Q1 – Q55. 課堂內容

Q56.

(a) $(x-3)^2+(y-4)^2=16$

(b) $(x+2)^2+y^2=1$

(c) $(x+1)^2+(y+4)^2=3$

Q57.

$(x-3)^2+(y+6)^2=c$

Sub $(3,-3)$,

$(3-3)^2+(-3+6)^2=c$

$c=9$

Q58.

$(x+2)^2+(y-4)^2=12$

Sub $(a,2)$,

$(a+2)^2+(2-4)^2=12$

$a=\pm\sqrt{8}-2$

Q59.

$(x-h)^2+(y-k)^2=41$

Sub $(3,5)$ and $k=0$,

$(3-h)^2+(5-0)^2=41$

$h=-1$ or $h=7$

Q60.

$(x-4)^2+y^2=16$

Q61.

(a) $x^2 + y^2 - 10x + 16 = 0$

(b) $8x + 6y - 4 = 0$

(c) 20 unit^2

Q62.

(a)	$2x^2 + 2y^2 - 4x + 16y + 16 = 0$	
	$x^2 + y^2 - 2x + 8y + 8 = 0$	
	Coordinates of the centre of C	
	$= \left(-\frac{-2}{2}, -\frac{8}{2}\right)$	1M
	$= (1, -4)$	
	Radius of C	
	$= \sqrt{1^2 + (-4)^2 - 8}$	1M
	$= 3$	
	Distance between Q and the centre of C	
	$= \sqrt{(1-2)^2 + [-4-(-3)]^2}$	1M
	$= \sqrt{2}$	
	< 3	
	$\therefore$ Q lies inside C.	
	$\therefore$ <u>Q does not lie outside C.</u>	1A
(b)	Coordinates of P'	
	$= (1 + 3, -4)$	1M
	$= (4, -4)$	
	Radius of C'	
	$= 2 \times 3$	
	$= 6$	
	The equation of C' is	
	$(x-4)^2 + [y-(-4)]^2 = 6^2$	
	$(x-4)^2 + (y+4)^2 = 36$	1A
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		(6)

(a) Note that Φ passes through two points (–4 , 12 – 3) and (8 , 3 – 3), i.e. (–4 , 9) and (8 , 0).

The equation of Φ is

$$\frac{y-0}{x-8} = \frac{9-0}{-4-8}$$ 1M

$$\frac{y}{x-8} = -\frac{3}{4}$$

$4y = -3x + 24$

$3x + 4y - 24 = 0$ 1A

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(2)

(b)

x-intercept of $\Phi = -\frac{-24}{3} = 8$

y-intercept of $\Phi = -\frac{-24}{4} = 6$ 1M either one

Coordinates of S = (8 , 0)

Coordinates of T = (0 , 6)

$\because$ $\angle SOT = 90°$

$\therefore$ ST is a diameter of the circle passing through S, T and the origin O.

Radius of the circle $= \frac{1}{2}\sqrt{(8-0)^2 + (0-6)^2}$

$= 5$ 1M

Area of the circle $= \pi(5)2$

= 78.5, cor. to 3 sig. fig.

> 75 1A

$\therefore$ The claim is disagreed.

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(3)

Q63.

(a) $2x + 3y - 13 = 0$

(b) $3x + 2y + 9 = 0$

Q64.

(a) $y - 2 = 0$, $4x + 3y - 6 = 0$

(b) $y - 5 = 0$, $24x + 7y - 35 = 0$

Q65. $x + y \pm 5\sqrt{2} = 0$

Q66. $x + y + 1 \pm \sqrt{42} = 0$

Q67. $x - y \pm 2\sqrt{2} = 0$

Q68. $2x + y = 0$, $2x + y + 10 = 0$

Q69. ± 5

Q70.

Let $P(p, q)$ be the point of contact of L and the circle.

Radius of the circle $= 4$

$\because$ $P(p, q)$ lies on L.

$\therefore$ $q = p$

Slope of $MP = \dfrac{p-4}{p-h}$

Slope of $L = 1$

$\because$ $MP \perp L$

$\therefore$ $\dfrac{p-4}{p-h} \cdot 1 = -1$ 1M

$$p - 4 = h - p$$

$$p = \frac{h+4}{2}$$

$\because$ P lies on the circle.

$\therefore$ $MP = 4$

$\sqrt{(p-h)^2 + (p-4)^2} = 4$ 1M

$$\left(\frac{h+4}{2} - h\right)^2 + \left(\frac{h+4}{2} - 4\right)^2 = 16$$

$$\left(\frac{4-h}{2}\right)^2 + \left(\frac{h-4}{2}\right)^2 = 16$$

$$(4-h)^2 + (h-4)^2 = 64$$

$$2h^2 - 16h - 32 = 0$$

$$h^2 - 8h - 16 = 0$$

$$h = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-16)}}{2(1)}$$

$= \underline{\underline{4 + 4\sqrt{2}}}$ or $4 - 4\sqrt{2}$ (rejected) 1A

Q71.

(a) **(i)** $\angle ACB = 90°$ ($\angle$ in semi-circle)

$\because$ *O* is the orthocentre of $\triangle OBD$.

$\therefore$ $\angle BOD = 90°$

$\therefore$ *O*, *C*, *B* and *D* are concyclic. (converse of $\angle$s in the same segment)

(ii) $\angle OBC = \angle OCD$ ($\angle$ in alt. segment)

$\angle OBC = \angle ODC$ ($\angle$s in the same segment)

$\because$ $\angle OCD = \angle ODC$

$\therefore$ *OD* = *OC* (sides opp. equal $\angle$s)

$\therefore$ $\triangle OCD$ is an isosceles triangle.

(b) **(i)** $\because$ *OD* = *OC* (proved in (a)(ii))

$\therefore$ *OD* = 4 units

$\therefore$ $AD = \sqrt{3^2 + 4^2}$ units $= 5$ units

$\because$ $\angle OBC = \angle ODC$ (proved in (a)(ii))

and $\angle CAB = \angle OAD$ (vert. opp. $\angle$s)

$\therefore$ $\triangle CAB \sim \triangle OAD$ (AAA)

$\therefore$ $\dfrac{AD}{AB} = \dfrac{OA}{CA}$ (corr. sides, ~$\triangle$s)

$\dfrac{5 \text{ units}}{AB} = \dfrac{3}{2.4}$

$AB = 4$ units

$\therefore$ *OB* = (3 + 4) units = 7 units

$\therefore$ The coordinates of *B* are (7, 0).

(ii) The coordinates of *D* are (0, 4).

Let the equation of the circle be $x^2 + y^2 + k_1x + k_2y + k_3 = 0$,

where k_1, k_2 and k_3 are constants.

$$\begin{cases} 0^2 + 0^2 + k_1(0) + k_2(0) + k_3 = 0 \\ 0^2 + 4^2 + k_1(0) + k_2(4) + k_3 = 0 \\ 7^2 + 0^2 + k_1(7) + k_2(0) + k_3 = 0 \end{cases}$$

By solving, we have $k_1 = -7$, $k_2 = -4$ and $k_3 = 0$.

$\therefore$ The equation of the circle is $x^2 + y^2 - 7x - 4y = 0$.

Q72.

(a) Since *D* is on the chord *BC*, *BC* is the diameter.

$\angle BAC = 90°$ ($\angle$ in semi-circle)

$\therefore$ $\triangle ABC$ is a right-angled triangle.

$\angle ACB = \angle CAM$ (alt. $\angle$s, CB // MN)

$\angle ABC = \angle CAM$ ($\angle$ in alt. segment)

$\therefore$ $\angle ACB = \angle ABC$

$\therefore$ $AC = AB$ (sides opp. equal $\angle$s)

$\therefore$ $\triangle ABC$ is an isosceles triangle.

$\therefore$ $\triangle ABC$ is a right-angled isosceles triangle.

(b) **(i)** From (a), C can be obtained by rotating B anti-clockwise about A through 90°.

$\therefore$ Coordinates of $C = (-2, 4)$

$\because$ D is the mid-point of BC.

$\therefore$ Coordinates of D

$$= \left(\frac{-2+4}{2}, \frac{4+2}{2} \right) = (1, 3)$$

$$CD = \sqrt{(-2-1)^2 + (4-3)^2} = \sqrt{10}$$

$\therefore$ The equation of the circle:

$$(x-1)^2 + (y-3)^2 = 10$$

$$x^2 - 2x + 1 + y^2 - 6y + 9 = 10$$

$$x^2 + y^2 - 2x - 6y = 0$$

(ii) $\because$ D is the mid-point of AE.

$\therefore$ Coordinates of $E = (2, 6)$

Slope of the required tangent $= \text{slope of } BC$

$$= \frac{4-2}{-2-4}$$

$$= -\frac{1}{3}$$

$\therefore$ The required equation of the tangent is:

$$\frac{y-6}{x-2} = -\frac{1}{3}$$

$$x + 3y - 20 = 0$$

Q73.

(a) $\because$ The x-axis divides C into two equal parts.

$\therefore$ The centre of C lies on the x-axis.

Let $(k , 0)$ be the coordinates of G.

$\sqrt{(12-k)^2+(6-0)^2} = \sqrt{(4-k)^2+(-10-0)^2}$

$144 - 24k + k^2 + 36 = 16 - 8k + k^2 + 100$

$-16k = -64$

$k = 4$

$\therefore$ <u>The coordinates of G are $(4 , 0)$.</u>

Radius of C

$= \sqrt{(12-4)^2+(6-0)^2}$ (or $\sqrt{(4-4)^2+(-10-0)^2}$)

$= 10$

The equation of C is

$(x-4)^2 + (y-0)^2 = 10^2$

$x^2 - 8x + 16 + y^2 = 100$

<u>$x^2 + y^2 - 8x - 84 = 0$</u>

Coordinates of G

$= \left(-\frac{-8}{2}, 0\right)$

$=$ <u>$(4 , 0)$</u>

(b) **(i)** Slope of $GS = \frac{0-6}{4-12} = \frac{3}{4}$

Slope of $L_1 = \frac{-1}{\frac{3}{4}} = -\frac{4}{3}$

The equation of L_1 is

$y - 6 = -\frac{4}{3}(x-12)$

$3y - 18 = -4x + 48$

<u>$4x + 3y - 66 = 0$</u>

(ii) $\because$ GT is a vertical line.

$\therefore$ L_2 is a horizontal line passing through $T(4, -10)$.

$\therefore$ The equation of L_2 is $y = -10$.

$$\begin{cases} 4x+3y-66=0 \text{ ....} \textbf{(3)} \\ y=-10 \text{ ....} \textbf{(4)} \end{cases}$$

Substitute **(4)** into **(3)**.

$$\begin{aligned} 4x + 3(-10) - 66 &= 0 \\ 4x &= 96 \\ x &= 24 \end{aligned}$$

$\therefore$ <u>The coordinates of P are $(24, -10)$.</u>

(iii) $\angle GSP = 90°$

$\angle GTP = 90°$

$$\begin{aligned} \angle GSP + \angle GTP &= 90° + 90° \\ &= 180° \end{aligned}$$

$\therefore$ G, S, P and T are concyclic.

$\therefore$ The circle passing through G, S, P and T is the circumcircle of $\triangle GST$

$\because$ GP is a diameter of the circumcircle.

$\therefore$ Q is the mid-point of GP.

Coordinates of Q

$= \left(\frac{4+24}{2}, \frac{0+(-10)}{2} \right)$

$= (14, -5)$

$GQ = \sqrt{(14-4)^2 + (-5-0)^2}$

$= \sqrt{125}$

$\therefore$ Area of $\triangle GSQ$: area of $\triangle PSR$ $= GQ : PR$

$= \sqrt{125} : \sqrt{20}$

$= \underline{\underline{5 : 2}}$

Q74.

(a) Let k be the y-intercept of L, where k is a positive constant.

Then the equation of L is $y = 2x + k$. 1M

$$\begin{cases} x^2 + y^2 + 6x - 8y - 20 = 0 \text{ .... (1)} \\ y = 2x + k \text{ .... (2)} \end{cases}$$

Substitute (2) into (1),

$$x^2+(2x+k)^2+6x-8(2x+k)-20=0$$
$$x^2+4x^2+4kx+k^2+6x-16x-8k-20=0$$
$$5x^2+(4k-10)x+(k^2-8k-20)=0$$

$\because$ L touches C.

$\therefore$ $(4k-10)^2-4(5)(k^2-8k-20)=0$ 1M

$$16k^2-80k+100-20k^2+160k+400=0$$
$$-4k^2+80k+500=0$$
$$k^2-20k-125=0$$
$$(k-25)(k+5)=0$$
$$k=25 \text{ or } -5 \text{ (rejected)}$$

$\therefore$ The equation of L is

$$y=2x+25$$
$$\underline{\underline{2x-y+25=0}}$$ 1A

(b) $\begin{cases} x^2+y^2+6x-8y-20=0 \text{ .... (1)} \\ y=2x+25 \text{ .... (2)} \end{cases}$

Substitute (2) into (1),

$x^2+(2x+25)^2+6x-8(2x+25)-20=0$ 1M

$$x^2+4x^2+100x+625+6x-16x-200-20=0$$
$$5x^2+90x+405=0$$
$$x^2+18x+81=0$$
$$(x+9)^2=0$$
$$x=-9$$

Substitute $x=-9$ into (2),

$$y=2(-9)+25$$
$$=7$$

$\therefore$ The coordinates of P are $(-9, 7)$. 1A

Q75.

(a) Let $G(h, k)$ be the centre of C_1.

$\because$ G lies on L.

$\therefore$ $h-k-3=0$

$h=k+3$ .... (1)

$\because$ A and B both lie on C_1.

$\therefore$ $$GA = GB$$

$$\sqrt{[h-(-4)]^2+[k-(-1)]^2} = \sqrt{(h-0)^2+(k-3)^2}$$ 1M

$$(h+4)^2+(k+1)^2 = h^2+(k-3)^2$$

$$h^2+8h+16+k^2+2k+1 = h^2+k^2-6k+9$$

$$8h+8k+8=0$$

$$h+k+1=0 \text{ .... (2)}$$

Substitute (1) into (2),

$k+3+k+1=0$ 1M

$2k=-4$

$k=-2$

Substitute $k=-2$ into (1),

$h=-2+3$

$=1$

$\therefore$ The coordinates of the centre of C_1 are (1, −2). 1A

(b) Radius $=\sqrt{(1-0)^2+(-2-3)^2}$

$=\sqrt{26}$

$\therefore$ The equation of C_1 is

$(x-1)^2+[y-(-2)]^2=(\sqrt{26})^2$ 1M

$x^2+y^2-2x+4y-21=0$ 1A

(c) Let $M(p, 0)$ be the centre of C_2.

$\because$ C_1 and C_2 touch each other at A.

$\therefore$ M, A and G are collinear.

$\therefore$ Slope of GA = Slope of MG

$$\frac{-2-(-1)}{1-(-4)} = \frac{0-(-2)}{p-1}$$ 1M

$$\frac{-1}{5} = \frac{2}{p-1}$$

$$-p+1=10$$

$$p=-9$$

$\therefore$ The equation of C_2 is

$[x-(-9)]^2+(y-0)^2=(\sqrt{26})^2$

$x^2+y^2+18x+55=0$

Q76.

(a) Let the required equation be $x^2 + y^2 + Dx + Ey + F = 0$.	1A
(0 , 0), (4 , 12) and (4 , –4) satisfy the above equation.	
$0^2 + 0^2 + D(0) + E(0) + F = 0$	
$F = 0$ ....**(1)**	
$4^2 + 12^2 + D(4) + E(12) + F = 0$	1M
$160 + 4D + 12E + F = 0$ ....**(2)**	
$4^2 + (-4)^2 + D(4) + E(-4) + F = 0$	
$32 + 4D - 4E + F = 0$ ....**(3)**	
Substitute **(1)** into **(2)**.	
$160 + 4D + 12E = 0$	
$40 + D + 3E = 0$ ....**(4)**	
Substitute **(1)** into **(3)**.	
$32 + 4D - 4E = 0$	
$8 + D - E = 0$ ....**(5)**	
(4) – **(5)**: $32 + 4E = 0$	
$4E = -32$	
$E = -8$	
Substitute $E = -8$ into **(5)**.	
$8 + D - (-8) = 0$	
$D = -16$	
$\therefore$ The required equation is $x^2 + y^2 - 16x - 8y = 0$.	1A
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	(3)

(b) $\begin{cases} x^2+y^2-16x-8y=0 \text{ .... } \textbf{(8)} \\ y=kx+10 \text{ .... } \textbf{(9)} \end{cases}$

Substitute **(9)** into **(8)**.

$x^2 + (kx + 10)^2 - 16x - 8(kx + 10) = 0$ | 1M

$x^2 + k^2x^2 + 20kx + 100 - 16x - 8kx - 80 = 0$

$(1 + k^2)x^2 + (12k - 16)x + 20 = 0$ ...**(10)**

$\because$ Γ and L do not intersect.

$\therefore$ Discriminant Δ of **(10)** < 0

$(12k - 16)^2 - 4(1 + k^2)(20) < 0$ | 1M

$144k^2 - 384k + 256 - 80 - 80k^2 < 0$

$64k^2 - 384k + 176 < 0$

$4k^2 - 24k + 11 < 0$

$(2k - 1)(2k - 11) < 0$

$\underline{\frac{1}{2} < k < \frac{11}{2}}$ | 1A

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(3)

(c) **(i)** Take $k = 5$.

Coordinates of P

$= (28 , 5 - 1)$

$= (28 , 4)$

Let m be the slope of L', where $m < 0$.

The equation of L' is

$y - 4 = m(x - 28)$

$y - 4 = mx - 28m$

$y = mx - (28m - 4)$ ....**(11)**

Substitute **(11)** into **(8)**.

$x^2 + [mx - (28m - 4)]^2 - 16x - 8[mx - (28m - 4)] = 0$ | 1M

$x^2 + m^2x^2 - 2(mx)(28m - 4) + (28m - 4)^2 - 16x - 8mx + 224m - 32 = 0$

$(1 + m^2)x^2 - 56m^2x + 8mx + 784m^2 - 224m + 16 - 16x - 8mx + 224m - 32 = 0$

$(1 + m^2)x^2 - (56m^2 + 16)x + 784m^2 - 16 = 0$ ....**(12)**

Since L' touches Γ,

$$\Delta \text{ of } \mathbf{(12)} = 0$$

$$[-(56m^2 + 16)]^2 - 4(1 + m^2)(784m^2 - 16) = 0$$ 1M

$$3\,136m^4 + 1\,792m^2 + 256 - 4(784m^4 + 768m^2 - 16) = 0$$

$$3\,136m^4 + 1\,792m^2 + 256 - 3\,136m^4 - 3\,072m^2 + 64 = 0$$

$$1\,280m^2 = 320$$

$$m^2 = \frac{1}{4}$$

$$m = -\frac{1}{2} \text{ or } \frac{1}{2}$$

(*rejected*)

Substitute $m = -\frac{1}{2}$ into **(12)**.

$$\left[1+\left(-\frac{1}{2}\right)^2\right]x^2 - \left[56\left(-\frac{1}{2}\right)^2+16\right]x + 784\left(-\frac{1}{2}\right)^2 - 16 = 0$$ 1M

$$\frac{5}{4}x^2 - 30x + 180 = 0$$

$$x^2 - 24x + 144 = 0$$

$$(x - 12)^2 = 0$$

$$x = 12$$

Substitute $x = 12$ and $m = -\frac{1}{2}$ into **(11)**.

$$y = \left(-\frac{1}{2}\right)(12) - \left[28\left(-\frac{1}{2}\right) - 4\right]$$

$$= 12$$

$\therefore$ The coordinates of S are (12 , 12). 1A

(ii) Let R be the mid-point of AB.	
Coordinates of R	
$= \left(\frac{4+4}{2}, \frac{12+(-4)}{2} \right)$	
$= (4, 4)$	
Slope of OR	
$= \frac{4-0}{4-0}$	
$= 1$	
Slope of OS	
$= \frac{12-0}{12-0}$	
$= 1$	
$\because$ Slope of OR = slope of OS	
$\therefore$ O, R and S are collinear.	
$\because$ G lies on OR.	
$\therefore$ O, G and S are collinear.	1M
$\therefore$ There does not exist a circle passing through O, G and S.	1A
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	(6)

Q77.

(a) Join OB.

In ΔOBD,

$\angle OBD = 90°$ (tangent $\perp$ radius)

$\angle BOD + \angle OBD + \angle ODB = 180°$ ($\angle$ sum of Δ)

$\angle BOD + 90° + 30° = 180°$

$\angle BOD = 60°$

$\angle OCB = \frac{\angle BOD}{2}$ ($\angle$ at centre = 2 $\angle$ at $\odot^{ce}$)

$= 30°$

$\angle OCE = 90°$ (tangent $\perp$ radius)

$\angle ODA = \angle ODB$ (tangents from ext. pt.)

$= 30°$

$\because$ $\angle BDE + \angle BCE = (30° + 30°) + (30° + 90°)$

$= 180°$

$\therefore$ $BDEC$ is a cyclic quadrilateral. (opp. $\angle$s supp.)

(b) **(i)** $\because$ $\angle BCD = \angle CDE = 30°$

$\therefore$ $BC // DE$ (alt. $\angle$s equal)

$\because$ BC is parallel to the y-axis.

$\therefore$ DE is parallel to the y-axis.

$\because$ $OA \perp DE$ (tangent $\perp$ radius)

$\therefore$ A lies on the x-axis. 1M

$$OB = \sqrt{(0-1)^2 + (0-\sqrt{3})^2}$$
$$= 2$$

$OA = OB$ (radii)

$= 2$

$\therefore$ The coordinates of A is $(-2, 0)$. 1A

(ii) In ΔDCE,

$\angle DEC + \angle DCE + \angle CDE = 180°$ ($\angle$ sum of Δ)

$\angle DEC + 90° + 30° = 180°$

$\angle DEC = 60°$

Join OE.

$\angle OEA = \angle OEC$ (tangents from ext. pt.)

$$= \frac{60°}{2}$$
$$= 30°$$

In ΔOAD and ΔOAE,

$OA = OA$ (common side)

$\angle ODA = \angle OEA = 30°$

$\angle OAD = \angle OAE = 90°$

$\therefore$ $\Delta OAD \cong \Delta OAE$ (A.A.S.)

$\therefore$ $AD = AE$ (corr. sides, $\cong$ Δs)

$\therefore$ A is the mid-point of DE.

(iii) $\because$ $\angle DCE = 90°$

$\therefore$ DE is a diameter of the circle $BDEC$. (converse of $\angle$ in semi-circle)

$\therefore$ A is the centre of the circle $BDEC$.

$$\tan\angle ODA = \frac{OA}{DA}$$

$$DA = \frac{OA}{\tan\angle ODA}$$

$$= \frac{2}{\tan 30^\circ}$$

$$= 2\sqrt{3}$$

$\therefore$ The equation of the circle $BDEC$ is

$[x-(-2)]^2 + (y-0)^2 = (2\sqrt{3})^2$ 1M

$\underline{\underline{x^2 + y^2 + 4x - 8 = 0}}$ 1A

Q78.

(a) Let the required equation be $x^2 + y^2 + Dx + Ey + F = 0$. 1M

(6 , 2), (1 , 2) and (5 , 4) satisfy the above equation.

$6^2 + 2^2 + D(6) + E(2) + F = 0$ 1M

$40 + 6D + 2E + F = 0$ ....**(1)**

$1^2 + 2^2 + D(1) + E(2) + F = 0$

$5 + D + 2E + F = 0$ ....**(2)**

$5^2 + 4^2 + D(5) + E(4) + F = 0$

$41 + 5D + 4E + F = 0$ ....**(3)**

(1) – **(2)**: $35 + 5D = 0$

$5D = -35$

$D = -7$

(3) – **(2)**: $36 + 4D + 2E = 0$ ....**(4)**

Substitute $D = -7$ into **(4)**.

$36 + 4(-7) + 2E = 0$

$2E = -8$

$E = -4$

Substitute $D = -7$ and $E = -4$ into **(2)**.

$5 + (-7) + 2(-4) + F = 0$

$F = 10$ 1M

$\therefore$ <u>The required equation is $x^2 + y^2 - 7x - 4y + 10 = 0$.</u> 1A

(b) (i) $\begin{cases} y = mx & \text{.......... (5)} \\ x^2 + y^2 - 7x - 4y + 10 = 0 & \text{.......... (6)} \end{cases}$

Substitute **(5)** into **(6)**.

$x^2 + (mx)^2 - 7x - 4mx + 10 = 0$ 1M

$(1 + m^2)x^2 + (-7 - 4m)x + 10 = 0$

$\because$ x_1 and x_2 are the roots of the above equation.

$\therefore$ $\underline{x_1 x_2 = \dfrac{10}{1+m^2}}$ 1

(ii) $OP = \sqrt{(x_1 - 0)^2 + (y_1 - 0)^2}$ 1M

$= \sqrt{x_1^2 + y_1^2}$

$= \sqrt{x_1^2 + (mx_1)^2}$

$= \sqrt{1+m^2}\, x_1$

$OQ = \sqrt{(x_2 - 0)^2 + (y_2 - 0)^2}$

$= \sqrt{x_2^2 + y_2^2}$ 1A either

$= \sqrt{x_2^2 + (mx_2)^2}$

$= \sqrt{1+m^2}\, x_2$

$\therefore$ $OP \times OQ = (\sqrt{1+m^2}\, x_1)(\sqrt{1+m^2}\, x_2)$

$= (1 + m^2)x_1 x_2$

$= (1 + m^2)\left(\dfrac{10}{1+m^2}\right)$ 1M

$= \underline{\underline{10}}$ 1A

Q79.

(a) Let k be the y-coordinate of G.

$\because$ G is a point on $y = 2x$.

$\therefore$ $k = 2h$ 1M

Radius $= \sqrt{(h-1)^2 + (k-4)^2}$

$= \sqrt{(h-1)^2 + (2h-4)^2}$ 1M

$= \sqrt{h^2 - 2h + 1 + 4h^2 - 16h + 16}$

$= \sqrt{5h^2 - 18h + 17}$

The equation of C is

$(x - h)^2 + (y - 2h)^2 = (\sqrt{5h^2 - 18h + 17})^2$ 1M

$x^2 - 2hx + h^2 + y^2 - 4hy + 4h^2 = 5h^2 - 18h + 17$

$\underline{x^2 + y^2 - 2hx - 4hy + 18h - 17 = 0}$ 1

(b) $\because$ C passes through $(-3, -8)$.

$\therefore$ $(-3)^2 + (-8)^2 - 2h(-3) - 4h(-8) + 18h - 17 = 0$

1M

$9 + 64 + 6h + 32h + 18h - 17 = 0$

$56h = -56$

$h = -1$

Area of C

$= \pi[\sqrt{5(-1)^2 - 18(-1) + 17}]^2$ 1M

$= \underline{\underline{40\pi}}$ 1A

(c) Radius $= \sqrt{5h^2 - 18h + 17}$

$= \sqrt{5\left(h^2 - \frac{18}{5}h\right) + 17}$

$= \sqrt{5\left[h^2 - \frac{18}{5}h + \left(\frac{-\frac{18}{5}}{2}\right)^2 - \left(\frac{-\frac{18}{5}}{2}\right)^2\right] + 17}$ 1M

$= \sqrt{5\left(h - \frac{9}{5}\right)^2 + \frac{4}{5}}$

∴ When $h = \frac{9}{5}$, the radius is minimum. 1A

Coordinates of $G = \left(\frac{9}{5}, 2 \times \frac{9}{5}\right) = \left(\frac{9}{5}, \frac{18}{5}\right)$

Slope of the straight line passing through G and $A(1, 4) = \frac{4 - \frac{18}{5}}{1 - \frac{9}{5}} = -\frac{1}{2}$

∵ The tangent to C at A is perpendicular to the straight line passing through G and A.

∴ Slope of the tangent to C at $A = \frac{-1}{-\frac{1}{2}}$ 1M

$= 2$

Slope of $L = 2$

∵ Slope of the tangent to C at A = slope of L

∴ <u>The tangent to C at A is parallel to L.</u> 1A

Q80.

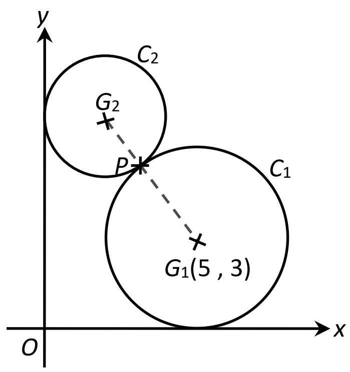
(a) Radius of $C_1 = 3$ 1A

∴ The equation of C_1 is

$(x - 5)^2 + (y - 3)^2 = 3^2$

$\underline{(x - 5)^2 + (y - 3)^2 = 9}$ 1A

(b) Let k be the y-coordinate of G_2.



Join G_1G_2.

$G_1G_2 = G_1P + PG_2$

$\sqrt{(2-5)^2+(k-3)^2} = 3 + 2$ 1M

$9 + (k-3)^2 = 25$

$(k-3)^2 = 16$

$k - 3 = \pm 4$ 1M

$k = 7$ *or* -1 (*rejected*)

$\therefore$ Coordinates of $G_2 = (2 , 7)$

Coordinates of $P = \left(\frac{2\times 5+3\times 2}{3+2}, \frac{2\times 3+3\times 7}{3+2}\right)$ 1M

$= \left(\frac{16}{5}, \frac{27}{5}\right)$ 1A

(c) Slope of $G_1G_2 = \frac{7-3}{2-5} = -\frac{4}{3}$ 1A

Let m be the slope of the required tangent.

$\because$ The required tangent $\perp G_1G_2$

$\therefore$ $m\left(-\frac{4}{3}\right) = -1$ 1M

$m = \frac{3}{4}$

The equation of the required tangent is

$y - \frac{27}{5} = \frac{3}{4}\left(x - \frac{16}{5}\right)$ 1M

$20y - 108 = 15x - 48$

$15x - 20y + 60 = 0$

$3x - 4y + 12 = 0$ 1A

Q81.

19. (a) (i)

$r^2 = \left(\frac{k-4}{2}\right)^2 + \left(\frac{2+k}{2}\right)^2 - (-3k-6)$

$r^2 = \frac{k^2-8k+16+4+4k+k^2+4(3k+6)}{4}$

$r^2 = \frac{2k^2+8k+44}{4}$

$r^2 = \frac{k^2+4k+22}{2}$

(ii) $r = \sqrt{\dfrac{k^2+4k+22}{2}}$

$= \sqrt{\dfrac{1}{2}(k^2+4k)+11}$

$= \sqrt{\dfrac{1}{2}\left[k^2+4k+\left(\dfrac{4}{2}\right)^2-\left(\dfrac{4}{2}\right)^2\right]+11}$

$= \sqrt{\dfrac{1}{2}(k^2+4k+4)-2+11}$

$= \sqrt{\dfrac{1}{2}(k+2)^2+9}$

When $k = -2$, the value of r is the least.

$\therefore$ The required equation is

$$x^2 + y^2 + (-2-4)x + [2+(-2)]y - 3(-2) - 6 = 0$$

$$x^2 + y^2 - 6x = 0$$

(b) The equation of L is $y = mx + h$.

(i) Substitute $y = mx + h$ into $x^2 + y^2 - 6x = 0$.

$$x^2 + (mx+h)^2 - 6x = 0$$

$$x^2 + m^2x^2 + 2mhx + h^2 - 6x = 0$$

$$(1+m^2)x^2 + (2mh-6)x + h^2 = 0$$

$\because$ The equation has only one real root.

$\therefore$ $\Delta = 0$

$$(2mh-6)^2 - 4(1+m^2)h^2 = 0$$

$$4m^2h^2 - 24mh + 36 - 4h^2 - 4m^2h^2 = 0$$

$$36 - 4h^2 = 24mh$$

$$m = \frac{9-h^2}{6h}$$

(ii) Let M be the mid-point of OA and G be the centroid of $\triangle OAP$.

Coordinates of A $= \left(-\frac{-6}{2}, 0\right)$

$= (3, 0)$

Coordinates of $M = \left(\frac{3}{2}, 0\right)$

$\because$ $PG : GM = 2 : 1$

$\therefore$ Coordinates of $G = \left(\frac{1(0) + 2\left(\frac{3}{2}\right)}{2+1}, \frac{1(h) + 2(0)}{2+1}\right)$

$= \left(1, \frac{h}{3}\right)$

Let M be the mid-point of OA and N be the mid-point of OP.

Coordinates of A $= \left(-\frac{-6}{2}, 0\right)$

$= (3, 0)$

Coordinates of $M = \left(\frac{3}{2}, 0\right)$

Coordinates of $N = \left(0, \frac{h}{2}\right)$

The equation of MP is

$y = \frac{h-0}{0-\frac{3}{2}}x + h$

$y = -\frac{2h}{3}x + h$ ………………… (1)

The equation of NA is

$y = \frac{\frac{h}{2}-0}{0-3}x + \frac{h}{2}$

$y = -\frac{h}{6}x + \frac{h}{2}$ ………………… (2)

Substitute (1) into (2).

$-\frac{2h}{3}x + h = -\frac{h}{6}x + \frac{h}{2}$

$-\frac{h}{2}x = -\frac{h}{2}$

$x = 1$

Substitute $x = 1$ into (1).

$y = -\frac{2h}{3}(1) + h = \frac{h}{3}$

$\therefore$ Coordinates of the centroid of $\triangle OAP = \left(1, \frac{h}{3}\right)$

Substitute x = 1 and y = $\frac{h}{3}$ into x2 + y2 – 6x = 0.

12 + $\left(\frac{h}{3}\right)^2$ – 6(1) = 0

$\frac{h^2}{9}$ = 5

h2 = 45

h = $\sqrt{45}$ or $-\sqrt{45}$ (rejected)

Slope of PQ = m

$\frac{0-\sqrt{45}}{q-0} = \frac{9-(\sqrt{45})^2}{6(\sqrt{45})}$

q = $\frac{15}{2}$

∴Coordinates of Q = $\left(\frac{15}{2}, 0\right)$

Area of △OPQ

= $\frac{1}{2}$ × OP × OQ

= $\frac{1}{2}$ × ($\sqrt{45}$ – 0) × $\left(\frac{15}{2} - 0\right)$

≈ 25.155 764 75

> 25

∴The claim is disagreed.

Q82.

(a) When $y = 0$, $x^2 + 0^2 + 5x - 0 - 6 = 0$

$x^2 + 5x - 6 = 0$

$(x+6)(x-1) = 0$

$x = -6$ or $x = 1$

∴ Coordinates of P = $(-6, 0)$

Coordinates of Q = $(1, 0)$

When $x = 0$, $0^2 + y^2 + 5(0) - y - 6 = 0$

$y^2 - y - 6 = 0$

$(y+2)(y-3) = 0$

$y = -2$ or $y = 3$ (rej.)

$\therefore$ Coordinates of $R = (0, -2)$

(b) Coordinates of the mid-point of $PR = \left(\frac{-6+0}{2}, \frac{0+(-2)}{2}\right) = (-3, -1)$

Let m be the slope of the perpendicular bisector of PR.

$$\text{Slope of } PR = \frac{-2-0}{0-(-6)} = -\frac{1}{3}$$

$$m \times \text{slope of } PR = -1$$

$$m \times \left(-\frac{1}{3}\right) = -1$$

$$m = 3$$

The equation of the perpendicular bisector of PR is

$$y-(-1) = 3[x-(-3)]$$

$$y + 1 = 3x + 9$$

$$3x - y + 8 = 0$$

(c) $\because$ SQ // PR

$\therefore$ Slope of SQ = slope of PR

$$= -\frac{1}{3}$$

The equation of SQ is

$$y - 0 = -\frac{1}{3}(x-1)$$

$$3y = -x+1$$

$$x + 3y - 1 = 0$$

Q83.

(a)

$$\text{Slope of } PQ = \frac{-2-6}{1-7} = \frac{4}{3}$$

(b) Coordinates of the mid-point of $PQ = \left(\frac{1+7}{2}, \frac{-2+6}{2}\right) = (4, 2)$

Let m be the slope of the perpendicular bisector of PQ.

$$\therefore m \times \text{slope of } PQ = -1$$

$$m \times \frac{4}{3} = -1$$

$$m = -\frac{3}{4}$$

The equation of the perpendicular bisector of PQ is

$$y-2 = -\frac{3}{4}(x-4)$$

$$4y - 8 = -3x+12$$

$3x + 4y - 20 = 0$

(c) $\begin{cases} x+2y-6=0 \text{ —— ①} \\ 3x+4y-20=0 \text{ —— ②} \end{cases}$

①×2, $2x+4y-12=0$ ——③

②−③, $0 = x-8$

$x = 8$

Substitute $x = 8$ into ①,

$8+2y-6 = 0$

$2y+2 = 0$

$y = -1$

Radius of C $= \sqrt{(8-7)^2+(-1-6)^2} = \sqrt{50}$

The general equation of C is

$(x-8)^2+[y-(-1)]^2 = (\sqrt{50})^2$

$(x^2-16x+64)+(y^2+2y+1) = 50$

$x^2+y^2-16x+2y+15 = 0$

∴ Coordinates of the centre of $C = (8,-1)$

Q84.

(a) C_1: $x^2+y^2+6x-4y+12 = 0$

$(x+3)^2-9+(y-2)^2-4+12 = 0$

$(x+3)^2+(y-2)^2 = 1$

∴ Centre of $C_1 = (-3, 2)$, radius of C_1 = 1

(b) Distance between two centres

$= \sqrt{(1+3)^2+(5-2)^2}$

$= 5$

∴ Radius of C_2

$= 5 - 1$ or $= 5 + 1$

$= 4$ $= 6$

∴ Equation of C_2 is

$(x-1)^2+(y-5)^2 = 4^2$

$x^2+y^2-2x-10y+10 = 0$

or

$(x-1)^2+(y-5)^2 = 6^2$

$x^2+y^2-2x-10y-10 = 0$

Q85.

(a) Radius = distance between the centre and (2 , 5)

$= \sqrt{(2-3)^2+(5-2)^2}$ 1M

$= \sqrt{10}$

∴ The equation of the circle is

$(x-3)^2 + (y-2)^2 = (\sqrt{10})^2$ 1M

$x^2 - 6x + 9 + y^2 - 4y + 4 = 10$

$\underline{x^2 + y^2 - 6x - 4y + 3 = 0}$ 1A

(b) $\begin{cases} y=3x+k \text{ .... } \mathbf{(1)} \\ x^2+y^2-6x-4y+3=0 \text{ .... } \mathbf{(2)} \end{cases}$

Substitute **(1)** into **(2)**.

$x^2 + (3x + k)^2 - 6x - 4(3x + k) + 3 = 0$ 1M

$x^2 + 9x^2 + 6kx + k^2 - 6x - 12x - 4k + 3 = 0$

$10x^2 + (6k - 18)x + k^2 - 4k + 3 = 0$

Since the straight line and the circle intersect, $\Delta \geq 0$.

$(6k - 18)^2 - 4(10)(k^2 - 4k + 3) \geq 0$ 1M

$36k^2 - 216k + 324 - 40k^2 + 160k - 120 \geq 0$

$4k^2 + 56k - 204 \leq 0$

$k^2 + 14k - 51 \leq 0$

$(k + 17)(k - 3) \leq 0$ 1M

$\underline{-17 \leq k \leq 3}$ 1A

Q86. C
Q87. B
Q88. C
Q89. A
Q90. D
Q91. B
Q92. D
Q93. A
Q94. D
Q95. B
Q96. C
Q97. B
Q98. B
Q99. D
Q100. A