Assignment 2

Due: 4pm on September 24, 2025[1]

1. **(Differentiability and Jacobian)** Consider a real-valued function f on $\mathbb{R}^2$, where $f(x,y) = 1$ if $x = 0$ or $y = 0$ and $f(x,y) = 0$ elsewhere. First, compute the Jacobian matrix of f at the origin $(0,0)$. Next, show the function f is not differentiable at $(0,0)$.

2. **(Budget Set)** When does one set contain another? Give conditions under which

   a. $\{x \in \mathbb{R}^n : p \cdot x \leq b_1\} \subset \{x \in \mathbb{R}^n : q \cdot x \leq b_2\}$.

   b. $\{x \in \mathbb{R}^n_+ : p \cdot x \leq b_1\} \subset \{x \in \mathbb{R}^n_+ : q \cdot x \leq b_2\}$.

where $p, q >> 0$ (all coordinates are positive) and $b_1, b_2 > 0$. Find conditions (on p, q, b_1, b_2) under which these two sets are equal.

3. **(Constraint Qualification)**

a. For the following minimization problem:

$$\min_{x_1, x_2, x_3 \in \mathbb{R}} x_3$$

subject to

$$2x_1 + x_2 = 1$$

$$x_2 = 0$$

$$x_2 + x_3^2 = 0$$

(a1) Find objective function and choice set.[2]

(a2) Find the set of minimizers.

(a3) Define the Lagrangian. (Write down the domain and range of the Lagrangian.)

(a4) Does the constraint qualification condition hold?

(a5) Does the first order condition hold at the minimizer for any choice of the Lagrange multiplier?

b. Consider the following parameterized choice set:

$$\{(x, y, z) \in \mathbb{R}^3_+ : x^2 + y^2 + z^2 \leq \alpha, z = 0\}$$

[1] Please submit the physical copy of your work. Write all your statement and deriviations as clearly as you can.

[2] Note to define a function, you need to write the domain, the range and the mapping relation of the function.

for some $\alpha \geq 0$. For which values of α do the Slater's condition hold? Justify your answer.

4. **(Linear Programming)** Consider the problem

$$\max_{x\geq 0, y\geq 0} 2x + y$$

subject to

$$x + 3y \leq 19$$

$$x + y \leq 7$$

$$3x + y \leq 11$$

(a) Draw the choice set.
(b) Solve the problem using graph by drawing the level sets of the objective function.
(c) Verify the Slater's condition holds.
(d) Write out the Lagrangian of this problem (Convert it into a minimization problem).
(e) Write out the Karush-Kuhn-Tucker condition.
(f) Solve the problem using the Karush-Kuhn-Tucker method.