Massachusetts Tutorials are designed specifically for the Learning Standards found in the Massachusetts Curriculum Frameworks to prepare students for the MCAS tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

Unit 1: Rational and Irrational Numbers

- **OPERATIONS ON RATIONAL AND IRRATIONAL NUMBERS**
  - MII.N-RN.B.3: Number and Quantity The Real Number System Use properties of rational and irrational numbers. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
- **LAWS OF EXPONENTS**
  - MII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Use the structure of an expression to identify ways to rewrite it.
  - MII.N-RN.A.1: Number and Quantity The Real Number System Extend the properties of exponents to rational exponents. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
  - MII.N-RN.A.2: Number and Quantity The Real Number System Extend the properties of exponents to rational exponents. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Unit 2: Expressions and Equations I

- **FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS**
  - MII.F-BF.A.1.a: Functions Building Functions Build a function that models a relationship between two quantities. Write linear, quadratic, and exponential functions that describe relationships between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
  - MII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - MII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - MII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Use the structure of an expression to identify ways to rewrite it.
  - MII.A-APR.A.1.b: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.
- **ONE-STEP EQUATIONS AND INEQUALITIES**
  - MII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.
  - MII.F-BF.A.1.a: Functions Building Functions Build a function that models a relationship between two quantities. Write linear, quadratic, and exponential functions that describe relationships between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Unit 3: Expressions and Equations II

- **MULTI-STEP EQUATIONS AND INEQUALITIES**
  - MII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.
- **LITERAL EQUATIONS**
  - MII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.
  - MII.A-CED.A.4: Algebra Creating Equations Create equations that describe numbers or relationships. Rearrange formulas, including formulas with quadratic terms, to highlight a quantity of interest using the same reasoning as in solving equations (Properties of equality).

Unit 4: Functions

- **FUNCTIONS AND RELATIONS**
  - MII.F-IF.B.4: Functions Interpreting Functions Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
  - MII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
  - MII.F-IF.C.7.b: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph piecewise-defined functions, including step functions and absolute value functions.
- **DOMAIN AND RANGE**
  - MII.F-IF.B.5: Functions Interpreting Functions Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**
  - MII.F-IF.B.4: Functions Interpreting Functions Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
  - MII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
  - MII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.
  - MII.A-CED.A.2: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Unit 5: Points, Lines, and Angles

- **PARALLEL LINES AND ANGLE RELATIONSHIPS**

- MII.G-CO.C.9: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segments endpoints.

- **PERPENDICULAR BISECTOR AND ANGLE BISECTOR THEOREMS**
  - MII.G-CO.C.9: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segments endpoints.
  - MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
  - MII.G-CO.C.9: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segments endpoints.
  - MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
  - MII.G-CO.C.11.a: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.
  - MII.G-CO.C.9: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segments endpoints.
  - MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely

prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

- MII.G-CO.C.9: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segments endpoints.
- MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

- **CONJECTURES IN COORDINATE GEOMETRY**
  - MII.G-GPE.B.4: Geometry Expressing Geometric Properties with Equations Use coordinates to prove simple geometric theorems algebraically. Use coordinates to prove simple geometric theorems algebraically including the distance formula and its relationship to the Pythagorean Theorem.

Unit 6: The Coordinate Plane

- **LENGTH AND THE DISTANCE FORMULA**
  - MII.G-GPE.B.6: Geometry Expressing Geometric Properties with Equations Use coordinates to prove simple geometric theorems algebraically. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
- **MIDPOINT FORMULA ON THE COORDINATE PLANE**
  - MII.G-GPE.B.6: Geometry Expressing Geometric Properties with Equations Use coordinates to prove simple geometric theorems algebraically. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Unit 7: Conic Sections

- **CIRCLES**
  - MII.G-GPE.A.1: Geometry Expressing Geometric Properties with Equations Translate between the geometric description and the equation for a conic section. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- **PARABOLAS**
  - MII.G-GPE.A.2: Geometry Expressing Geometric Properties with Equations Translate between the geometric description and the equation for a conic section. Derive the equation of a parabola given a focus and directrix.
  - MII.A-CED.A.2: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Unit 8: Geometric Transformations

- **TRANSFORMATIONS ON THE COORDINATE PLANE**
  - MII.G-CO.C.11.a: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.
  - MII.G-SR.A.2: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
  - MII.G-SR.A.1.a: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations. Verify experimentally the properties of dilations given by a center and a scale factor: A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
  - MII.G-SR.A.1.b: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations. Verify experimentally the properties of dilations given by a center and a scale factor: The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
- **DILATIONS, TRANSLATIONS, ROTATIONS, AND REFLECTIONS**
  - MII.G-CO.C.11.a: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.
  - MII.G-SR.A.2: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
  - MII.G-SR.A.1.a: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations. Verify experimentally the properties of dilations given by a center and a scale factor: A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
  - MII.G-SR.A.1.b: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations. Verify experimentally the properties of dilations given by a center and a scale factor: The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Unit 9: Congruence and Similarity

- **TRIANGLES AND CONGRUENCE TRANSFORMATIONS**
    - MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
    - MII.G-SR.B.5: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity using a variety of ways of writing proofs, showing validity of underlying reasoning. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
    - MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
    - MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
    - MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- **TRIANGLES AND SIMILARITY TRANSFORMATIONS**
    - MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
    - MII.G-SR.A.2: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
    - MII.G-SR.B.4: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity using a variety of ways of writing proofs, showing validity of underlying reasoning. Prove

theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

- MII.G-SR.B.5: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity using a variety of ways of writing proofs, showing validity of underlying reasoning. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- MII.G-SR.A.3: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations. Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar.
- MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

- **SIMILARITY OF OTHER POLYGONS**
  - MII.G-CO.C.11.a: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.
  - MII.G-SR.A.2: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Unit 10: Triangles and Quadrilaterals

- **TRIANGLE ANGLE THEOREMS**
  - MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior

angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

- MII.G-CO.C.11.a: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.
- MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- MII.G-C.A.3: Geometry Circles Understand and apply theorems about circles. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral and other polygons inscribed in a circle.

- **TRIANGLE BISECTORS**
  - MII.G-CO.C.9: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about lines and angles. Theorems include: vertical angles are

congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segments endpoints.

- MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- MII.G-CO.C.11.a: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.
- MII.G-CO.C.9: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segments endpoints.
- MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

- MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- MII.G-SR.B.4: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity using a variety of ways of writing proofs, showing validity of underlying reasoning. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
- MII.G-SR.B.5: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity using a variety of ways of writing proofs, showing validity of underlying reasoning. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- MII.G-C.A.3: Geometry Circles Understand and apply theorems about circles. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral and other polygons inscribed in a circle.

- **MEDIANS AND ALTITUDES OF TRIANGLES**
  - MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
  - MII.G-CO.C.11.a: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.
  - MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
  - MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
  - MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior

angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Unit 11: Right Triangles and Trigonometric Ratios

- **PYTHAGOREAN THEOREM**
  - MII.G-SR.C.8: Geometry Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
  - MII.G-SR.B.4: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity using a variety of ways of writing proofs, showing validity of underlying reasoning. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
  - MII.G-SR.B.5: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity using a variety of ways of writing proofs, showing validity of underlying reasoning. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
  - MII.G-CO.C.10: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- **TRIGONOMETRIC RATIOS**
  - MII.G-SR.C.6: Geometry Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
  - MII.G-SR.C.8: Geometry Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
  - MII.G-SR.C.7: Geometry Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles. Explain and use the relationship between the sine and cosine of complementary angles.
  - MII.G-SR.B.5: Geometry Similarity, Right Triangles, and Trigonometry Prove theorems involving similarity using a variety of ways of writing proofs, showing validity of underlying reasoning. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Unit 12: Trigonometry and Constructions

- **RADIANS AND THE UNIT CIRCLE**

- MII.G-C.B.5: Geometry Circles Find arc lengths and areas of sectors of circles. Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
- MII.G-SR.C.8: Geometry Similarity, Right Triangles, and Trigonometry Define trigonometric ratios and solve problems involving right triangles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

- **TRIGONOMETRIC FUNCTIONS**
- **CONSTRUCTIONS**
  - MII.G-C.A.4: Geometry Circles Understand and apply theorems about circles. Construct a tangent line from a point outside a given circle to the circle.

Unit 13: Circles I

- **CIRCLE BASICS**
  - MII.G-C.A.2: Geometry Circles Understand and apply theorems about circles. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
- **CENTRAL ANGLES, INSCRIBED ANGLES, AND CHORDS**
  - MII.G-C.A.2: Geometry Circles Understand and apply theorems about circles. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
  - MII.G-CO.C.9: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segments endpoints.
  - MII.G-C.B.5: Geometry Circles Find arc lengths and areas of sectors of circles. Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Unit 14: Circles II

- **SECANTS, ANGLES, AND INTERCEPTED ARCS**
  - MII.G-CO.C.9: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segments endpoints.

- MII.G-C.A.2: Geometry Circles Understand and apply theorems about circles. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
- MII.G-CO.C.9: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segments endpoints.

- **TANGENTS, ANGLES, AND INTERCEPTED ARCS**
  - MII.G-CO.C.9: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segments endpoints.
  - MII.G-CO.C.9: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segments endpoints.
  - MII.G-C.A.2: Geometry Circles Understand and apply theorems about circles. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
  - MII.G-CO.C.9: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segments endpoints.
  - MII.G-CO.C.9: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segments endpoints.
  - MII.G-CO.C.9: Geometry Congruence Prove geometric theorems and, when appropriate, the converse of theorems. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segments endpoints.

Unit 15: Advanced Circle Properties

- **CONGRUENT AND SIMILAR CIRCLES**
  - MII.G-C.A.1: Geometry Circles Understand and apply theorems about circles. Prove that all circles are similar.
  - MII.G-SR.A.2: Geometry Similarity, Right Triangles, and Trigonometry Understand similarity in terms of similarity transformations. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- **CIRCUMFERENCE AND ARC LENGTH**
  - MII.G-GMD.A.1: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieris principle, and informal limit arguments.
- **AREA OF CIRCLES AND SECTORS**
  - MII.G-GMD.A.1: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieris principle, and informal limit arguments.
  - MII.G-C.B.5: Geometry Circles Find arc lengths and areas of sectors of circles. Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Unit 16: Exponential Functions, Equations, and Inequalities

- **EXPONENTIAL FUNCTIONS**
  - MII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - MII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - MII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.
  - MII.F-IF.C.8.b: Functions Interpreting Functions Analyze functions using different representations. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as Identifying appreciation/depreciation rate for the value of a house or car some time after its initial purchase: subscript n = (1+r).

- MII.F-IF.B.6: Functions Interpreting Functions Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- MII.F-IF.C.8.b: Functions Interpreting Functions Analyze functions using different representations. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as Identifying appreciation/depreciation rate for the value of a house or car some time after its initial purchase: subscript n = (1+r).
- MII.F-LE.A.3: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- MII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- MII.F-IF.C.8.b: Functions Interpreting Functions Analyze functions using different representations. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as Identifying appreciation/depreciation rate for the value of a house or car some time after its initial purchase: subscript n = (1+r).
- MII.A-SSE.B.3.c: Algebra Seeing Structure in Expressions Write quadratic and exponential expressions in equivalent forms to solve problems. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.
- MII.F-IF.B.4: Functions Interpreting Functions Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- MII.F-IF.B.5: Functions Interpreting Functions Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- MII.F-IF.C.8.b: Functions Interpreting Functions Analyze functions using different representations. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as Identifying appreciation/depreciation rate for the value of a house or car some time after its initial purchase: subscript n = (1+r).

- MII.F-BF.A.1.a: Functions Building Functions Build a function that models a relationship between two quantities. Write linear, quadratic, and exponential functions that describe relationships between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

- **EXPONENTIAL GROWTH AND DECAY**
  - MII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - MII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - MII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.
  - MII.F-IF.C.8.b: Functions Interpreting Functions Analyze functions using different representations. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as Identifying appreciation/depreciation rate for the value of a house or car some time after its initial purchase: subscript n = (1+r).
  - MII.F-IF.C.8.b: Functions Interpreting Functions Analyze functions using different representations. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as Identifying appreciation/depreciation rate for the value of a house or car some time after its initial purchase: subscript n = (1+r).
  - MII.A-CED.A.2: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - MII.F-LE.A.3: Functions Linear, Quadratic, and Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

- **SOLVING EXPONENTIAL INEQUALITIES**
  - MII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - MII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.

- MII.A-CED.A.2: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Unit 17: Polynomials I

- **POLYNOMIAL BASICS**
  - MII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - MII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - MII.A-APR.A.1.b: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.
- **ADDITION AND SUBTRACTION OF POLYNOMIALS**
  - MII.A-APR.A.1.a: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. Perform operations on polynomial expressions (addition, subtraction, multiplication), and compare the system of polynomials to the system of integers when performing operations.
  - MII.A-APR.A.1.b: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.

Unit 18: Polynomials II

- **MULTIPLICATION OF POLYNOMIALS**
  - MII.A-APR.A.1.a: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. Perform operations on polynomial expressions (addition, subtraction, multiplication), and compare the system of polynomials to the system of integers when performing operations.
  - MII.A-APR.A.1.b: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.
- **ARITHMETIC OPERATIONS ON FUNCTIONS**

- MII.F-BF.A.1.b: Functions Building Functions Build a function that models a relationship between two quantities. Write linear, quadratic, and exponential functions that describe relationships between two quantities. Combine standard function types using arithmetic operations.

Unit 19: Factoring

- **FACTORING QUADRATIC TRINOMIALS**
  - MII.A-SSE.B.3.a: Algebra Seeing Structure in Expressions Write quadratic and exponential expressions in equivalent forms to solve problems. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
  - MII.A-APR.A.1.b: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.
  - MII.A-REI.B.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
  - MII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - MII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **FACTORING SPECIAL CASES**
  - MII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - MII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - MII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Use the structure of an expression to identify ways to rewrite it.
  - MII.A-APR.A.1.b: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.

Unit 20: Complex Numbers

- **COMPLEX NUMBERS**
  - MII.N-CN.A.1: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number i such that i = 1, and every complex number has the form a + bi with a and b real.
  - MII.N-CN.A.2: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Use the relation i = 1 and the Commutative, Associative, and Distributive properties to add, subtract, and multiply complex numbers.
- **POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS**
  - MII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - MII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - MII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Use the structure of an expression to identify ways to rewrite it.
  - MII.A-APR.A.1.b: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.
  - MII.N-CN.A.1: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number i such that i = 1, and every complex number has the form a + bi with a and b real.
  - MII.N-CN.C.7: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
  - MII.A-REI.B.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.

Unit 21: Representations of Quadratic Functions

- **QUADRATIC FUNCTIONS**
  - MII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.
  - MII.F-IF.B.4: Functions Interpreting Functions Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context. For a function that models a

relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

- MII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- MII.A-CED.A.2: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- MII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- MII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- MII.A-SSE.B.3.b: Algebra Seeing Structure in Expressions Write quadratic and exponential expressions in equivalent forms to solve problems. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- MII.F-IF.C.7.a: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph quadratic functions and show intercepts, maxima, and minima.
- MII.F-IF.C.8.a: Functions Interpreting Functions Analyze functions using different representations. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum values, and symmetry of the graph and interpret these in terms of a context.
- MII.F-BF.A.1.a: Functions Building Functions Build a function that models a relationship between two quantities. Write linear, quadratic, and exponential functions that describe relationships between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

- **ANALYZING GRAPHS OF QUADRATIC FUNCTIONS**
  - MII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Use the structure of an expression to identify ways to rewrite it.
  - MII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations

arising from quadratic and exponential functions.

- MII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- MII.F-IF.B.5: Functions Interpreting Functions Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- MII.F-IF.B.4: Functions Interpreting Functions Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- MII.F-IF.C.7.a: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph quadratic functions and show intercepts, maxima, and minima.
- MII.F-IF.C.8.a: Functions Interpreting Functions Analyze functions using different representations. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum values, and symmetry of the graph and interpret these in terms of a context.

- **REPRESENTATIONS OF QUADRATIC FUNCTIONS**
  - MII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Use the structure of an expression to identify ways to rewrite it.
  - MII.A-REI.B.4.a: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x p) = q that has the same solutions. Derive the quadratic formula from this form.
  - MII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.
  - MII.F-IF.B.4: Functions Interpreting Functions Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

- MII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- MII.F-BF.A.1.a: Functions Building Functions Build a function that models a relationship between two quantities. Write linear, quadratic, and exponential functions that describe relationships between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- MII.A-SSE.B.3.c: Algebra Seeing Structure in Expressions Write quadratic and exponential expressions in equivalent forms to solve problems. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.
- MII.A-CED.A.2: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- MII.F-IF.C.8.a: Functions Interpreting Functions Analyze functions using different representations. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum values, and symmetry of the graph and interpret these in terms of a context.

Unit 22: Quadratic Parent Functions and Transformations

- **QUADRATIC PARENT FUNCTION**
  - MII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.
  - MII.F-IF.C.7.a: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph quadratic functions and show intercepts, maxima, and minima.
  - MII.F-IF.B.5: Functions Interpreting Functions Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **TRANSFORMATIONS OF THE QUADRATIC PARENT FUNCTION**
  - MII.F-BF.B.3: Functions Building Functions Build new functions from existing functions. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Include exponential, quadratic, and absolute value functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph.

- MII.F-IF.B.5: Functions Interpreting Functions Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- MII.F-BF.B.3: Functions Building Functions Build new functions from existing functions. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Include exponential, quadratic, and absolute value functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph.

Unit 23: Solving Quadratic Equations

- **SOLVING QUADRATIC EQUATIONS BY FACTORING**
  - MII.A-SSE.B.3.a: Algebra Seeing Structure in Expressions Write quadratic and exponential expressions in equivalent forms to solve problems. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
  - MII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.
  - MII.A-REI.B.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
  - MII.F-IF.C.8.a: Functions Interpreting Functions Analyze functions using different representations. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum values, and symmetry of the graph and interpret these in terms of a context.
  - MII.A-APR.A.1.b: Algebra Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.
  - MII.A-REI.B.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
  - MII.A-REI.B.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula, and factoring, as

appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.

- MII.A-REI.B.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- MII.F-IF.C.7.a: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph quadratic functions and show intercepts, maxima, and minima.
- MII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- MII.F-BF.A.1.a: Functions Building Functions Build a function that models a relationship between two quantities. Write linear, quadratic, and exponential functions that describe relationships between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

- **COMPLETING THE SQUARE**
  - MII.A-SSE.B.3.b: Algebra Seeing Structure in Expressions Write quadratic and exponential expressions in equivalent forms to solve problems. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
  - MII.A-REI.B.4.a: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x p) = q that has the same solutions. Derive the quadratic formula from this form.
  - MII.A-REI.B.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
  - MII.F-IF.C.8.a: Functions Interpreting Functions Analyze functions using different representations. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum values, and symmetry of the graph and interpret these in terms of a context.
  - MII.A-SSE.A.2: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Use the structure of an expression to identify ways to rewrite it.

- MII.A-REI.B.4.a: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x p) = q that has the same solutions. Derive the quadratic formula from this form.
- MII.A-REI.B.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- MII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.
- MII.A-REI.B.4.a: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x p) = q that has the same solutions. Derive the quadratic formula from this form.
- MII.A-REI.B.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- MII.F-IF.C.7.a: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph quadratic functions and show intercepts, maxima, and minima.

Unit 24: Quadratic Formula and Complex Numbers

- **QUADRATIC FORMULA**
  - MII.A-SSE.A.1.a: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
  - MII.A-SSE.A.1.b: Algebra Seeing Structure in Expressions Interpret the structure of quadratic and exponential expressions. Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  - MII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.
  - MII.A-REI.B.4.a: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Use the method of completing the square to

transform any quadratic equation in x into an equation of the form (x p) = q that has the same solutions. Derive the quadratic formula from this form.

- MII.A-REI.B.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- MII.A-REI.B.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- MII.F-IF.B.4: Functions Interpreting Functions Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- MII.F-BF.A.1.a: Functions Building Functions Build a function that models a relationship between two quantities. Write linear, quadratic, and exponential functions that describe relationships between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.

- **COMPLEX NUMBERS AND QUADRATIC FUNCTIONS**
  - MII.N-CN.A.2: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Use the relation i = 1 and the Commutative, Associative, and Distributive properties to add, subtract, and multiply complex numbers.
  - MII.N-CN.C.7: Number and Quantity The Complex Number System Use complex numbers in polynomial identities and equations. Solve quadratic equations with real coefficients that have complex solutions.
  - MII.A-REI.B.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
  - MII.N-CN.A.1: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number i such that i = 1, and every complex number has the form a + bi with a and b real.
  - MII.A-REI.B.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula, and factoring, as

appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.

- MII.N-CN.A.1: Number and Quantity The Complex Number System Perform arithmetic operations with complex numbers. Know there is a complex number i such that i = 1, and every complex number has the form a + bi with a and b real.
- MII.A-REI.B.4.b: Algebra Reasoning with Equations and Inequalities Solve equations and inequalities in one variable. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for x = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a bi for real numbers a and b.
- MII.F-IF.C.9: Functions Interpreting Functions Analyze functions using different representations. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

Unit 25: Nonlinear Functions

- **INVERSE FUNCTIONS**
  - MII.F-BF.B.4.a: Functions Building Functions Build new functions from existing functions. Find inverse functions algebraically and graphically. Solve an equation of the form f(x) = c for a linear function f that has an inverse and write an expression for the inverse.
- **ABSOLUTE VALUE FUNCTIONS**
  - MII.F-IF.B.5: Functions Interpreting Functions Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
  - MII.F-IF.C.7.b: Functions Interpreting Functions Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph piecewise-defined functions, including step functions and absolute value functions.
  - MII.F-BF.B.3: Functions Building Functions Build new functions from existing functions. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Include exponential, quadratic, and absolute value functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph.
- **SYSTEMS OF NONLINEAR EQUATIONS**
  - MII.A-REI.C.7: Algebra Reasoning with Equations and Inequalities Solve systems of equations. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
  - MII.A-CED.A.1: Algebra Creating Equations Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations

arising from quadratic and exponential functions.

Unit 26: Volume

- **VOLUME OF PRISMS AND PYRAMIDS**
  - MII.G-GMD.A.2: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Give an informal argument using Cavalieris principle for the formulas for the volume of a sphere and other solid figures.
  - MII.G-GMD.A.1: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieris principle, and informal limit arguments.
  - MII.G-GMD.A.2: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Give an informal argument using Cavalieris principle for the formulas for the volume of a sphere and other solid figures.
  - MII.G-GMD.A.3: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
  - MII.G-GMD.A.1: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieris principle, and informal limit arguments.
  - MII.G-GMD.A.2: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Give an informal argument using Cavalieris principle for the formulas for the volume of a sphere and other solid figures.
- **VOLUME OF CYLINDERS AND CONES**
  - MII.G-GMD.A.1: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieris principle, and informal limit arguments.
  - MII.G-GMD.A.2: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Give an informal argument using Cavalieris principle for the formulas for the volume of a sphere and other solid figures.
  - MII.G-GMD.A.3: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
  - MII.G-GMD.A.1: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieris principle, and informal limit arguments.

- MII.G-GMD.A.2: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Give an informal argument using Cavalieris principle for the formulas for the volume of a sphere and other solid figures.
- MII.G-GMD.A.1: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieris principle, and informal limit arguments.
- MII.G-GMD.A.2: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Give an informal argument using Cavalieris principle for the formulas for the volume of a sphere and other solid figures.

- **VOLUME OF COMPOSITE SOLIDS**
  - MII.G-GMD.A.3: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
  - MII.G-GMD.A.2: Geometry Geometric Measurement and Dimension Explain volume formulas and use them to solve problems. Give an informal argument using Cavalieris principle for the formulas for the volume of a sphere and other solid figures.

Unit 27: Basic Probability Concepts

- **INTRODUCTION TO PROBABILITY**
  - MII.S-CP.A.2: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data from simulations or experiments. Understand that two events and are independent if the probability of and occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
  - MII.S-CP.B.8: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model. Apply the general Multiplication Rule in a uniform probability model, (and) = ()() = ()(), and interpret the answer in terms of the model.
  - MII.S-CP.A.2: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data from simulations or experiments. Understand that two events and are independent if the probability of and occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
  - MII.S-CP.A.5: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data from simulations or experiments. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
  - MII.S-CP.B.8: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Apply the general Multiplication Rule in a uniform probability model, (and) = ()() = ()(), and interpret the answer in terms of the model.

- MII.S-CP.B.7: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model. Apply the Addition Rule, (or) = () + () (and), and interpret the answer in terms of the model.

- **COMBINATIONS AND PERMUTATIONS**
  - MII.S-CP.B.9: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model. Use permutations and combinations to compute probabilities of compound events and solve problems.

Unit 28: Advanced Probability Concepts

- **CONDITIONAL PROBABILITY**
  - MII.S-CP.A.3: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data from simulations or experiments. Understand the conditional probability of given as (and)/(), and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
  - MII.S-CP.A.5: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data from simulations or experiments. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
  - MII.S-CP.B.6: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model. Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.
  - MII.S-CP.A.2: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data from simulations or experiments. Understand that two events and are independent if the probability of and occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
  - MII.S-CP.A.3: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data from simulations or experiments. Understand the conditional probability of given as (and)/(), and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
  - MII.S-CP.B.6: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model. Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.

- MII.S-CP.A.4: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data from simulations or experiments. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
- MII.S-CP.A.1: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data from simulations or experiments. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
- MII.S-CP.A.3: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data from simulations or experiments. Understand the conditional probability of given as (and)/(), and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
- MII.S-CP.B.6: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model. Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.
- MII.S-CP.A.3: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data from simulations or experiments. Understand the conditional probability of given as (and)/(), and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
- MII.S-CP.B.6: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model. Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.
- MII.S-CP.A.3: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data from simulations or experiments. Understand the conditional probability of given as (and)/(), and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .
- MII.S-CP.B.6: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model. Find the conditional probability of given as the fraction of s outcomes that also belong to , and interpret the answer in terms of the model.

- **GEOMETRIC PROBABILITIES**
  - MII.S-CP.A.1: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data from simulations or experiments.

experiments. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).

- MII.S-CP.B.7: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model. Apply the Addition Rule, (or) = () + () (and), and interpret the answer in terms of the model.
- MII.S-CP.A.1: Statistics and Probability Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data from simulations or experiments. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
- MII.S-CP.B.7: Statistics and Probability Conditional Probability and the Rules of Probability Use the rules of probability to compute probabilities of compound events in a uniform probability model. Apply the Addition Rule, (or) = () + () (and), and interpret the answer in terms of the model.

- **ANALYZING DECISIONS IN PROBABILITY**