Assignment 5

Due: 4pm on November 5, 2025[1]

1. **(Preference)**

   (a) Prove if a relation is complete, it must be reflexive. Show by example that a relation that is reflexive can be incomplete.

   (b) Say that a relation $\succeq$ is acyclic if the associated strict relation $\succ$ exhibits no cycles. Show that every transitive relation is acyclic, but show by example that a relation can be acyclic without being transitive.

2. **(Revealed Preference)** In each of the first two questions, you are given a data set D of consumption bundles purchased at specific price vectors. In each case, first, check if D satisfies the weak axiom of revealed preference. Second, either show D can not be the purchase of a rational consumer or find a utility function for a rational consumer who makes the purchases.

   (a) $D = \{(p^i, x^i) : i = 1, 2, 3\}$

   - $p^1 = (2, 1, 3), x^1 = (3, 1, 1)$
   - $p^2 = (3, 2, 1), x^2 = (1, 3, 1)$
   - $p^3 = (1, 2, 2), x^3 = (1, 1, 3)$

   (b) $D = \{(p^i, x^i) : i = 1, 2, 3\}$

   - $p^1 = (1, 2, 3), x^1 = (3, 3, 1)$
   - $p^2 = (3, 2, 1), x^2 = (1, 3, 3)$
   - $p^3 = (3, 1, 2), x^3 = (2, 12, 3)$

   (c) we know the observations in data set $D = \{(p^i, x^i), i = 1, 2, 3, 4, 5\}$ are from two consumers. Partition the observations into two groups such that the observation in each group are consistent with the maximization of a single continuous, quasi-concave utility function.

   - $x^1 = (4, 8), p^1 = (1, 2)$
   - $x^2 = (8, 4), p^2 = (2, 1)$

[1] Please submit the physical copy of your work. Write all your statement and deriviations as clearly as you can.

- $x^3 = (4, 6), p^3 = (1, 3)$
- $x^4 = (6, 4), p^4 = (3, 1)$
- $x^5 = (4, 7), p^5 = (2, 5)$

3. **(Utility Representation)** Write whether the following statements are true or false, and justify your answers:

a. All continuous preferences do not have discontinuous utility representations.

b. Some continuous preference does not have a discontinuous utility representation.

c. All continuous preferences can be represented by a utility function, which has a range given by $[a^*, b^*]$ for $a^* < b^*$.

d. A utility function $u : \mathbb{R}^L_+ \to \mathbb{R}$ satisfies the law of decreasing marginal utility on good i if $\frac{du(x)}{dx_i}$ is decreasing in x_i. Show that increasing transformation of utility functions need not preserve decreasing marginal utility.

4. **(Debreu's Theorem)** There are two equivalent definitions for the continuity of preference $\succeq$ on a set X:

Def 1 for any sequence x_n, y_n in set X, $x_n \succeq y_n$ for all $n \in \mathbb{N}$ implies $x \succeq y$.

Def 2 All the better than sets and worse than sets are closed. Formally, for all $x \in X$, $B(x) = \{y \in X : y \succeq x\}$ and $W(x) = \{y \in X : x \succeq y\}$ are closed.

A binary relation $\succeq$ on $\mathbb{R}^2_+$ defines a preference as follows: for any pairs of vectors $(x, y), (x', y') \in \mathbb{R}^2_+$,

- if $x \neq y$ and $x' \neq y'$, $(x, y) \succeq (x', y')$ if and only if $xy \geq x'y'$;
- if $x = y$ but $x' \neq y'$, $(x, y) \succeq (x', y')$ if and only if $xy \geq x'y' - 1$;
- if $x \neq y$ but $x' = y'$, $(x, y) \succeq (x', y')$ if and only if $xy \geq x'y' + 1$;
- if $x = y$ and $x' = y'$, $(x, y) \succeq (x', y')$ if and only if $x \geq x'$.

Answer the following questions:

(a) Is $\succeq$ defined above rational?

(b) Use Def 1 to check if $\succeq$ is continuous.

(c) Use Def 2 to check if $\succeq$ is continuous.

(d) Find a utility representation of $\succeq$.[2]

(e) Is $\succeq$ convex?

[2]If your answer in either b or c is $\succeq$ is discontinuous, it means a discontinuous preference could have a utility representation (though maybe discontinuous).