

Assignment B

Due: 7pm on September 24, 2025

This is a *mandatory* assignment for everyone with optional submission requirement: these exercises will be used later in the course, and the assumption is everyone has learn the material through working on this assignment.

If you choose to submit your work, please submit the physical copy of your work. Write all your statement and deriviations as clearly as you can. And mark your file with your unique character that is not your name. For this assignment, I will not grade any work with a student name on it. In addition, I will grade an assignment only if all problems have been properly attempted.

1. **(Budget Set)** When does one set contain another? Give conditions under which

a. $\{x \in \mathbb{R}^2 : p \cdot x \leq b_1\} \subset \{x \in \mathbb{R}^2 : q \cdot x \leq b_2\}.$

b. $\{x \in \mathbb{R}_+^2 : p \cdot x \leq b_1\} \subset \{x \in \mathbb{R}_+^2 : q \cdot x \leq b_2\}.$

where $p, q \gg 0$ (all coordinates are positive) and $b_1, b_2 > 0$. Find conditions (on p, q, b_1, b_2) under which these two sets are equal.

2. **(Constraint Qualification)**

a. For the following minimization problem:

$$\min_{x_1, x_2, x_3 \in \mathbb{R}} x_3$$

subject to

$$2x_1 + x_2 = 1$$

$$x_2 = 0$$

$$x_2 + x_3^2 = 0$$

(a1) Find objective function and choice set.¹

(a2) Find the set of minimizers.

(a3) Define the Lagrangian. (Write down the domain and range of the Lagrangian.)

(a4) Does the constraint qualification condition hold?

(a5) Does the first order condition hold at the minimizer for any choice of the Lagrange

¹Note to define a function, you need to write the domain, the range and the mapping relation of the function.

multiplier?

3. **(Slater's Condition)** Consider the following parameterized choice set:

$$\{(x, y, z) \in \mathbb{R}_+^3 : x^2 + y^2 + z^2 \leq \alpha, z = 0\}$$

for some $\alpha \geq 0$. For which values of α do the Slater's condition hold? Justify your answer.

4. **(Linear Programming)** Consider the problem

$$\max_{x \geq 0, y \geq 0} 2x + y$$

subject to

$$x + 3y \leq 19$$

$$x + y \leq 7$$

$$3x + y \leq 11$$

- (a) Draw the choice set.
- (b) Solve the problem using graph by drawing the level sets of the objective function.
- (c) Verify the Slater's condition holds.
- (d) Write out the Lagrangian of this problem (Convert it into a minimization problem).
- (e) Write out the Karush-Kuhn-Tucker condition.
- (f) Solve the problem using the Karush-Kuhn-Tucker method.