HSF.IF.7f: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- OH.Math.HSA.CED.2b: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying simple quadratic expressions.
- OH.Math.HSF.BF.1a.i: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from context. Focus on linear and exponential functions.
- OH.Math.HSF.BF.1a.ii: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from context. Focus on situations that exhibit quadratic or exponential relationships.

## Unit 7: Logarithmic Expressions and Functions

### • EVALUATING LOGARITHMIC EXPRESSIONS

- OH.Math.HSA.SSE.1a: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- OH.Math.HSA.SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- OH.Math.HSF.LE.4: For exponential models, express as a logarithm the solution to  $ay = b$  where  $a$ ,  $b$ , and  $y$  are numbers and the base is 2, 10, or  $e$ ; evaluate the logarithm using technology.

### • LOGARITHMIC FUNCTIONS

- OH.Math.HSF.IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

- OH.Math.HSF.IF.7f: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- OH.Math.HSF.BF.4b: Find inverse functions. Read values of an inverse function from a graph or a table, given that the function has an inverse.
- OH.Math.HSF.BF.4d: Find inverse functions. Find the inverse of a function algebraically, given that the function has an inverse.
- OH.Math.HSF.BF.5: Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
- OH.Math.HSF.LE.4: For exponential models, express as a logarithm the solution to  $ay = b$  where  $a$ ,  $b$ , and  $y$  are numbers and the base is 2, 10, or  $e$ ; evaluate the logarithm using technology.
- OH.Math.HSF.IF.4a: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear and exponential functions.
- OH.Math.HSF.IF.4b: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear, quadratic, and exponential functions.
- OH.Math.HSF.IF.5c: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Emphasize the selection of a type of function for a model based on behavior of data and context.
- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- OH.Math.HSF.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- OH.Math.HSF.IF.7h: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph logarithmic functions, indicating intercepts and end behavior.

## Unit 8: Solving Exponential and Logarithmic Equations and Inequalities

### • SOLVING EXPONENTIAL EQUATIONS

- OH.Math.HSA.SSE.3c: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.



- OH.Math.HSF.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- OH.Math.HSF.IF.7f: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- OH.Math.HSF.LE.4: For exponential models, express as a logarithm the solution to  $ax = by + c$  where  $a$ ,  $b$ , and  $c$  are numbers and the base is 2, 10, or  $e$ ; evaluate the logarithm using technology.
- OH.Math.HSA.CED.1c: Create equations and inequalities in one variable and use them to solve problems. Extend to include more complicated function situations with the option to solve with technology.
- OH.Math.HSA.CED.2a: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying linear and simple exponential expressions.
- OH.Math.HSA.CED.2c: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Extend to include more complicated function situations with the option to graph with technology.
- OH.Math.HSF.IF.4a: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear and exponential functions.
- OH.Math.HSF.IF.4b: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear, quadratic, and exponential functions.
- OH.Math.HSF.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- OH.Math.HSF.BF.1a.i: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from context. Focus on linear and exponential functions.
- OH.Math.HSF.BF.1a.ii: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from context. Focus on situations that exhibit quadratic or exponential relationships.
- **SOLVING EXPONENTIAL INEQUALITIES**
  - OH.Math.HSA.CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - OH.Math.HSF.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading



these from a table).

- OH.Math.HSA.CED.2a: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying linear and simple exponential expressions.
- OH.Math.HSA.CED.2c: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Extend to include more complicated function situations with the option to graph with technology.
- OH.Math.HSA.CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- OH.Math.HSA.SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- OH.Math.HSA.CED.3a: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
- OH.Math.HSA.CED.4d: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
- OH.Math.HSF.LE.1c: Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- OH.Math.HSA.CED.2b: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying simple quadratic expressions.
- **SOLVING LOGARITHMIC EQUATIONS**
  - OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - OH.Math.HSF.LE.4: For exponential models, express  $y = ab^{cx+d}$  as a logarithm the solution to  $y = a \cdot b^{cx+d}$  to the power  $=$  where  $a$ ,  $b$ ,  $c$ , and  $d$  are numbers and the base is 2, 10, or  $e$ ; evaluate the logarithm using technology.
  - OH.Math.HSA.CED.2c: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Extend to include more complicated function situations with the option to graph with technology.
  - OH.Math.HSA.REI.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

- OH.Math.HSF.LE.4: For exponential models, express as a logarithm the solution to to the power = where , , and are numbers and the base is 2, 10, or ; evaluate the logarithm using technology.

### Unit 9: Arithmetic with Polynomials 1

#### • POLYNOMIAL BASICS

- OH.Math.HSA.SSE.1a: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- OH.Math.HSA.SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- OH.Math.HSA.SSE.2: Use the structure of an expression to identify ways to rewrite it.
- OH.Math.HSA.APR.1: Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- OH.Math.HSA.APR.1b: Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic.

#### • ADDITION AND SUBTRACTION OF POLYNOMIALS

- OH.Math.HSA.APR.1b: Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic.
- OH.Math.HSA.APR.1a: Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Focus on polynomial expressions that simplify to forms that are linear or quadratic.
- OH.Math.HSA.SSE.2: Use the structure of an expression to identify ways to rewrite it.

### Unit 10: Arithmetic with Polynomials 2

#### • MULTIPLICATION OF POLYNOMIALS

- OH.Math.HSA.APR.1b: Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic.
- OH.Math.HSA.APR.1a: Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Focus on polynomial expressions that simplify to forms that are linear or quadratic.
- OH.Math.HSA.SSE.2: Use the structure of an expression to identify ways to rewrite it.

- **DIVISION OF POLYNOMIALS**

- OH.Math.HSA.SSE.2: Use the structure of an expression to identify ways to rewrite it.
- OH.Math.HSA.APR.1b: Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic.
- OH.Math.HSA.APR.6: Rewrite simple rational expressions in different forms; write  $\frac{p(x)}{q(x)}$  in the form  $\frac{r(x)}{s(x)} + \frac{t(x)}{q(x)}$ , where  $r(x)$ ,  $t(x)$ ,  $s(x)$ , and  $q(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $q(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
- OH.Math.HSA.APR.6: Rewrite simple rational expressions in different forms; write  $\frac{p(x)}{q(x)}$  in the form  $\frac{r(x)}{s(x)} + \frac{t(x)}{q(x)}$ , where  $r(x)$ ,  $t(x)$ ,  $s(x)$ , and  $q(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $q(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
- OH.Math.HSA.SSE.1a: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- OH.Math.HSA.SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.

## Unit 11: Graphs and Representations of Quadratic Functions

- **ANALYZING GRAPHS OF QUADRATIC FUNCTIONS**

- OH.Math.HSA.SSE.2: Use the structure of an expression to identify ways to rewrite it.
- OH.Math.HSF.IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- OH.Math.HSF.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- OH.Math.HSF.IF.4b: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear, quadratic, and exponential functions.
- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- OH.Math.HSF.IF.5b: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on linear, quadratic, and exponential functions.
- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- OH.Math.HSF.IF.7b: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph quadratic functions and indicate intercepts, maxima, and minima.
- OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f\left(\frac{x}{k}\right)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $f(x) = x^2$ .
- OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f\left(\frac{x}{k}\right)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $f(x) = x^2$ .
- OH.Math.HSA.REI.4b: Solve quadratic equations in one variable. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x^2 = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- OH.Math.HSF.IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- OH.Math.HSA.APR.3: Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **REPRESENTATIONS OF QUADRATIC FUNCTIONS**
  - OH.Math.HSA.SSE.2: Use the structure of an expression to identify ways to rewrite it.
  - OH.Math.HSA.REI.4a: Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $ax^2 + bx + c = 0$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions.
  - OH.Math.HSA.REI.4c: Solve quadratic equations in one variable. Derive the quadratic formula using the method of completing the square.
  - OH.Math.HSA.CED.1c: Create equations and inequalities in one variable and use them to solve problems. Extend to include more complicated function situations with the option to solve with technology.
  - OH.Math.HSA.CED.2b: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying simple quadratic expressions.

- OH.Math.HSA.CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- OH.Math.HSA.CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- OH.Math.HSF.IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- OH.Math.HSF.IF.8a: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- OH.Math.HSA.CED.2c: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Extend to include more complicated function situations with the option to graph with technology.
- OH.Math.HSF.IF.4b: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear, quadratic, and exponential functions.
- OH.Math.HSF.IF.8a.i: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Focus on completing the square to quadratic functions with the leading coefficient of 1.
- OH.Math.HSF.BF.1a.ii: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from context. Focus on situations that exhibit quadratic or exponential relationships.
- OH.Math.HSF.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- OH.Math.HSA.CED.2a: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying linear and simple exponential expressions.
- OH.Math.HSA.SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- OH.Math.HSA.SSE.3a: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.

- OH.Math.HSA.REI.4a: Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in into an equation of the form  $(x - h)^2 = k$  that has the same solutions.
- OH.Math.HSF.BF.3: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f(x/k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f(x/k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $f(x) = x^2$ .
- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**
  - OH.Math.HSA.CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - OH.Math.HSF.IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
  - OH.Math.HSF.IF.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
  - OH.Math.HSF.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
  - OH.Math.HSA.CED.2c: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Extend to include more complicated function situations with the option to graph with technology.
  - OH.Math.HSF.IF.4a: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear and exponential functions.
  - OH.Math.HSF.IF.4b: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear, quadratic, and exponential functions.
  - OH.Math.HSF.IF.9a: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Focus on linear and exponential functions.
  - OH.Math.HSF.IF.9b: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Focus on linear, quadratic, and exponential functions.
  - OH.Math.HSA.CED.2a: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying linear and



simple exponential expressions.

- OH.Math.HSA.CED.2b: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying simple quadratic expressions.
- OH.Math.HSF.LE.1a: Distinguish between situations that can be modeled with linear functions and with exponential functions. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

## Unit 12: Solving Quadratic Equations

### • SOLVING QUADRATIC EQUATIONS BY FACTORING

- OH.Math.HSA.APR.3: Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.
- OH.Math.HSA.SSE.3a: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
- OH.Math.HSA.REI.4b: Solve quadratic equations in one variable. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- OH.Math.HSA.APR.4: Prove polynomial identities and use them to describe numerical relationships.
- OH.Math.HSA.REI.4b: Solve quadratic equations in one variable. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- OH.Math.HSA.REI.4b: Solve quadratic equations in one variable. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- OH.Math.HSF.IF.8a.i: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Focus on completing the square to quadratic functions with the leading coefficient of 1.
- OH.Math.HSA.REI.4b: Solve quadratic equations in one variable. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- OH.Math.HSA.REI.4b: Solve quadratic equations in one variable. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x = 49$ ; taking square roots;



completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.

- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- OH.Math.HSF.IF.7b: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph quadratic functions and indicate intercepts, maxima, and minima.
- OH.Math.HSF.IF.8a: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- OH.Math.HSA.REI.4b: Solve quadratic equations in one variable. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x^2 = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- OH.Math.HSA.CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- OH.Math.HSA.CED.2b: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying simple quadratic expressions.
- OH.Math.HSA.CED.2c: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Extend to include more complicated function situations with the option to graph with technology.
- OH.Math.HSF.BF.1a.ii: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from context. Focus on situations that exhibit quadratic or exponential relationships.
- **COMPLETING THE SQUARE**
  - OH.Math.HSA.SSE.3b: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
  - OH.Math.HSA.REI.4a: Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $ax^2 + bx + c = 0$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions.
  - OH.Math.HSA.REI.4b: Solve quadratic equations in one variable. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x^2 = 49$ ; taking square roots;

completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.

- OH.Math.HSF.IF.8a: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- OH.Math.HSF.IF.8a.i: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Focus on completing the square to quadratic functions with the leading coefficient of 1.
- OH.Math.HSA.SSE.2: Use the structure of an expression to identify ways to rewrite it.
- OH.Math.HSA.REI.4a: Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in into an equation of the form  $( ) =$  that has the same solutions.
- OH.Math.HSA.REI.4b: Solve quadratic equations in one variable. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $= 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- OH.Math.HSA.CED.2b: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying simple quadratic expressions.
- OH.Math.HSA.REI.4a: Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in into an equation of the form  $( ) =$  that has the same solutions.
- OH.Math.HSA.REI.4b: Solve quadratic equations in one variable. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $= 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- OH.Math.HSA.REI.4c: Solve quadratic equations in one variable. Derive the quadratic formula using the method of completing the square.
- OH.Math.HSF.IF.7b: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph quadratic functions and indicate intercepts, maxima, and minima.
- OH.Math.HSA.REI.4a: Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in into an equation of the form  $( ) =$  that has the same solutions.

### Unit 13: Factoring Polynomials

- **FACTORING SPECIAL CASES**

- OH.Math.HSA.SSE.2: Use the structure of an expression to identify ways to rewrite it.
- OH.Math.HSA.SSE.1a: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- OH.Math.HSA.SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- OH.Math.HSA.SSE.3a: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
- OH.Math.HSA.APR.3: Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.
- OH.Math.HSA.APR.4: Prove polynomial identities and use them to describe numerical relationships.

- **FACTORING CUBIC POLYNOMIALS**

- OH.Math.HSA.SSE.2: Use the structure of an expression to identify ways to rewrite it.
- OH.Math.HSA.APR.3: Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.
- OH.Math.HSA.APR.4: Prove polynomial identities and use them to describe numerical relationships.
- OH.Math.HSA.SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.

#### Unit 14: Factoring Higher-Order Polynomials

- **FACTORING HIGHER-ORDER POLYNOMIALS**

- OH.Math.HSA.SSE.2: Use the structure of an expression to identify ways to rewrite it.
- OH.Math.HSA.APR.3: Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.
- OH.Math.HSA.SSE.1a: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- OH.Math.HSA.SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- OH.Math.HSA.APR.4: Prove polynomial identities and use them to describe numerical relationships.
- OH.Math.HSA.APR.1b: Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic.

- **FACTOR THEOREM AND REMAINDER THEOREM**

- OH.Math.HSA.APR.3: Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.
- OH.Math.HSA.APR.2: Understand and apply the Remainder Theorem: For a polynomial  $f(x)$  and a number  $r$ , the remainder on division by  $(x - r)$  is  $f(r)$ . In particular,  $f(r) = 0$  if and only if  $(x - r)$  is a factor of  $f(x)$ .
- OH.Math.HSA.APR.6: Rewrite simple rational expressions in different forms; write  $\frac{p(x)}{q(x)}$  in the form  $\frac{a(x)}{b(x)} + \frac{r(x)}{b(x)}$ , where  $a(x)$ ,  $b(x)$ ,  $r(x)$ , and  $q(x)$  are polynomials with the degree of  $a(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
- OH.Math.HSF.IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- OH.Math.HSA.APR.2: Understand and apply the Remainder Theorem: For a polynomial  $f(x)$  and a number  $r$ , the remainder on division by  $(x - r)$  is  $f(r)$ . In particular,  $f(r) = 0$  if and only if  $(x - r)$  is a factor of  $f(x)$ .

### Unit 15: Polynomial Functions and Complex Numbers

#### • GRAPHS OF POLYNOMIAL FUNCTIONS

- OH.Math.HSA.APR.3: Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.
- OH.Math.HSF.BF.3: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f\left(\frac{x}{k}\right)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f\left(\frac{x}{k}\right)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $f(x) = x^2$ .
- OH.Math.HSF.IF.7d: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior.
- OH.Math.HSF.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

#### • COMPLEX NUMBERS

- OH.Math.HSN.CN.1: Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
- OH.Math.HSN.CN.2: Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

- OH.Math.HSN.CN.1: Know there is a complex number such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
- OH.Math.HSN.CN.2: Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

## Unit 16: Polynomial Identities and Complex Numbers

### • POLYNOMIAL IDENTITIES

- OH.Math.HSA.APR.4: Prove polynomial identities and use them to describe numerical relationships.
- OH.Math.HSA.REI.4a: Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions.
- OH.Math.HSA.REI.4b: Solve quadratic equations in one variable. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x^2 = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- OH.Math.HSA.REI.4c: Solve quadratic equations in one variable. Derive the quadratic formula using the method of completing the square.
- OH.Math.HSA.SSE.2: Use the structure of an expression to identify ways to rewrite it.
- OH.Math.HSA.REI.4b: Solve quadratic equations in one variable. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x^2 = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- OH.Math.HSA.APR.5: Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $a$  and  $b$  are any numbers.

### • POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS

- OH.Math.HSA.SSE.2: Use the structure of an expression to identify ways to rewrite it.
- OH.Math.HSA.SSE.1a: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- OH.Math.HSA.SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- OH.Math.HSN.CN.7: Solve quadratic equations with real coefficients that have complex solutions.
- OH.Math.HSN.CN.8: Extend polynomial identities to the complex numbers.
- OH.Math.HSA.REI.4b: Solve quadratic equations in one variable. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x^2 = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.

- OH.Math.HSN.CN.2: Use the relation  $= 1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- OH.Math.HSN.CN.9: Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

## Unit 17: Radical Functions and Equations

### • ANALYZING GRAPHS OF SQUARE ROOT FUNCTIONS

- OH.Math.HSF.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- OH.Math.HSG.CO.6: Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $(x)$  by  $(x) + h$ ,  $(x) - h$ ,  $(k)(x)$ , and  $(x) + k$  for specific values of  $h$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $(x)$ .
- OH.Math.HSF.IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- OH.Math.HSF.IF.4a: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear and exponential functions.
- OH.Math.HSF.IF.4b: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear, quadratic, and exponential functions.
- OH.Math.HSF.IF.5c: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Emphasize the selection of a type of function for a model based on behavior of data and context.
- OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $(x)$  by  $(x) + h$ ,  $(x) - h$ ,  $(k)(x)$ , and  $(x) + k$  for specific values of  $h$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $(x)$ .
- OH.Math.HSF.IF.7c: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.



- OH.Math.HSF.BF.4b: Find inverse functions. Read values of an inverse function from a graph or a table, given that the function has an inverse.
- OH.Math.HSF.BF.4d: Find inverse functions. Find the inverse of a function algebraically, given that the function has an inverse.
- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **SOLVING SQUARE ROOT EQUATIONS**
  - OH.Math.HSA.REI.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
  - OH.Math.HSA.REI.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
  - OH.Math.HSF.IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

## Unit 18: Rational Expressions and Equations

- **OPERATIONS WITH RATIONAL EXPRESSIONS**
  - OH.Math.HSA.SSE.1a: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - OH.Math.HSA.SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - OH.Math.HSA.APR.7: Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
  - OH.Math.HSA.SSE.2: Use the structure of an expression to identify ways to rewrite it.
  - OH.Math.HSA.APR.6: Rewrite simple rational expressions in different forms; write  $\frac{A(x)}{B(x)}$  in the form  $\frac{A(x)}{B(x)} + \frac{C(x)}{B(x)}$ , where  $A(x)$ ,  $C(x)$ , and  $B(x)$  are polynomials with the degree of  $C(x)$  less than the degree of  $B(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
- **SOLVING RATIONAL EQUATIONS**
  - OH.Math.HSA.REI.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
  - OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .



- OH.Math.HSA.REI.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- OH.Math.HSA.CED.3a: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.

## Unit 19: Rational Functions

### • ANALYZING GRAPHS OF RATIONAL FUNCTIONS

- OH.Math.HSF.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f(x/k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $f(x) = x^2$ .
- OH.Math.HSF.IF.7g: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior.
- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

### • MODELING SITUATIONS WITH RATIONAL FUNCTIONS

- OH.Math.HSA.CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- OH.Math.HSA.CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- OH.Math.HSA.SSE.1a: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- OH.Math.HSA.SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- OH.Math.HSF.IF.7g: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function

model appropriate. Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior.

- OH.Math.HSA.REI.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- OH.Math.HSA.CED.2c: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Extend to include more complicated function situations with the option to graph with technology.
- OH.Math.HSA.CED.3a: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
- OH.Math.HSF.IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

## Unit 20: Working with Functions

### • LINEAR VERSUS NONLINEAR FUNCTIONS

- OH.Math.HSF.IF.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- OH.Math.HSF.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- OH.Math.HSF.BF.1b: Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations.
- OH.Math.HSF.BF.1c: Write a function that describes a relationship between two quantities. Compose functions.
- OH.Math.HSF.LE.1a: Distinguish between situations that can be modeled with linear functions and with exponential functions. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
- OH.Math.HSF.LE.1b: Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- OH.Math.HSF.LE.1c: Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- OH.Math.HSF.IF.4a: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear and exponential functions.
- OH.Math.HSF.IF.4b: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features

given a verbal description of the relationship. Focus on linear, quadratic, and exponential functions.

- OH.Math.HSF.BF.1a.i: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from context. Focus on linear and exponential functions.
- OH.Math.HSA.REI.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- OH.Math.HSF.LE.5: Interpret the parameters in a linear or exponential function in terms of a context.
- OH.Math.HSF.IF.5a: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on linear and exponential functions.
- OH.Math.HSA.CED.2a: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying linear and simple exponential expressions.
- OH.Math.HSF.IF.5c: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Emphasize the selection of a type of function for a model based on behavior of data and context.
- OH.Math.HSF.IF.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- OH.Math.HSF.IF.9b: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Focus on linear, quadratic, and exponential functions.
- **ABSOLUTE VALUE FUNCTIONS**
  - OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
  - OH.Math.HSF.BF.3: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
  - OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $f(x) = x^2$ .
  - OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

- OH.Math.HSF.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- OH.Math.HSF.IF.7c: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- OH.Math.HSF.IF.4b: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear, quadratic, and exponential functions.
- **ARITHMETIC OPERATIONS ON FUNCTIONS**
- OH.Math.HSF.BF.1b: Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations.

## Unit 21: Parent Functions and Transformations

- **PARENT FUNCTIONS**
- OH.Math.HSF.IF.7c: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- OH.Math.HSF.IF.4b: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear, quadratic, and exponential functions.
- OH.Math.HSF.IF.5c: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Emphasize the selection of a type of function for a model based on behavior of data and context.
- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- OH.Math.HSF.IF.7g: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior.
- OH.Math.HSF.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- OH.Math.HSF.IF.7f: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- OH.Math.HSF.IF.4a: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear and exponential functions.
- OH.Math.HSF.IF.7h: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph logarithmic functions, indicating intercepts and end behavior.
- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- OH.Math.HSF.BF.3: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- OH.Math.HSF.IF.7d: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior.
- OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $f(x) = x^2$ .
- **TRANSFORMATIONS OF PARENT FUNCTIONS**
  - OH.Math.HSF.BF.3: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
  - OH.Math.HSG.CO.2: Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.

- OH.Math.HSF.BF.3: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f\left(\frac{x}{k}\right)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- OH.Math.HSG.CO.6: Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- OH.Math.HSG.SRT.1a: Verify experimentally the properties of dilations given by a center and a scale factor: A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged.
- OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f\left(\frac{x}{k}\right)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $f(x) = x^2$ .
- OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f\left(\frac{x}{k}\right)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $f(x) = x^2$ .
- OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f\left(\frac{x}{k}\right)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $f(x) = x^2$ .
- OH.Math.HSF.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- OH.Math.HSF.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- OH.Math.HSF.IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- OH.Math.HSF.IF.7c: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.



- OH.Math.HSF.IF.7f: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- OH.Math.HSF.IF.7d: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior.
- OH.Math.HSF.IF.4a: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear and exponential functions.
- OH.Math.HSF.IF.5c: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Emphasize the selection of a type of function for a model based on behavior of data and context.
- OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f\left(\frac{x}{k}\right)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $f(x) = x^2$ .
- **MULTIPLE TRANSFORMATIONS OF PARENT FUNCTIONS**
  - OH.Math.HSG.CO.2: Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.
  - OH.Math.HSG.CO.6: Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
  - OH.Math.HSG.CO.5: Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
  - OH.Math.HSG.SRT.1a: Verify experimentally the properties of dilations given by a center and a scale factor: A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged.
  - OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f\left(\frac{x}{k}\right)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $f(x) = x^2$ .

- OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f\left(\frac{x}{k}\right)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $f(x) = x^2$ .
- OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f\left(\frac{x}{k}\right)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $f(x) = x^2$ .
- OH.Math.HSF.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- OH.Math.HSF.IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- OH.Math.HSF.BF.3: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f\left(\frac{x}{k}\right)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- OH.Math.HSF.IF.7c: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- OH.Math.HSF.IF.7f: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- OH.Math.HSF.IF.7d: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior.
- OH.Math.HSF.IF.4a: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Focus on linear and exponential functions.
- OH.Math.HSF.IF.5c: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Emphasize the selection of a type of function for a model based on behavior of data and context.

- OH.Math.HSF.BF.3a: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f\left(\frac{x}{k}\right)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on transformations of graphs of quadratic functions, except for  $f(x) = x^2$ .
- OH.Math.HSF.BF.3: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $f(x) - k$ ,  $f(kx)$ , and  $f\left(\frac{x}{k}\right)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

## Unit 22: Systems of Equations

### • SOLVING SYSTEMS OF LINEAR EQUATIONS: GRAPHING

- OH.Math.HSA.CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- OH.Math.HSA.CED.2a: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying linear and simple exponential expressions.
- OH.Math.HSA.CED.2b: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying simple quadratic expressions.
- OH.Math.HSA.CED.3a: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
- OH.Math.HSA.REI.11: Explain why the  $x$ -coordinates of the points where the graphs of the equation  $f(x) = k$  and  $f(x) = c$  intersect are the solutions of the equation  $f(x) = c - k$ ; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.
- OH.Math.HSA.REI.6a: Solve systems of linear equations algebraically and graphically. Limit to pairs of linear equations in two variables.
- OH.Math.HSA.REI.6: Solve systems of linear equations algebraically and graphically.
- OH.Math.HSA.REI.11: Explain why the  $x$ -coordinates of the points where the graphs of the equation  $f(x) = k$  and  $f(x) = c$  intersect are the solutions of the equation  $f(x) = c - k$ ; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.
- OH.Math.HSA.REI.11: Explain why the  $x$ -coordinates of the points where the graphs of the equation  $f(x) = k$  and  $f(x) = c$  intersect are the solutions of the equation  $f(x) = c - k$ ; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.
- OH.Math.HSA.REI.11: Explain why the  $x$ -coordinates of the points where the graphs of the equation  $f(x) = k$  and  $f(x) = c$  intersect are the solutions of the equation  $f(x) = c - k$ ; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.

- OH.Math.HSA.CED.2c: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Extend to include more complicated function situations with the option to graph with technology.
- **SOLVING THREE-VARIABLE SYSTEMS OF LINEAR EQUATIONS**
  - OH.Math.HSA.CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
  - OH.Math.HSA.REI.5: Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
  - OH.Math.HSA.CED.2a: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Focus on applying linear and simple exponential expressions.
  - OH.Math.HSA.CED.2c: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Extend to include more complicated function situations with the option to graph with technology.
  - OH.Math.HSA.CED.3a: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
  - OH.Math.HSA.REI.6b: Solve systems of linear equations algebraically and graphically. Extend to include solving systems of linear equations in three variables, but only algebraically.
  - OH.Math.HSA.CED.4d: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
- **SYSTEMS OF NONLINEAR EQUATIONS**
  - OH.Math.HSA.REI.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
  - OH.Math.HSA.REI.5: Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
  - OH.Math.HSA.REI.11: Explain why the  $x$ -coordinates of the points where the graphs of the equation  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.
  - OH.Math.HSA.CED.3a: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.

- OH.Math.HSA.CED.4d: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
- OH.Math.HSA.REI.11: Explain why the  $x$ -coordinates of the points where the graphs of the equation  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.
- OH.Math.HSA.REI.11: Explain why the  $x$ -coordinates of the points where the graphs of the equation  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.
- OH.Math.HSF.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

### Unit 23: Radians and Trigonometric Functions

#### • RADIANS AND THE UNIT CIRCLE

- OH.Math.HSG.C.6: Derive formulas that relate degrees and radians, and convert between the two.
- OH.Math.HSF.TF.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- OH.Math.HSF.TF.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- OH.Math.HSG.C.5a: Find arc lengths and areas of sectors of circles. Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems.
- OH.Math.HSF.TF.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$ , and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\theta$ ,  $\pi + \theta$ , and  $2\pi - \theta$  in terms of their values for  $\theta$ , where  $\theta$  is any real number.
- OH.Math.HSF.TF.4: Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- OH.Math.HSG.SRT.8a: Solve problems involving right triangles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given.

#### • TRIGONOMETRIC FUNCTIONS

- OH.Math.HSF.IF.7f: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- OH.Math.HSF.TF.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed

counterclockwise around the unit circle.

- OH.Math.HSF.TF.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- OH.Math.HSF.TF.8: Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$ , and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.
- OH.Math.HSF.TF.8: Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$ , and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.
- OH.Math.HSA.CED.2c: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Extend to include more complicated function situations with the option to graph with technology.

## Unit 24: Triangles and Trigonometry

### • PYTHAGOREAN THEOREM

- OH.Math.HSG.SRT.8a: Solve problems involving right triangles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given.
- OH.Math.HSG.SRT.8b: Solve problems involving right triangles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- OH.Math.HSG.MG.1: Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.
- OH.Math.HSG.CO.10: Prove and apply theorems about triangles.
- OH.Math.HSG.SRT.4: Prove and apply theorems about triangles.
- OH.Math.HSG.SRT.5: Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.

### • TRIGONOMETRIC RATIOS

- OH.Math.HSF.TF.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\frac{\pi}{3}$ ,  $\frac{\pi}{4}$ , and  $\frac{\pi}{6}$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\theta$ ,  $\pi + \theta$ , and  $2\pi - \theta$  in terms of their values for  $\theta$ , where  $\theta$  is any real number.
- OH.Math.HSG.SRT.6: Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- OH.Math.HSG.SRT.7: Explain and use the relationship between the sine and cosine of complementary angles.
- OH.Math.HSG.SRT.8a: Solve problems involving right triangles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given.
- OH.Math.HSG.SRT.8b: Solve problems involving right triangles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.



- OH.Math.HSF.TF.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$ , and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\theta$ ,  $\theta + \pi$ , and  $\theta + 2\pi$  in terms of their values for  $\theta$ , where  $\theta$  is any real number.
- OH.Math.HSF.TF.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$ , and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\theta$ ,  $\theta + \pi$ , and  $\theta + 2\pi$  in terms of their values for  $\theta$ , where  $\theta$  is any real number.
- OH.Math.HSG.SRT.5: Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.
- OH.Math.HSG.MG.1: Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.
- **LAWS OF SINE AND COSINE**
  - OH.Math.HSG.SRT.9: Derive the formula  $A = \frac{1}{2}ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
  - OH.Math.HSG.SRT.8a: Solve problems involving right triangles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given.
  - OH.Math.HSG.SRT.8b: Solve problems involving right triangles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
  - OH.Math.HSG.SRT.10: Explain proofs of the Laws of Sines and Cosines and use the Laws to solve problems.
  - OH.Math.HSG.SRT.11: Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles, e.g., surveying problems, resultant forces.

## Unit 25: Properties of Circles

- **CIRCUMFERENCE AND ARC LENGTH**
  - OH.Math.HSG.CO.1: Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length.
  - OH.Math.HSG.GPE.4: Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles.
  - OH.Math.HSG.GMD.1: Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone.
  - OH.Math.HSG.C.6: Derive formulas that relate degrees and radians, and convert between the two.
  - OH.Math.HSG.MG.1: Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.
- **AREA OF CIRCLES AND SECTORS**
  - OH.Math.HSG.GMD.1: Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone.

- OH.Math.HSG.C.5b: Find arc lengths and areas of sectors of circles. Derive the formula for the area of a sector, and use it to solve problems.
- OH.Math.HSG.C.6: Derive formulas that relate degrees and radians, and convert between the two.
- OH.Math.HSG.MG.1: Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.
- OH.Math.HSG.CO.1: Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length.

## Unit 26: Scatterplots and Regression

### • SCATTERPLOTS

- OH.Math.HSS.ID.5: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
- OH.Math.HSS.ID.6a: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
- OH.Math.HSS.ID.6b: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Informally assess the fit of a function by discussing residuals.
- OH.Math.HSS.ID.6c: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a linear function for a scatterplot that suggests a linear association.
- OH.Math.HSS.ID.9: Distinguish between correlation and causation.
- OH.Math.HSS.ID.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- OH.Math.HSS.ID.8: Compute (using technology) and interpret the correlation coefficient of a linear fit.
- OH.Math.HSF.IF.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

### • SCATTERPLOTS AND MODELING

- OH.Math.HSS.ID.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- OH.Math.HSS.ID.6a: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
- OH.Math.HSS.ID.6b: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Informally assess the fit of a function by discussing residuals.
- OH.Math.HSS.ID.6c: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a linear function for a scatterplot that suggests a linear association.
- OH.Math.HSS.ID.8: Compute (using technology) and interpret the correlation coefficient of a linear fit.

- OH.Math.HSF.LE.1a: Distinguish between situations that can be modeled with linear functions and with exponential functions. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
- OH.Math.HSF.LE.1c: Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

### Unit 27: Statistical Design and Analysis

#### • ANALYZING STATISTICAL SAMPLES

- OH.Math.HSS.IC.1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
- OH.Math.HSS.IC.2: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
- OH.Math.HSS.IC.4: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

#### • EXPERIMENTAL AND OBSERVATIONAL DESIGN

- OH.Math.HSS.IC.3: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

#### • CONCLUSIONS IN DATA

- OH.Math.HSS.IC.5: Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between sample statistics are statistically significant.
- OH.Math.HSS.IC.6: Evaluate reports based on data.

### Unit 28: Statistics and Probability

#### • NORMAL DISTRIBUTION

- OH.Math.HSS.ID.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
- OH.Math.HSS.IC.4: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

#### • ANALYZING DECISIONS IN PROBABILITY

- OH.Math.HSS.MD.6: Use probabilities to make fair decisions, e.g., drawing by lots, using a random number generator.
- OH.Math.HSS.MD.7: Analyze decisions and strategies using probability concepts, e.g., product testing, medical testing, pulling a hockey goalie at the end of a game.
- OH.Math.HSS.IC.2: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.