Rationale Exponenten

Theorie

Allgemeine Wurzeln können auch als Potenzen mit rationalem Exponenten dargestellt werden.

$$\sqrt{a} = a^{\frac{1}{2}} \qquad \sqrt[n]{a} = a^{\frac{1}{n}} \qquad \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$$

Dabei gelten die bekannten Potenzgesetze:

$$a^m \cdot a^n = a^{m+n} \qquad a^n \cdot b^n = (a \cdot b)^n \qquad (a^m)^n = a^{m \cdot n}$$

$$\frac{a^m}{a^n} = a^{m-n} \qquad \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

1. **Rationale Exponenten.** Vereinfachen Sie und stellen Sie als Potenz mit rationalem Exponenten dar.

   Beispiel. $\sqrt[4]{144} = \sqrt[4]{12^2} = 12^{\frac{2}{4}} = \boxed{12^{\frac{1}{2}}}$

   (a) $\sqrt[6]{625} = \sqrt[6]{5^4} = 5^{\frac{4}{6}} = \boxed{5^{\frac{2}{3}}}$

   (b) $\sqrt[9]{125} = \sqrt[9]{5^3} = 5^{\frac{3}{9}} = \boxed{5^{\frac{1}{3}}}$

   (c) $\sqrt[10]{32} = \sqrt[10]{2^5} = 2^{\frac{5}{10}} = \boxed{2^{\frac{1}{2}}}$

   (d) $\sqrt[3]{27} = \sqrt[3]{3^3} = 3^{\frac{3}{3}} = \boxed{3}$

   (e) $\sqrt[4]{81} = \sqrt[4]{3^4} = 3^{\frac{4}{4}} = \boxed{3}$

   (f) $\sqrt[5]{32} = \sqrt[5]{2^5} = 2^{\frac{5}{5}} = \boxed{2}$

   (g) $\sqrt[3]{64} = \sqrt[3]{2^6} = 2^{\frac{6}{3}} = 2^2 = \boxed{4}$

   (h) $\sqrt[6]{4} = \sqrt[6]{2^2} = 2^{\frac{2}{6}} = \boxed{2^{\frac{1}{3}}}$

   (i) $\sqrt[4]{36} = \sqrt[4]{6^2} = 6^{\frac{2}{4}} = \boxed{6^{\frac{1}{2}}}$

2. **Als Wurzel.** Vereinfachen Sie und stellen Sie die Zahl als Wurzel mit möglichst kleinem Wurzelexponenten dar.

   Beispiel. $8^{\frac{1}{6}} = \left(2^3\right)^{\frac{1}{6}} = 2^{3 \cdot \frac{1}{6}} = 2^{\frac{3}{6}} = 2^{\frac{1}{3}} = \boxed{\sqrt[3]{2}}$

   (a) $25^{\frac{1}{8}} = \left(5^2\right)^{\frac{1}{8}} = 5^{2 \cdot \frac{1}{8}} = 5^{\frac{2}{8}} = 5^{\frac{1}{4}} = \boxed{\sqrt[4]{5}}$

   (b) $125^{\frac{1}{6}} = \left(5^3\right)^{\frac{1}{6}} = 5^{\frac{3}{6}} = 5^{\frac{1}{2}} = \boxed{\sqrt{5}}$

   (c) $27^{\frac{1}{9}} = \left(3^3\right)^{\frac{1}{9}} = 3^{3 \cdot \frac{1}{9}} = 3^{\frac{3}{9}} = 3^{\frac{1}{3}} = \boxed{\sqrt[3]{3}}$

(d) $16^{\frac{1}{12}} = \left(2^4\right)^{\frac{1}{12}} = 2^{4\cdot\frac{1}{12}} = 2^{\frac{4}{12}} = 2^{\frac{1}{3}} = \boxed{\sqrt[3]{2}}$

(e) $81^{\frac{1}{10}} = \left(3^4\right)^{\frac{1}{10}} = 3^{4\cdot\frac{1}{10}} = 3^{\frac{4}{10}} = 3^{\frac{2}{5}} = \boxed{\sqrt[5]{3^2} = \sqrt[5]{9}}$

(f) $81^{\frac{1}{20}} = \left(3^4\right)^{\frac{1}{20}} = 3^{4\cdot\frac{1}{20}} = 3^{\frac{4}{20}} = 3^{\frac{1}{5}} = \boxed{\sqrt[5]{3}}$

(g) $216^{\frac{1}{6}} = \left(6^3\right)^{\frac{1}{6}} = 6^{3\cdot\frac{1}{6}} = 6^{\frac{3}{6}} = 6^{\frac{1}{2}} = \boxed{\sqrt{6}}$

(h) $32^{\frac{1}{15}} = \left(2^5\right)^{\frac{1}{15}} = 2^{5\cdot\frac{1}{15}} = 2^{\frac{5}{15}} = 2^{\frac{1}{3}} = \boxed{\sqrt[3]{2}}$

(i) $256^{\frac{1}{8}} = \left(2^8\right)^{\frac{1}{8}} = 2^{\frac{8}{8}} = \boxed{2}$

3. Berechnen Sie ohne Taschenrechner.

(a) $\sqrt{2} \cdot \sqrt[3]{2} \cdot \sqrt[6]{2}$

Lösung:

$$
\begin{aligned}
&= 2^{\frac{1}{2}} \cdot 2^{\frac{1}{3}} \cdot 2^{\frac{1}{6}} \\
&= 2^{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}} \\
&= 2^{\frac{3}{6}+\frac{2}{6}+\frac{1}{6}} \\
&= 2^{\frac{6}{6}} \\
&= 2^1 \\
&= \boxed{2}
\end{aligned}
$$

(b) $\sqrt{6} \cdot \sqrt[3]{12} \cdot \sqrt[6]{96}$

Lösung:

$$
\begin{aligned}
&= \sqrt{2 \cdot 3} \cdot \sqrt[3]{2^2 \cdot 3} \cdot \sqrt[6]{2^5 \cdot 3} \\
&= (2 \cdot 3)^{\frac{1}{2}} \cdot (2^2 \cdot 3)^{\frac{1}{3}} \cdot \left(2^5 \cdot 3\right)^{\frac{1}{6}} \\
&= 2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} \cdot 2^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} \cdot 2^{\frac{5}{6}} \cdot 3^{\frac{1}{6}} \\
&= 2^{\frac{1}{2}+\frac{2}{3}+\frac{5}{6}} \cdot 3^{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}} \\
&= 2^{\frac{3}{6}+\frac{4}{6}+\frac{5}{6}} \cdot 3^{\frac{3}{6}+\frac{2}{6}+\frac{1}{6}} \\
&= 2^{\frac{12}{6}} \cdot 3^{\frac{6}{6}} \\
&= 2^2 \cdot 3 \\
&= \boxed{12}
\end{aligned}
$$

(c) $\sqrt{10} \cdot \sqrt[3]{5} \cdot \sqrt[4]{2} \cdot \sqrt[12]{200}$

Lösung:

$$
\begin{aligned}
&= 10^{\frac{1}{2}} \cdot 5^{\frac{1}{3}} \cdot 2^{\frac{1}{4}} \cdot 200^{\frac{1}{12}} \\
&= (2 \cdot 5)^{\frac{1}{2}} \cdot 5^{\frac{1}{3}} \cdot 2^{\frac{1}{4}} \cdot \left(2^3 \cdot 5^2\right)^{\frac{1}{12}} \\
&= 2^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} \cdot 5^{\frac{1}{3}} \cdot 2^{\frac{1}{4}} \cdot \left(2^3\right)^{\frac{1}{12}} \cdot \left(5^2\right)^{\frac{1}{12}} \\
&= 2^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} \cdot 5^{\frac{1}{3}} \cdot 2^{\frac{1}{4}} \cdot 2^{\frac{3}{12}} \cdot 5^{\frac{2}{12}} \\
&= 2^{\frac{1}{2}+\frac{1}{4}+\frac{3}{12}} \cdot 5^{\frac{1}{2}+\frac{1}{3}+\frac{2}{12}} \\
&= 2^{\frac{2}{4}+\frac{1}{4}+\frac{1}{4}} \cdot 5^{\frac{3}{6}+\frac{2}{6}+\frac{1}{6}} \\
&= 2^{\frac{4}{4}} \cdot 5^{\frac{6}{6}} \\
&= 2^1 \cdot 5^1 \\
&= \boxed{10}
\end{aligned}
$$

(d) $\sqrt{2} \cdot \sqrt[6]{2} - \sqrt[3]{4}$

Lösung:

$$
\begin{aligned}
&= 2^{\frac{1}{2}} \cdot 2^{\frac{1}{6}} - 4^{\frac{1}{3}} \\
&= 2^{\frac{1}{2}+\frac{1}{6}} - \left(2^2\right)^{\frac{1}{3}} \\
&= 2^{\frac{3}{6}+\frac{1}{6}} - 2^{2 \cdot \frac{1}{3}} \\
&= 2^{\frac{4}{6}} - 2^{\frac{2}{3}} \\
&= 2^{\frac{2}{3}} - 2^{\frac{2}{3}} \\
&= \boxed{0}
\end{aligned}
$$

(e) $\left(\sqrt[3]{54} - \sqrt[3]{16}\right) \cdot \sqrt[6]{16}$

Lösung:

$$
\begin{aligned}
&= \left(54^{\frac{1}{3}} - 16^{\frac{1}{3}}\right) \cdot 16^{\frac{1}{6}} \\
&= \left(\left(2 \cdot 3^3\right)^{\frac{1}{3}} - \left(2^4\right)^{\frac{1}{3}}\right) \cdot \left(2^4\right)^{\frac{1}{6}} \\
&= \left(2^{\frac{1}{3}} \cdot 3^{\frac{3}{3}} - 2^{\frac{4}{3}}\right) \cdot 2^{\frac{4}{6}} \\
&= \left(2^{\frac{1}{3}} \cdot 3 - 2^{\frac{4}{3}}\right) \cdot 2^{\frac{2}{3}} \\
&= 2^{\frac{1}{3}} \cdot 3 \cdot 2^{\frac{2}{3}} - 2^{\frac{4}{3}} \cdot 2^{\frac{2}{3}} \\
&= 2^{\frac{1}{3}+\frac{2}{3}} \cdot 3 - 2^{\frac{4}{3}+\frac{2}{3}} \\
&= 2^{\frac{3}{3}} \cdot 3 - 2^{\frac{6}{3}} \\
&= 2^1 \cdot 3 - 2^2 \\
&= 6 - 4 \\
&= \boxed{2}
\end{aligned}
$$

(f) $\left(\sqrt[4]{512} - \sqrt[4]{32} - \sqrt[4]{2}\right) \cdot \sqrt[4]{8}$

Lösung:

$$
\begin{aligned}
&= \left(512^{\frac{1}{4}} - 32^{\frac{1}{4}} - 2^{\frac{1}{4}}\right) \cdot 8^{\frac{1}{4}} \\
&= \left(\left(2^9\right)^{\frac{1}{4}} - \left(2^5\right)^{\frac{1}{4}} - 2^{\frac{1}{4}}\right) \cdot \left(2^3\right)^{\frac{1}{4}} \\
&= \left(2^{\frac{9}{4}} - 2^{\frac{5}{4}} - 2^{\frac{1}{4}}\right) \cdot 2^{\frac{3}{4}} \\
&= 2^{\frac{9}{4}} \cdot 2^{\frac{3}{4}} - 2^{\frac{5}{4}} \cdot 2^{\frac{3}{4}} - 2^{\frac{1}{4}} \cdot 2^{\frac{3}{4}} \\
&= 2^{\frac{9}{4}+\frac{3}{4}} - 2^{\frac{5}{4}+\frac{3}{4}} - 2^{\frac{1}{4}+\frac{3}{4}} \\
&= 2^{\frac{12}{4}} - 2^{\frac{8}{4}} - 2^{\frac{4}{4}} \\
&= 2^3 - 2^2 - 2^1 \\
&= 8 - 4 - 2 \\
&= \boxed{2}
\end{aligned}
$$

4. **Vereinfachen** Sie und stellen Sie das Resultat sowohl in Potenz- als auch in Wurzelschreibweise dar.

(a) $3^{0.5} \cdot 9^{0.75}$

Lösung:

$$
\begin{aligned}
&= 3^{\frac{1}{2}} \cdot \left(3^2\right)^{\frac{3}{4}} \\
&= 3^{\frac{1}{2}} \cdot 3^{\frac{6}{4}} \\
&= 3^{\frac{1}{2}+\frac{6}{4}} \\
&= 3^{\frac{1}{2}+\frac{3}{2}} \\
&= 3^2 \\
&= \boxed{9}
\end{aligned}
$$

(b) $4^{\frac{1}{5}} \cdot 2^{\frac{1}{10}} : 8^{\frac{1}{4}}$

Lösung:

$$
\begin{aligned}
&= \left(2^2\right)^{\frac{1}{5}} \cdot 2^{\frac{1}{10}} : \left(2^3\right)^{\frac{1}{4}} \\
&= 2^{\frac{2}{5}} \cdot 2^{\frac{1}{10}} : 2^{\frac{3}{4}} \\
&= 2^{\frac{2}{5}+\frac{1}{10}-\frac{3}{4}} \\
&= 2^{\frac{8}{20}+\frac{2}{20}-\frac{15}{20}} \\
&= 2^{-\frac{5}{20}} \\
&= \boxed{2^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{2}}}
\end{aligned}
$$

(c) $\sqrt[4]{27} \cdot \sqrt[5]{9} \cdot \sqrt[20]{27}$

Lösung:

$$
\begin{aligned}
&= 27^{\frac{1}{4}} \cdot 9^{\frac{1}{5}} \cdot 27^{\frac{1}{20}} \\
&= \left(3^3\right)^{\frac{1}{4}} \cdot \left(3^2\right)^{\frac{1}{5}} \cdot \left(3^3\right)^{\frac{1}{20}} \\
&= 3^{\frac{3}{4}} \cdot 3^{\frac{2}{5}} \cdot 3^{\frac{3}{20}} \\
&= 3^{\frac{3}{4}+\frac{2}{5}+\frac{3}{20}} \\
&= 3^{\frac{15}{20}+\frac{8}{20}+\frac{3}{20}} \\
&= 3^{\frac{26}{20}} \\
&= \boxed{3^{\frac{13}{10}} = \sqrt[10]{3^{13}}}
\end{aligned}
$$

(d) $\sqrt[3]{4} : \sqrt[3]{36}$

Lösung:

$$
\begin{aligned}
&= 4^{\frac{1}{3}} : 36^{\frac{1}{3}} \\
&= \left(2^2\right)^{\frac{1}{3}} : \left(6^2\right)^{\frac{1}{3}} \\
&= 2^{\frac{2}{3}} : 6^{\frac{2}{3}} \\
&= (2:6)^{\frac{2}{3}} \\
&= \boxed{\left(\tfrac{1}{3}\right)^{\frac{2}{3}} = \frac{1}{3^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{9}}}
\end{aligned}
$$

(e) $\sqrt[3]{\frac{4}{5}} : \sqrt[6]{25}$

Lösung:

$$
\begin{aligned}
&= \left(\tfrac{4}{5}\right)^{\frac{1}{3}} : 25^{\frac{1}{6}} \\
&= \left(\tfrac{4}{5}\right)^{\frac{1}{3}} : \left(5^2\right)^{\frac{1}{6}} \\
&= \left(\tfrac{4}{5}\right)^{\frac{1}{3}} : 5^{\frac{2}{6}} \\
&= \left(\tfrac{4}{5}\right)^{\frac{1}{3}} : 5^{\frac{1}{3}} \\
&= \left(\tfrac{4}{5} : 5\right)^{\frac{1}{3}} \\
&= \left(\tfrac{4}{2 \cdot 5}\right)^{\frac{1}{3}} \\
&= \left(\tfrac{2^2}{5^2}\right)^{\frac{1}{3}} \\
&= \left(\left(\tfrac{2}{5}\right)^2\right)^{\frac{1}{3}} \\
&= \boxed{\left(\tfrac{2}{5}\right)^{\frac{2}{3}} = \sqrt[3]{\left(\tfrac{2}{5}\right)^2}}
\end{aligned}
$$

(f) $\sqrt[5]{\frac{8}{9}} : \sqrt[5]{\frac{27}{128}}$

Lösung:

$$
\begin{aligned}
&= \left(\tfrac{8}{9} : \tfrac{27}{128}\right)^{\frac{1}{5}} \\
&= \left(\tfrac{8}{9} \cdot \tfrac{128}{27}\right)^{\frac{1}{5}} \\
&= \left(\tfrac{1024}{243}\right)^{\frac{1}{5}} \\
&= \frac{\sqrt[5]{1024}}{\sqrt[5]{243}} \\
&= \boxed{\tfrac{4}{3}}
\end{aligned}
$$

5. **Vereinfachen** Sie so weit wie möglich und stellen Sie in Potenz- und Wurzelschreibweise dar.

Beispiel. $\left(\sqrt[4]{16}\right)^3 = 16^{\frac{3}{4}} = \left(2^4\right)^{\frac{3}{4}} = 2^{4 \cdot \frac{3}{4}} = 2^3 = \boxed{8}$

(a) $\sqrt[4]{3^{16}}$

Lösung: $= 3^4 = \boxed{81}$

(b) $\sqrt{\sqrt[3]{2}}$

Lösung: $= 2^{\frac{1}{2} \cdot \frac{1}{3}} = \boxed{2^{\frac{1}{6}} = \sqrt[6]{2}}$

(c) $\sqrt[3]{\sqrt[4]{2}}$

Lösung: $= 2^{\frac{1}{3}\cdot\frac{1}{4}} = \boxed{2^{\frac{1}{12}} = \sqrt[12]{2}}$

(d) $\sqrt{\sqrt[3]{32}}$

Lösung: $= \boxed{2^{\frac{5}{6}} = \sqrt[6]{32}}$

(e) $\sqrt[3]{\sqrt[5]{\sqrt[4]{16}}}$

Lösung: $= \boxed{2^{\frac{1}{15}} = \sqrt[15]{2}}$

(f) $\sqrt{\sqrt{625}}$

Lösung: $= \left(625^{\frac{1}{2}}\right)^{\frac{1}{2}} = 625^{\frac{1}{4}} = \left(5^4\right)^{\frac{1}{4}} = 5^{\frac{4}{4}} = \boxed{5}$

(g) $\sqrt[3]{4 \cdot \sqrt[4]{8}}$

Lösung: $= \boxed{2^{\frac{11}{12}} = \sqrt[12]{2^{11}}}$

(h) $\sqrt[5]{9 \cdot \sqrt[3]{3 \cdot \sqrt{27}}}$

Lösung: $= \boxed{3^{\frac{17}{30}} = \sqrt[30]{3^{17}}}$

(i) $\sqrt[6]{6 \cdot \sqrt[4]{6 \cdot \sqrt[3]{6}}}$

Lösung: $= \boxed{6^{\frac{2}{9}} = \sqrt[9]{6^2}}$

6. Wurzelterme. Vereinfachen Sie.

(a) $a^2 \cdot \sqrt{a}$

Lösung: $= a^2 \cdot a^{\frac{1}{2}} = a^{2+\frac{1}{2}} = \boxed{a^{\frac{5}{2}}}$

(b) $c \cdot \sqrt[3]{c}$

Lösung: $= c \cdot c^{\frac{1}{3}} = c^{1+\frac{1}{3}} = c^{\frac{4}{3}} = \boxed{\sqrt[3]{c^4}}$

(c) $\sqrt{k} \cdot \sqrt[3]{k}$

Lösung: $= k^{\frac{1}{2}} \cdot k^{\frac{1}{3}} = k^{\frac{1}{2}+\frac{1}{3}} = k^{\frac{3}{6}+\frac{2}{6}} = k^{\frac{5}{6}} = \boxed{\sqrt[6]{k^5}}$

(d) $\sqrt[5]{s^3} \cdot \sqrt[5]{s^2}$

Lösung: $= s^{\frac{3}{5}} \cdot s^{\frac{2}{5}} = s^{\frac{3}{5}+\frac{2}{5}} = \boxed{s}$

(e) $\dfrac{e}{\sqrt{e}}$

Lösung: $= \dfrac{e}{e^{\frac{1}{2}}} = e^{1-\frac{1}{2}} = e^{\frac{1}{2}} = \boxed{\sqrt{e}}$

(f) $\dfrac{b^2}{\sqrt[4]{b}}$

Lösung: $= \dfrac{b^2}{b^{\frac{1}{4}}} = b^{2-\frac{1}{4}} = b^{\frac{7}{4}} = \boxed{\sqrt[4]{b^7}}$

(g) $\dfrac{\sqrt[3]{d}}{\sqrt[4]{d}}$

Lösung: $= \dfrac{d^{\frac{1}{3}}}{d^{\frac{1}{4}}} = d^{\frac{1}{3}-\frac{1}{4}} = d^{\frac{4}{12}-\frac{3}{12}} = d^{\frac{1}{12}} = \boxed{\sqrt[12]{d}}$

(h) $\dfrac{\sqrt[3]{y^2}}{y}$

Lösung: $= \dfrac{y^{\frac{2}{3}}}{y} = y^{\frac{2}{3}-1} = y^{\frac{-1}{3}} = \boxed{\dfrac{1}{\sqrt[3]{y}}}$

(i) $\left(\sqrt[4]{a}\right)^2$

Lösung: $= a^{\frac{2}{4}} = a^{\frac{1}{2}} = \boxed{\sqrt{a}}$

(j) $\left(\sqrt[3]{x}\right)^9$

Lösung: $= x^{\frac{9}{3}} = \boxed{x^3}$

(k) $\sqrt[3]{\sqrt{u}}$

Lösung: $= \left(u^{\frac{1}{2}}\right)^{\frac{1}{3}} = u^{\frac{1}{2}\cdot\frac{1}{3}} = u^{\frac{1}{6}} = \boxed{\sqrt[6]{u}}$

(l) $\sqrt{\sqrt[5]{z^4}}$

Lösung: $= \left(z^{\frac{4}{5}}\right)^{\frac{1}{2}} = z^{\frac{4}{5}\cdot\frac{1}{2}} = z^{\frac{2}{5}} = \boxed{\sqrt[5]{z^2}}$

7. Vereinfachen Sie so weit wie möglich und stellen Sie als Wurzel dar.

(a) $a^{\frac{3}{4}} : \left(a^{\frac{2}{3}} : a\right)$

Lösung:

$$a^{\frac{3}{4}} : \left(a^{\frac{2}{3}} : a\right) = a^{\frac{3}{4}} : \left(a^{\frac{2}{3}} \cdot a^{-1}\right) = a^{\frac{3}{4}} : a^{\frac{2}{3}-1} = a^{\frac{3}{4}} : a^{-\frac{1}{3}} = a^{\frac{3}{4}} \cdot a^{\frac{1}{3}}$$
$$= a^{\frac{3}{4}+\frac{1}{3}} = a^{\frac{9}{12}+\frac{4}{12}} = a^{\frac{13}{12}} = \boxed{\sqrt[12]{a^{13}}}$$

(b) $\left(b^{\frac{1}{4}} : b^{\frac{1}{5}}\right) \cdot b^{\frac{1}{10}}$

Lösung:

$$\left(b^{\frac{1}{4}} : b^{\frac{1}{5}}\right) \cdot b^{\frac{1}{10}} = \left(b^{\frac{1}{4}} \cdot b^{-\frac{1}{5}}\right) \cdot b^{\frac{1}{10}} = b^{\frac{1}{4}-\frac{1}{5}} \cdot b^{\frac{1}{10}} = b^{\frac{5}{20}-\frac{4}{20}+\frac{2}{20}} = b^{\frac{3}{20}} = \boxed{\sqrt[20]{b^3}}$$

(c) $c^{\frac{2}{3}} \cdot c^{\frac{3}{2}}$

Lösung:

$$c^{\frac{2}{3}} \cdot c^{\frac{3}{2}} = c^{\frac{2}{3}+\frac{3}{2}} = c^{\frac{4}{6}+\frac{9}{6}} = c^{\frac{13}{6}} = \boxed{\sqrt[6]{c^{13}}}$$

(d) $d^{1.4} \cdot d^{1.5} \cdot d^{0.6}$

Lösung:

$$d^{1.4} \cdot d^{1.5} \cdot d^{0.6} = d^{1.4+1.5+0.6} = d^{3.5} = d^{\frac{7}{2}} = \boxed{\sqrt{d^7}}$$

(e) $\sqrt[4]{e^9} \cdot \sqrt[5]{e^9}$

Lösung:

$$\sqrt[4]{e^9} \cdot \sqrt[5]{e^9} = e^{\frac{9}{4}} \cdot e^{\frac{9}{5}} = e^{\frac{45}{20}} \cdot e^{\frac{36}{20}} = e^{\frac{81}{20}} = \boxed{\sqrt[20]{e^{81}}}$$

(f) $(2f)^{0.25} \cdot \left(8f^2\right)^{0.25} \cdot f^{0.25}$

Lösung:

$$(2f)^{0.25} \cdot \left(8f^2\right)^{0.25} \cdot f^{0.25} = 2^{0.25} \cdot f^{0.25} \cdot 8^{0.25} \cdot f^{0.5} \cdot f^{0.25}$$
$$= 2^{0.25} \cdot \left(2^3\right)^{0.25} \cdot f^{0.25+0.5+0.25} \quad = 2^{0.25+0.75} \cdot f = \boxed{2f}$$