Assignment 7

Due: 4pm on December 3, 2025[1]

1. **(Theory)** Briefly answer the following questions:

   a. What are the differences between an axiom, an assumption and a hypothesis?

   b. What are the differences between a theorem and a theory?

   c. What are the differences between a theory and a model?

   d. Is the word "hypothesis" relevant to this course?

2. **(Consumer's and Producer's Problem)** You are given the following optimization problem

$$\max_{x\in[0,1]} x,$$

answer the following questions.

   a. Describe the problem by its objective function, choice variable, and choice set.

   b. Write down the "First-order Condition" methodology for solving this problem.

   c. Explain why the "First-order Condition" methodology is inappropriate for solving this optimization problem.

   d. Describe two methodologies for solving this problem correctly. Justify the applicability for each methodology.

   e. Is this problem a consumer's problem? If so, justify your answer. If not, explain why.

   f. Is this problem a producer's problem? If so, justify your answer. If not, explain why.

3. **(Preference)** Given a utility function $u(x, y) = \min(x^{1/2}y^{1/2}, x + y)$, we would like to know the convexity of the preference it represented. A student, Xinyang, thought this task is related to the concavity of u, and used chatGPT to see u is concave. ChatGPT generated the following argument:

[1] Please submit the physical copy of your work. Write all your statement and deriviations as clearly as you can.

To determine whether the function $u(x, y) = \min(\sqrt{x}\sqrt{y}, x + y)$ is concave, we need to examine its second-order partial derivatives. If the second-order partial derivatives satisfy a specific condition, the function is concave.

First, we calculate the first-order partial derivatives:

$$\frac{\partial u}{\partial x} = \begin{cases} \frac{\sqrt{y}}{2\sqrt{x}} & \text{if } \sqrt{x}\sqrt{y} \leq x + y \\ 1 & \text{if } \sqrt{x}\sqrt{y} > x + y \end{cases}$$

$$\frac{\partial u}{\partial y} = \begin{cases} \frac{\sqrt{x}}{2\sqrt{y}} & \text{if } \sqrt{x}\sqrt{y} \leq x + y \\ 1 & \text{if } \sqrt{x}\sqrt{y} > x + y \end{cases}$$

Next, we differentiate again to find the second-order partial derivatives:

$$\frac{\partial^2 u}{\partial x^2} = \begin{cases} -\frac{\sqrt{y}}{4x^{3/2}} & \text{if } \sqrt{x}\sqrt{y} \leq x + y \\ 0 & \text{if } \sqrt{x}\sqrt{y} > x + y \end{cases}$$

$$\frac{\partial^2 u}{\partial y^2} = \begin{cases} -\frac{\sqrt{x}}{4y^{3/2}} & \text{if } \sqrt{x}\sqrt{y} \leq x + y \\ 0 & \text{if } \sqrt{x}\sqrt{y} > x + y \end{cases}$$

$$\frac{\partial^2 u}{\partial x \partial y} = \begin{cases} \frac{1}{4\sqrt{x}\sqrt{y}} - \frac{1}{2\sqrt{x}\sqrt{y}} & \text{if } \sqrt{x}\sqrt{y} \leq x + y \\ 0 & \text{if } \sqrt{x}\sqrt{y} > x + y \end{cases}$$

To determine concavity, we examine the sign of the second-order partial derivatives. If both second-order partial derivatives are non-positive, the function is concave.

Considering the cases separately:

Case 1: If $\sqrt{x}\sqrt{y} \leq x + y$:

In this case, all second-order partial derivatives are negative or zero. Therefore, $u(x, y)$ is concave within this region.

Case 2: If $\sqrt{x}\sqrt{y} > x + y$:

In this case, the second-order partial derivatives are zero. This does not provide any information about concavity.

Overall, $u(x, y)$ is concave in the region where $\sqrt{x}\sqrt{y} \leq x+y$. However, concavity cannot be determined when $\sqrt{x}\sqrt{y} > x + y$.

(a) The argument generated by ChatGPT has a handful of mistakes. Identify as many of them as you can.

(b) Determine if u is concave or quasi-concave in its domain.

(c) Explain why do we care about quasi-concavity rather than concavity in the setting of this problem.

4. **(Modeling)** On one day, you get out from the main entrance of the Rio Hondo campus. You went east and need to pass the car exit of ITAM. It is during the busy hour of the day: there are a lot of students going towards the same direction, a lot of cars waiting to get out from ITAM, and a lot of cars queuing on the main road in the city traffic.

When you attempt to cross, you found a car is moving slowly towards outside, with an attempt to join the city traffic. The driver moves her car because she will not be able to see you before she got part of her car out. The driver has no intention to crush you, but may do so by mistake in rare cases.

Now, you need to make a decision about what to do. There can be many options, I give you only three to illustrate: you can stop and wait for the car to pass before moving; you can pass the car from the front of the car (so necessarily the car needs to stop if the driver does not want to hurt you); and pass the car from the back of the car (so the car can keep moving without harming you, but you need to walk extra).

In this problem, you are asked to model this decision scenario and discuss your optimal choice under your description of the scenario. You must model the trade off of all options so that making a choice is not a tautology, and you must take into account the uncertainties involved in this decision problem.