

Assignment 2

Due: 7pm on October 27, 2025

This is a *mandatory* assignment for everyone with a *mandatory* submission requirement. Please submit the physical copy of your work and indicate your full name on the file. Write all your statements and derivations as clearly as you can.

Provided all problems have been attempted properly (with a solution or a description of what you have tried), these problems weight equally in your grading.

1. **(Commodity Type)** A utility function is *homothetic* if

$$u(ax) = au(x) \text{ for all } a > 0$$

- a Prove, when the utility function is homothetic and the Walrasian demand is single-valued, the demand is in the form $\xi(p, w) = g(p)w$ for some function g .¹
 - b Prove that if the utility function is homothetic, then there is no Giffen good.
 - c Explain briefly why you would or would not expect utility functions to be homothetic.
2. **(Consumer Welfare)** Consider a price change from the initial price p to a new price p' in which only the price of commodity i decreases. Show if commodity i is inferior, compare the compensating variation (CV) and the equivalence variation (EV).²

3. **(Preference)**

- a. Prove if a relation is complete, it must be reflexive.
 - b. Provide an example of preference that is reflexive but incomplete.
 - c. Is the Lexicographic preference on $\mathbb{R}_+^2$ convex?
4. **(Utility Representation)** Write whether the following statements are true or false, and justify your answers:

- a. All continuous preferences do not have discontinuous utility representations.
- b. Some continuous preference does not have a discontinuous utility representation.

¹Due to this property, homothetic utility functions are very convenient in some applications.

²You would need to use and prove that $e(p, v)$ is strictly increasing in v and the equivalence between two types of demand here.

- c. All continuous preferences can be represented by a utility function, which has a range given by an arbitrarily chosen non-trivial interval³ of numbers.

³i.e. this interval is not degenerated to a point.