Q1 – Q46. 課堂內容

Q47.

$AB = AC$

$\angle ACB = \angle ABC$

$= 52^\circ$

In $\triangle ABC$,

$\angle BAC + \angle ACB + \angle ABC = 180^\circ$

$\angle BAC + 52^\circ + 52^\circ = 180^\circ$

$\angle BAC = 76^\circ$

$\angle CDE = \angle BAC$

$= 76^\circ$

Q48.

(a) Join OA.

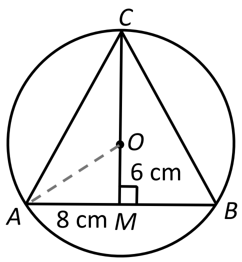
In $\triangle OAM$,

$OA^2 = AM^2 + OM^2$

$OA = \sqrt{8^2 + 6^2}$ cm

$= 10$ cm

$\therefore$ The radius of the circle is 10 cm.



(b) $\because$ $CM \perp AB$

$\therefore$ $BM = AM$

$= 8$ cm

$CM = CO + OM$

$= (10 + 6)$ cm

$= 16$ cm

In $\triangle BCM$,

$BC^2 = CM^2 + BM^2$

$BC = \sqrt{16^2 + 8^2}$ cm

$= \sqrt{320}$ cm (*or* $8\sqrt{5}$ cm)

Q49.

$\angle PQR = \angle POR$ *(property of rhombus)*

Reflex $\angle POR = 360^\circ - \angle POR$ *($\angle$s at a pt.)* 1M

$2\angle PQR$ = Reflex $\angle POR$ *($\angle$ at centre twice $\angle$ at $\odot^{ce}$)* 1M

$2\angle PQR = 360^\circ - \angle POR$

$3\angle PQR = 360^\circ$

$\angle PQR = 120^\circ$

$OM \perp PQ$ and $ON \perp QR$ *(line from centre to mid-pt. of chord $\perp$ chord)* 1M

In *OMQN*,

$$\angle MON + \angle OMQ + \angle PQR + \angle ONQ = 360^\circ \text{ (}\angle\text{ sum of polygon)}$$

$$\angle MON + 90^\circ + 120^\circ + 90^\circ = 360^\circ \quad \text{1M}$$

$$\angle MON = \underline{\underline{60^\circ}} \quad \text{1A}$$

Q50.

$\because$ $AB = AC$

$\therefore$ $\angle ABC = \angle ACB$

In $\triangle ABC$,

$\angle BAC + \angle ABC + \angle ACB = 180^\circ$

$45^\circ + 2\angle ACB = 180^\circ$

$\angle ACB = 67.5^\circ$

$\angle AEB = 90^\circ$

In $\triangle BCE$,

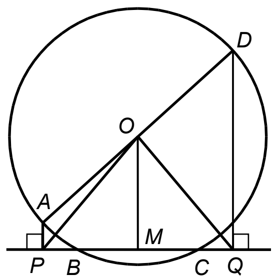
$\angle EBC + \angle ACB = \angle AEB$

$\angle EBC + 67.5^\circ = 90^\circ$

$\angle EBC = 22.5^\circ$

Q51.

(a) Join *OM*, where *M* is the mid-point of *BC*.



$OM \perp BC$ *(line from centre to mid-pt. of chord $\perp$ chord)* 1M

In ΔOMP and ΔOMQ,

$\because$ $BP = CQ$ *(given)*

$\therefore$ $MP = MQ$ 1M

$OP^2 = OM^2 + MP^2$ *(Pyth. theorem)*

$OQ^2 = OM^2 + MQ^2$ *(Pyth. theorem)*

$\therefore$ $OP = OQ$ 1M

(b) Radius of the circle = $\frac{34}{2}$ cm = 17 cm

In ΔOBM,

$OM^2 + BM^2 = OB^2$ (Pyth. theorem)

$OM = \sqrt{17^2 - \left(\frac{16}{2}\right)^2}$ cm

$= 15$ cm 1M

$\because$ $OP = OQ$ and $OM \perp PQ$

$\therefore$ $\angle MOP = \angle MOQ$ (property of isos. Δ) 1M

$= \frac{100°}{2}$

$= 50°$

In ΔOMP,

$\frac{MP}{OM} = \tan 50°$

$MP = 15 \tan 50°$ cm 1M

$\therefore$ Area of $\Delta OPQ = 2\left[\frac{1}{2}(MP)(OM)\right]$

$= 2\left[\frac{1}{2}(15\tan 50°)(15)\right]$ cm^2

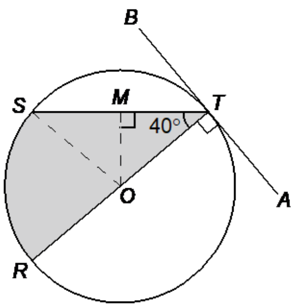
$= \underline{\underline{268 \text{ cm}^2}}$ (cor. to the nearest cm^2) 1A

Q52.

$\because AB$ 是圓在 T 的切線且 $RT \perp AB$

$\therefore$ RT 通過圓心。 (垂直切線且通過切點的直線通過圓心) 1M

設 O 為該圓的圓心，M 為 ST 上的一點使得 $OM \perp ST$ 及 r cm 為該圓的半徑。 連接 OM 和連接 OS。



$\angle ROS = 2\angle RTS$ (圓心角兩倍於圓周角)

$= 2 \times 40°$ 1M

$= 80°$

$$\frac{OM}{OT} = \sin \angle OTM$$

$$\frac{OM}{r \text{ cm}} = \sin 40°$$

$$OM = r \sin 40° \text{ cm}$$

$\because$ $OM \perp ST$ (作圖)

$\therefore$ $TM = SM$ (圓心至弦的垂線平分弦)

$= \frac{1}{2} ST$

$$\frac{TM}{OT} = \cos \angle OTM$$

$$\frac{\frac{1}{2} ST}{r \text{ cm}} = \cos 40°$$

$$ST = 2r \cos 40° \text{ cm}$$

$\because$ 陰影區域的面積 $= 27 \text{ cm}^2$

$\therefore$ 扇形 ORS 的面積 + ΔOST 的面積 $= 27 \text{ cm}^2$

$$\pi \times OR^2 \times \frac{80°}{360°} + \frac{1}{2} \times OM \times ST = 27 \text{ cm}^2$$ 1M

$$\frac{2\pi}{9} r^2 + \frac{1}{2} \times r \sin 40° \times 2r \cos 40° = 27$$

$$r^2 \left(\frac{2\pi}{9} + \sin 40° \cos 40° \right) = 27$$

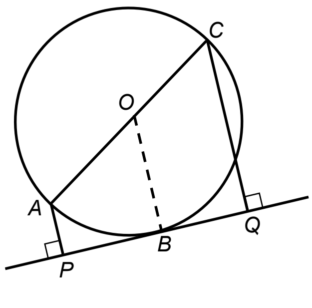
$$r^2 = \frac{27}{\frac{2\pi}{9} + \sin 40° \cos 40°}$$

$r = 4.76$ (準確至三位有效數字) 或 -4.76 (準確至三位有效數字) (捨去)

$\therefore$ 該圓的半徑是 4.76 cm。 1A

Q53.

(a) Join *OB*.



$OB \perp PQ$ *(tangent ⊥ radius)* 1M

$\therefore$ $AP \,//\, OB \,//\, CQ$ *(corr. ∠s equal)* 1M

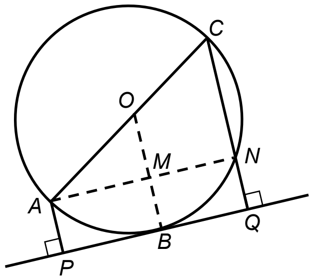
$\because$ $OA = OC$ (radii)

$\therefore$ $BP = BQ$ (intercept theorem) 1M

Thus, B is the mid-point of PQ.

(b) Let N be the point of intersection of the circle and CQ,

and AN cuts OB at M.



Note that $AN \perp OB$, $AN \perp CQ$ and $APQN$ is a rectangle.

$NQ = AP = 9$ cm

$CN = (25 - 9)$ cm $= 16$ cm

Note that $\Delta AMO \sim \Delta ANC$.

$$\frac{OM}{CN} = \frac{AO}{AC} \quad \textit{(corr. sides, ~}\Delta\textit{s)}$$

$$OM = \frac{1}{2}(16 \text{ cm})$$

$$OM = 8 \text{ cm}$$

$\therefore$ $OA = OB = (9 + 8)$ cm $= 17$ cm 1M

In ΔAMO,

$$AM^2 + OM^2 = OA^2 \quad \textit{(Pyth. theorem)}$$

$$AM = \sqrt{17^2 - 8^2} \text{ cm}$$

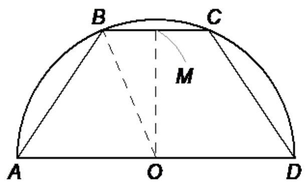
$$= 15 \text{ cm}$$ 1M

Area of $OAPB = \frac{1}{2}(9 + 17)(15)$ cm^2

$= \underline{\underline{195 \text{ cm}^2}}$ 1A

Q54.

Let O be the centre of the semi-circle. Join OB and join OM, where M is the mid-point of BC.



$\because$ $BM = MC$ (construction)

$\therefore\quad \angle OMB = 90^\circ$ (line joining centre and mid-pt. of chord $\perp$ chord) 1M

$OB = OA$ (radii)

$= \frac{1}{2}AD$

$= \frac{1}{2} \times 52 \text{ cm}$

$= 26 \text{ cm}$

In ΔOBM,

$OB^2 = OM^2 + BM^2$ (Pyth. theorem)

$BM = \sqrt{OB^2 - OM^2}$

$= \sqrt{26^2 - 24^2} \text{ cm}$ 1M

$= 10 \text{ cm}$

$BC = 2BM$

$= 2 \times 10 \text{ cm}$

$= 20 \text{ cm}$ 1A

$\therefore$ Area of the trapezium $ABCD$

$= \frac{1}{2}(BC + AD)(OM)$

$= \frac{1}{2}(20 + 52)(24) \text{ cm}^2$

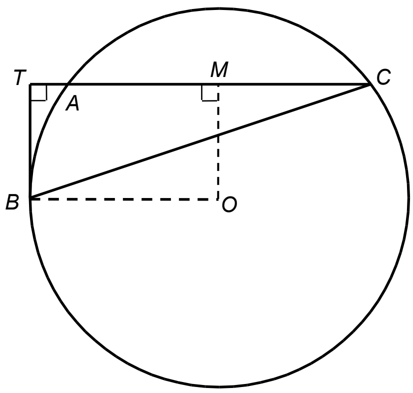
$= \underline{\underline{864 \text{ cm}^2}}$ 1A

Q55.

(a) Let *O* be the centre of the circle.

Let *M* be a point on *AC* such that $OM \perp AC$.

Join *OB*.



$OB \perp TB$ *(tangent $\perp$ radius)* 1M

$\therefore$ *OMTB* is a rectangle.

$\therefore$ $MT = OB = r$ cm 1M

$MC = MA$ *(line from centre ⊥ chord bisects chord)*

$= \frac{8}{2}$ cm

= 4 cm 1M

∴ $TC = \underline{\underline{(r + 4) \text{ cm}}}$ 1A

In ΔTBC,

$\frac{TB}{TC} = \tan \angle ACB$

$TB = \underline{\underline{(r + 4) \tan 18^\circ \text{ cm}}}$ 1A

(b) $OM = TB = (r + 4) \tan 18^\circ$ cm

In ΔOCM,

$OM^2 + CM^2 = OC^2$ *(Pyth. theorem)*

$[(r + 4) \tan 18^\circ]^2 + 4^2 = r^2$ 1M

$(1 - \tan^2 18^\circ)r^2 - (8 \tan^2 18^\circ)r - 16(1 + \tan^2 18^\circ) = 0$

∴ $r = \underline{\underline{4.94}}$ or -4 (rejected) 1A

(cor. to 3 sig. fig.)

Q56.

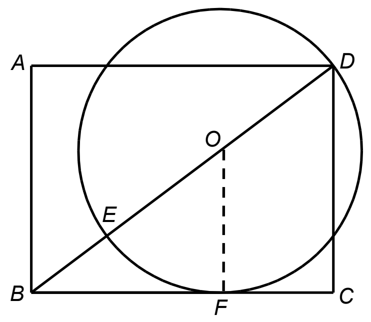
(a) In ΔBCD,

$BD^2 = BC^2 + DC^2$ *(Pyth. theorem)*

$BD = \sqrt{16^2 + 12^2}$ cm

= 20 cm 1M

Let O be the centre of the circle. Join OF.



$OF \perp BC$ *(tangent ⊥ radius)*

$OF \parallel DC$ *(corr. ∠s equal)*

In ΔBCD and ΔBFO,

$\angle CBD = \angle FBO$ *(common ∠)*

$\angle BDC = \angle BOF$ *(corr. ∠s, OF // DC)*

$\angle BCD = \angle BFO = 90^\circ$ *(proved)*

∴ $\Delta BCD \sim \Delta BFO$ *(AAA)* 1M

Let r cm be the radius of the circle.

$$\frac{OF}{DC} = \frac{BO}{BD} \quad \textit{(corr. sides, ~}\Delta\textit{s)}$$

$$\frac{r}{12} = \frac{20-r}{20} \qquad \text{1M}$$

$$20r = 240 - 12r$$

$$r = 7.5$$

$\therefore$ The radius of the circle is 7.5 cm. 1A

(b) $BE = (20 - 7.5 \times 2)$ cm = 5 cm

$BT = \frac{12}{2}$ cm = 6 cm

In ΔBET, by the cosine formula,

$$TE^2 = BE^2 + BT^2 - 2(BE)(BT)\cos\angle ABD$$

$$TE = \sqrt{5^2 + 6^2 - 2(5)(6)\left(\frac{12}{20}\right)} \text{ cm} \qquad \text{1M}$$

$$= 5 \text{ cm}$$

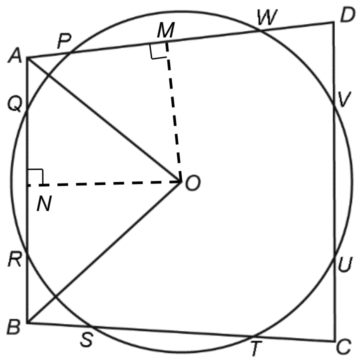
$\therefore$ $BT^2 \neq BE^2 + TE^2$

i.e., $\angle BET \neq 90°$ 1M

$\therefore$ TE is not a tangent to the circle. 1A

Q57.

(a) Let M and N be the points on PW and QR respectively such that $OM \perp PW$ and $ON \perp QR$.



In ΔOAM and ΔOAN,

$\angle OMA = \angle ONA = 90°$ *(by construction)*

$OA = OA$ *(common side)*

$OM = ON$ *(equal chords, equidistant from centre)* 1M

$\therefore$ $\Delta OAM \cong \Delta OAN$ *(RHS)* 1M

$\angle OAM = \angle OAN$ *(corr. $\angle$s, $\cong\Delta$s)* 1M

$\therefore$ OA bisects $\angle BAD$.

(b) Similar to **(a)**, *OB* bisects $\angle ABC$, i.e., $\angle OBA = \angle OBC$.

In ΔOAB,

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ \quad (\angle \text{ sum of } \Delta)$$

$$\angle OAB + \angle OBA + 78^\circ = 180^\circ$$

$$\angle OAB + \angle OBA = 102^\circ \qquad \text{1M}$$

$\because$ $\angle OAB = \angle OAD$ and $\angle OBA = \angle OBC$

$\therefore$ $\angle ABC + \angle BAD = 102^\circ \times 2$

$= 204^\circ$

In *ABCD*,

$$\angle ABC + \angle BAD + \angle ADC + \angle BCD = 360^\circ \quad (\angle \text{ sum of polygon})$$

$$204^\circ + 86^\circ + \angle BCD = 360^\circ \qquad \text{1M}$$

$$\angle BCD = \underline{\underline{70^\circ}} \qquad \text{1A}$$

Q58.

(a) $\angle ACB = \angle ADB$	1M
$= a$	
$\angle ABC = 90^\circ$	1M
In $\triangle ABC$,	
$\angle CAB + \angle ACB + \angle ABC = 180^\circ$	
$\angle CAB + a + 90^\circ = 180^\circ$	
$\angle CAB = 90^\circ - a$	
In $\triangle ABE$,	
$\angle CEB = \angle CAB + \angle ABD$	
$x = 90^\circ - a + b$	1A
(b) $x = 2b$	
$90^\circ - a + b = 2b$	1M
$a = 90^\circ - b$	
In $\triangle ABD$,	
$\angle BDA + \angle ABD + \angle DAB = 180^\circ$	
$a + b + \angle DAB = 180^\circ$	
$90^\circ - b + b + \angle DAB = 180^\circ$	
$\angle DAB = 90^\circ$	
$\therefore$ *BD* is a diameter of the circle.	1A

Q59.

(a) Note that the radii of the two circles are the same.

AC, *AD* and *CD* are radii of the circles.

$\therefore$ $AC = AD = CD$

$\therefore$ $\angle ACD = 60°$ *(property of equil. Δ)* 1M

$\angle ABC = \dfrac{\angle ACD}{2}$ *(∠ at centre twice ∠ at ⊙ce)* 1M

$= \underline{\underline{30°}}$ 1A

(b) $\angle BAD = 90°$ *(∠ in semicircle)* 1M

$\therefore$ *AB* is a tangent to the circle. *(converse of tangent ⊥ radius)* 1

(c) In ΔABD,

$\dfrac{AD}{AB} = \tan \angle ABD$

$AD = 12 \tan 30°$ cm

$BE = 3AD = 36 \tan 30°$ cm 1M

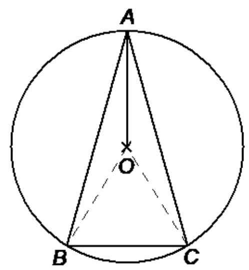
Area of $\Delta ABE = \dfrac{1}{2}(AB)(BE) \sin \angle ABD$

$= \dfrac{1}{2}(12)(36 \tan 30°) \sin 30°$ cm^2 1M

$= \underline{\underline{36\sqrt{3} \text{ cm}^2}}$ 1A

Q60.

(a) Join *OB* and join *OC*.



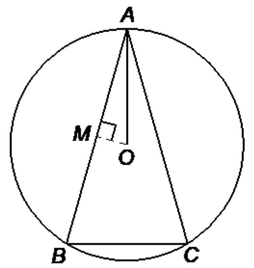
In ΔABO and ΔACO,

$OA = OA$	(common side)
$OB = OC$	(radii)
$AB = AC$	(given)
$\therefore \Delta ABO \cong \Delta ACO$	(S.S.S.)
$\therefore \angle BAO = \angle CAO$	(corr. ∠s, ≅ Δs)

$\therefore$ *OA* is the angle bisector of $\angle BAC$.

Marking Scheme	Case 1: Any correct proof with correct reasons.	3
	Case 2: Any correct proof without reasons.	2
	Case 3: Incomplete proof with any one correct step and one correct reason.	1

(b) Join OM, where M is a point on AB such that $OM \perp AB$.



$\because$ $OM \perp AB$ (construction)

$\therefore$ $AM = \frac{1}{2}AB$ (⊥ from centre bisects chord)

$= \frac{1}{2} \times 12$ cm

$= 6$ cm 1A

$\angle MAO = \frac{1}{2}\angle BAC$

$= \frac{1}{2} \times 45°$

$= 22.5°$ 1A

$\cos\angle MAO = \frac{AM}{OA}$

$\cos 22.5° = \frac{6 \text{ cm}}{OA}$

$OA = \frac{6}{\cos 22.5°}$ cm

$= \underline{\underline{6.49 \text{ cm}}}$ (corr. to 3 sig. fig.) 1A

Q61.

(a) In ΔABE and ΔCDE,

$\angle BAE = \angle DCE$ (ext. ∠, cyclic quad.)

$\angle ABE = \angle CDE$ (ext. ∠, cyclic quad.)

$\angle AEB = \angle CED$ (common ∠)

$\therefore$ $\Delta ABE \sim \Delta CDE$ (AAA) 1A

Let $AE = x$ cm.

$\because$ BD is a median of ΔABE.

$\therefore \quad DE = \frac{x}{2}$ cm

$\frac{AE}{CE} = \frac{BE}{DE}$ *(corr. sides, ~Δs)*

$\frac{x}{8} = \frac{17+8}{\frac{x}{2}}$ 1M

$x^2 = 400$

$x = 20$ or -20 (rejected)

$\therefore \quad AE = \underline{\underline{20 \text{ cm}}}$ 1A

(b) In ΔCDE,

$CD^2 + CE^2 = 6^2 + 8^2 = 100$

$DE^2 = \left(\frac{20}{2}\right)^2 = 100$

$\because \quad CD^2 + CE^2 = DE^2$

$\therefore \quad \angle DCE = 90°$ *(converse of Pyth. theorem)* 1M

$\angle BAE = \angle DCE = 90°$ *(ext. ∠, cyclic quad.)* 1M

$\therefore$ *BD* is a diameter of the circle. *(converse of ∠ in semicircle)* 1A

Q62.

(a) Let $\angle ABC = x$.

In ΔABC,

$\angle ACB = 90°$ *(∠ in semicircle)*

$\angle BAC = 180° - \angle ACB - \angle ABC$ *(∠ sum of Δ)*

$= 90° - x$

In ΔACD,

$\angle ACD = 180° - \angle ADC - \angle BAC$ *(∠ sum of Δ)*

$= x$

$\angle BCD = 90° - x$

$\therefore \quad \Delta ABC \sim \Delta ACD \sim \Delta CBD$ *(AAA)* 1A + 1A

(b) **(i)** $\because \quad \Delta ACD \sim \Delta CBD$

$\therefore \quad \left(\frac{CD}{BD}\right)^2 = \frac{3}{1}$ 1M

$\frac{CD}{BD} = \sqrt{3}$

$\tan \angle ABC = \sqrt{3}$ 1M

$\angle ABC = \underline{\underline{60°}}$ 1A

(ii) $\angle BAC = 90° - \angle ABC = 30°$

$\overset{\frown}{AC} : \overset{\frown}{BC} = \angle ABC : \angle BAC$ *(arcs prop. to ∠s at ⊙ce)*

$= 60° : 30°$ 1M

$= \underline{\underline{2 : 1}}$ 1A

Q63.

(a) $\angle ACB : \angle BAC = \overset{\frown}{AB} : \overset{\frown}{BC}$	
$\dfrac{\angle ACB}{25°} = \dfrac{8}{5}$	1M
$\angle ACB = 40°$	1A
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	(2)
(b) In $\triangle ABC$,	
$\angle ABC + \angle BAC + \angle ACB = 180°$	
$\angle ABC + 25° + 40° = 180°$	
$\angle ABC = 115°$	
Reflex $\angle AOC = 2\angle ABC$	
$= 2 \times 115°$	1M
$= 230°$	
$\therefore \quad \angle AOC = 360° - 230°$	1M
$= 130°$	1A
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	(3)
(c) $\angle ADC + \angle AOC = 180°$	
$\angle ADC + 130° = 180°$	1M
$\angle ADC = 50°$	1A
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	(2)

Q64.

		Marks
(a)	$\angle BAC = \angle BDC$	
	$= 30°$	1A
	$\angle ACB = 90°$	1A
	In $\triangle ABC$,	
	$\angle ABC + \angle ACB + \angle BAC = 180°$	
	$\angle ABC + 90° + 30° = 180°$	
	$\angle ABC = 60°$	1A
		----------
		(3)
(b)	In $\triangle ACE$,	
	$\angle ACD + \angle AEC + \angle BAC = 180°$	
	$\angle ACD + 100° + 30° = 180°$	1M
	$\angle ACD = 50°$	
	$\angle BCD = \angle ACB - \angle ACD$	
	$= 90° - 50°$	
	$= 40°$	
	$\overset{\frown}{AD} : \overset{\frown}{DB} = \angle ACD : \angle BCD$	
	$= 50° : 40°$	1M
	$= 5 : 4$	1A
		----------
		(3)
(c)	$\overset{\frown}{ADB} : \overset{\frown}{BC} = \angle ACB : \angle BDC$	
	$\dfrac{\overset{\frown}{ADB}}{3\text{ cm}} = \dfrac{90°}{30°}$	
	$\overset{\frown}{ADB} = 9\text{ cm}$	
	Circumference	
	$= 2 \times 9$ cm	1M
	$= 18$ cm	1A
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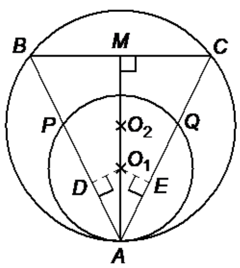
		(2)

Q65.

(a) In ΔAMB and ΔAMC,

$\because \quad AM \perp BC$	(given)
$\therefore \quad BM = CM$	($\perp$ from centre bisects chord)
$AM = AM$	(common side)
$\angle AMB = \angle AMC = 90°$	(given)
$\therefore \quad \Delta AMB \cong \Delta AMC$	(S.A.S.)
$\therefore \quad \angle MAB = \angle MAC$	(corr. $\angle$s, $\cong$ Δs)
$\therefore \quad AM$ bisects $\angle BAC$.	

(b) Join O_1D and join O_1E, where D is a point on AB such that $O_1D \perp AB$ and E is a point on AC such that $O_1E \perp AC$.



In ΔO_1AD and ΔO_1AE,

$O_1A = O_1A$	(common side)
$\angle O_1DA = \angle O_1EA = 90°$	(construction)
$\angle O_1AD = \angle O_1AE$	(proved)
$\therefore \quad \Delta O_1AD \cong \Delta O_1AE$	(A.A.S.)
$\therefore \quad O_1D = O_1E$	(corr. sides, $\cong$ Δs)
$\therefore \quad AP = AQ$	(chords equidistant from centre equal)
$\because \quad \Delta AMB \cong \Delta AMC$	(proved)
$\therefore \quad AB = AC$	(corr. sides, $\cong$ Δs)

$$PB = AB - AP$$
$$= AC - AQ$$
$$= QC$$

Q66.

(a) $\angle ABD : \angle CAB = \overset{\frown}{AD} : \overset{\frown}{BC}$ (arcs prop. to $\angle$s at $\odot^{ce}$) 1M

$$\frac{\angle ABD}{\angle CAB} = \frac{7}{4}$$

$$\angle ABD = \frac{7}{4}\angle CAB$$

$\angle AEB = \angle DEC$ (vert. opp. $\angle$s)

$= 147°$

In ΔAEB,

$\angle AEB + \angle ABE + \angle EAB = 180°$ ($\angle$ sum of Δ)

$147° + \frac{7}{4}\angle CAB + \angle CAB = 180°$ 1M

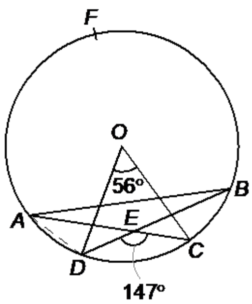
$\frac{11}{4}\angle CAB = 33°$

$\angle CAB = \underline{\underline{12°}}$ 1A

$\angle ABD = \frac{7}{4} \times 12°$

$= \underline{\underline{21°}}$ 1A

(b) Join AD.



$\angle DAC = \frac{1}{2}\angle DOC$ ($\angle$ at centre $= 2\ \angle$ at $\odot^{ce}$)

$= \frac{1}{2} \times 56°$ 1M

$= 28°$

$\angle DAE + \angle ADE = \angle DEC$ (ext. $\angle$ of Δ)

$28° + \angle ADB = 147°$ 1M

$\angle ADB = 119°$

$\overset{\frown}{AFB} : \overset{\frown}{DC} = \angle ADB : \angle DAC$ (arcs prop. to $\angle$s at $\odot^{ce}$)

$= 119° : 28°$

$= \underline{\underline{17 : 4}}$ 1A

Q67.

(a) $\angle BCD = \angle BAD$ (菱形性質)

$= 76°$

$\angle BCH = \frac{1}{2}\angle BCD$ (菱形性質)

$= \frac{1}{2} \times 76°$

$= 38°$

$\angle ECH = \frac{1}{2}\angle BCH$ (角平分線)

$= \frac{1}{2} \times 38°$

$= 19°$

$\angle ECD = \angle ECH + \angle HCD$

$= 19° + 38°$

$= 57°$

$\because \quad \angle ECD = \angle EFD = 57°$

$\therefore \quad$ C、F、D 和 E 共圓。 (同弓形內的圓周角的逆定理)

(b) $\angle CEF = \angle CDF$ (同弓形內的圓周角) 1M

$= 19°$

$\angle ECB = \angle ECH$ (角平分線)

$= 19°$

$\because \quad \angle ECB = \angle CEF = 19°$

$\therefore \quad$ BC 平行於 EF。 (錯角相等) 1A

(c) $\angle BAH = \frac{1}{2}\angle BAD$ (菱形性質)

$= \frac{1}{2} \times 76°$

$= 38°$

$\angle AEG + \angle EAG = \angle AGD$ (三角形的外角)

$\angle AEG + 38° = 107°$ 1M

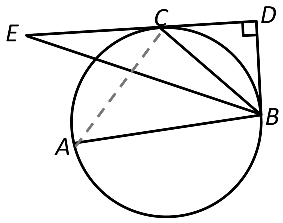
$\angle AEG = 69°$

$\because \quad \angle BCD = 76° \neq \angle AED$

$\therefore \quad$ B、C、D 和 E 不是共圓。 1A

Q68.

(a) **(i)** Join *AC*.



$\angle BCD = \angle BAC$	*$\angle$ in alt. segment*
BC bisects $\angle ABD$.	*given*
$\therefore \quad \angle CBD = \angle ABC$	
$\angle CDB = 90°$	*given*
In $\triangle CBD$,	
$\angle BCD + \angle CBD + 90° = 180°$	*$\angle$ sum of $\triangle$*
$\angle BCD + \angle CBD = 90°$	
i.e. $\quad \angle BAC + \angle ABC = 90°$	
In $\triangle ABC$,	
$\angle BAC + \angle ABC + \angle ACB = 180°$	*$\angle$ sum of $\triangle$*
$90° + \angle ACB = 180°$	
$\angle ACB = 90°$	
$\therefore$ *AB* is a diameter of the circle.	*converse of $\angle$ in semi-circle*

(ii)

$\angle ACB = \angle CDB = 90°$	*proved*
$\angle ABC = \angle CBD$	*proved*
$\angle BAC = \angle BCD$	*proved*
$\therefore \quad \triangle ABC \sim \triangle CBD$	*AAA*
$\dfrac{AB}{CB} = \dfrac{BC}{BD}$	*corr. sides, ~$\triangle$s*
$\therefore \quad BC^2 = AB \cdot BD$	

(b) Area of $\triangle ECB = \frac{1}{2} EC \cdot DB$

Area of $\triangle EBD = \frac{1}{2} ED \cdot DB$

Area of $\triangle ECB = \frac{1}{2} \times$ area of $\triangle EBD$

$\therefore \quad \frac{1}{2} EC \cdot DB = \frac{1}{2} \cdot \frac{1}{2} ED \cdot DB$

$$EC = \frac{1}{2} ED$$

$\therefore$ *C* is the mid-point of *DE*.

$\therefore$ The claim is agreed.

Q69.

(a)	**(i)**	$\angle PAO = 90°$	*tangent $\perp$ radius*
		$\angle PBO = 90°$	*tangent $\perp$ radius*
		$\angle PAO + \angle PBO = 180°$	
		$\therefore$ A, O, B and P are concyclic.	*opp. ∠s supp.*
	(ii)	A, O, B and P are concyclic.	*proved*
		$\angle CBA = \angle OPA$	*∠s in the same segment*
		$\angle BAC = 90°$	*∠ in semi-circle*
		$\therefore$ $\angle BAC = \angle PAO$	
		$\angle ACB = 180° - \angle CBA - \angle BAC$	*∠ sum of △*
		$\angle AOP = 180° - \angle OPA - \angle PAO$	*∠ sum of △*
		$\therefore$ $\angle ACB = \angle AOP$	
		$\therefore$ $\triangle ABC \sim \triangle APO$	*AAA*

(b) $BC = 2 \times 2 = 4$

$\triangle ABC \sim \triangle APO$

$\therefore \quad \frac{AC}{AO} = \frac{BC}{PO}$

$\frac{a}{2} = \frac{4}{b}$

$ab = 8$

$\because \quad a < 4$ and $b > 2$

$\therefore \quad a = 1, b = 8$ *or*

$a = 2, b = 4$

i.e. There are 2 pairs of possible values of a and b.

Q70.

連接 BD。

$\angle DAT = \angle BCD$ (圓內接四邊形外角)

$= 86°$

$\because \quad AT = AD$ (已知)

$\therefore \quad \angle ATD = \angle ADT$ (等腰 Δ 底角)

在 ΔATD 中，

$\angle ATD + \angle ADT + \angle DAT = 180°$ (三角形內角和)

$\angle ADT + \angle ADT + 86° = 180°$

$2\angle ADT = 94°$

$\angle ADT = 47°$

$\angle ABD = \angle ADT$ (內錯弓形的圓周角)

$= 47°$

$\angle ADB + \angle ABD = \angle DAT$ (三角形的外角)

$\angle ADB + 47^\circ = 86^\circ$

$\angle ADB = 39^\circ$

$\because \quad \angle BAS = \angle ADB = 39^\circ$

$\therefore \quad AS$ 是圓在 A 的切線。 (內錯弓形的圓周角的逆定理)

Q71. C

Q72. B

Q73. C

Q74. C

Q75. A

Q76. D

Q77. D

Q78. D

Q79. C

Q80. B