Assignment 1

Due: 7pm on October 13, 2025

This is a *mandatory* assignment for everyone with a *mandatory* submission requirement. Please submit the physical copy of your work and indicate your full name on the file. Write all your statements and deriviations as clearly as you can.

Provided all problems have been attempted properly (with a solution or a description of what you have tried), these problems weight equally in your grading.

1. **(Consumer's Problem)** Let $u(x_1, x_2) = x_1 + 2\sqrt{x_2}$ be a utility function for quantities x_1 and x_2 of commodities 1 and 2, respectively. Let p_1, p_2 be the prices of commodities 1 and 2, respectively. Assume $p_1 > 0$ and $p_2 > 0$.

   a. Compute the Marshallian demand $\xi(p_1, p_2, w)$.

   b. Compute the indirect utility function $V(p_1, p_2, w) = \max\{u(x) : p \cdot x \leq w, x \geq 0\}$.

   c. Compute the expenditure function $e(p_1, p_2, v)$.

   d. Compute the Hicksian demand $h(p_1, p_2, v)$

   e. Compute the substitution matrix $S(p_1, p_2, w)$ at $p_1 = p_2 = 2$, $w = 10$.

2. **(Marshallian Demand)** Each of the following four functions is a possible Marshallian demand function for two commodities at prices p_1 and p_2, respectively and when wealth is w. In each case, determine whether it is the demand function of a consumer with a locally non-satiated, continuous, and strictly quasi-concave utility function. If it is, say what the utility function is. Otherwise, give a reason.

   a. $\xi(p_1, p_2, w) = \left(\frac{wp_2}{2p_1^2}, \frac{wp_1}{2p_2^2}\right)$.

   b. $\xi(p_1, p_2, w) = \left(\frac{3}{4}\frac{w}{p_1}, \frac{1}{4}\frac{w}{p_2}\right)$.

   c. $\xi(p_1, p_2, w) = \left(\frac{w}{p_1} - \frac{p_2}{p_1^3}, \frac{p_2}{p_1^2}\right)$.

   d. $\xi(p_1, p_2, w) = \left(\frac{w\sqrt{p_1}}{p_1^{3/2} + p_2^{3/2}}, \frac{w\sqrt{p_2}}{p_1^{3/2} + p_2^{3/2}}\right)$.

3. **(Impossibility of Linear Demand)** Many economists like to work with linear/affine demand. In this exercise, I guide you to prove it is not possible to have a linear/affine demand when there are at least two commodities. By linear/affine demand, I mean the demand function $\xi(p, w) = (\xi_1(p, w), \xi_2(p, w)) \in \mathbb{R}^2_+$ has the form

$$\xi_1(p, w) = a_1 p_1 + a_2 p_2 + g_1(w)$$

$$\xi_2(p, w) = b_1 p_1 + b_2 p_2 + g_2(w)$$

for $a_1, a_2, b_1, b_2 > 0$ and some differentiable functions $g_1, g_2 : \mathbb{R} \to \mathbb{R}$.
The proof goes as follows: we start by assuming there are affine demand. Then,

a. Write out the Walras' law.

b. Differentiate both side of the equation with respect to w. Note the equality you obtained hold for any choice of p and w.

c. Check what happens by fixing $w > 0$ and taking both p_1 and p_2 goes to zero. Find a contradiction by checking the limit on both sides of the equation.

4. **(Lagrange Multiplier)** Given a concave utility function u, for the cost minimization problem

$$e(p, v) = \min_{x \geq 0 : u(x) \geq v} p \cdot x$$

Write out the Lagrangian as a function of the choice variable and the Lagrange multiplier λ on the constraint $u(x) \geq v$.

a. What is the dimension of λ?

b. Argue the cost minimization problem is a convex optimization problem and state when the Slater's condition hold.
Therefore, there is a number λ^* such that the pair $(h(p, v), \lambda^*)$ satisfies the KKT condition.

c. Give an economic interpretation to the Lagrange multiplier at the optimum λ^*.