Rationale Exponenten

Theorie

Allgemeine Wurzeln können auch als Potenzen mit rationalem Exponenten dargestellt werden.

$$\sqrt{a} = a^{\frac{1}{2}} \qquad \sqrt[n]{a} = a^{\frac{1}{n}} \qquad \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$$

Dabei gelten die bekannten Potenzgesetze:

$$a^m \cdot a^n = a^{m+n} \qquad a^n \cdot b^n = (a \cdot b)^n \qquad (a^m)^n = a^{m \cdot n}$$

$$\frac{a^m}{a^n} = a^{m-n} \qquad \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

1. **Rationale Exponenten.** Vereinfachen Sie und stellen Sie als Potenz mit rationalem Exponenten dar.

   Beispiel. $\sqrt[4]{144} = \sqrt[4]{12^2} = 12^{\frac{2}{4}} = \boxed{12^{\frac{1}{2}}}$

   (a) $\sqrt[6]{625}$

   (b) $\sqrt[9]{125}$

   (c) $\sqrt[10]{32}$

   (d) $\sqrt[3]{27}$

   (e) $\sqrt[4]{81}$

   (f) $\sqrt[5]{32}$

   (g) $\sqrt[3]{64}$

   (h) $\sqrt[6]{4}$

   (i) $\sqrt[4]{36}$

2. **Als Wurzel.** Vereinfachen Sie und stellen Sie die Zahl als Wurzel mit möglichst kleinem Wurzelexponenten dar.

   Beispiel. $8^{\frac{1}{6}} = \left(2^3\right)^{\frac{1}{6}} = 2^{3 \cdot \frac{1}{6}} = 2^{\frac{3}{6}} = 2^{\frac{1}{3}} = \boxed{\sqrt[3]{2}}$

   (a) $25^{\frac{1}{8}}$

   (b) $125^{\frac{1}{6}}$

   (c) $27^{\frac{1}{9}}$

   (d) $16^{\frac{1}{12}}$

   (e) $81^{\frac{1}{10}}$

   (f) $81^{\frac{1}{20}}$

   (g) $216^{\frac{1}{6}}$

   (h) $32^{\frac{1}{15}}$

   (i) $256^{\frac{1}{8}}$

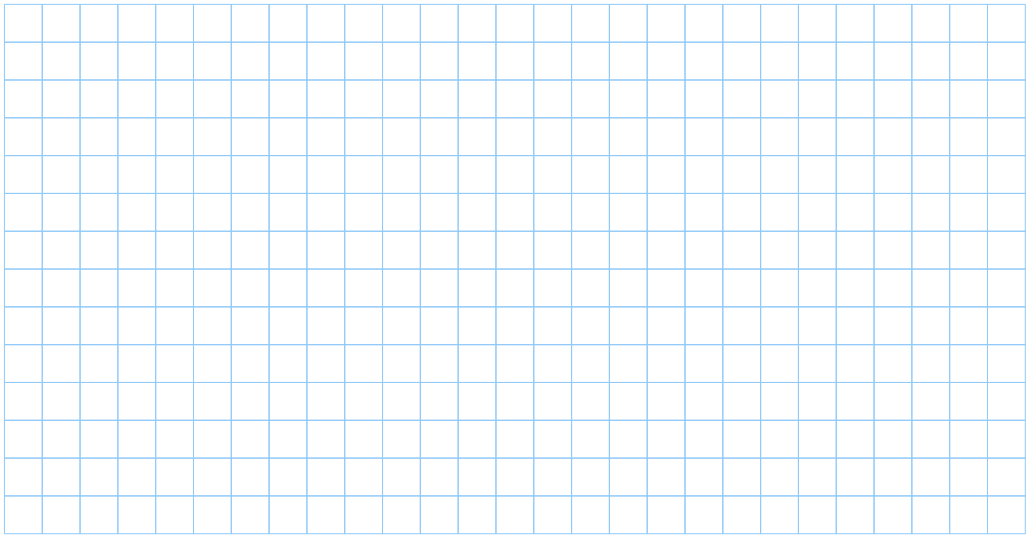
3. Berechnen Sie ohne Taschenrechner.

(a) $\sqrt{2} \cdot \sqrt[3]{2} \cdot \sqrt[6]{2}$

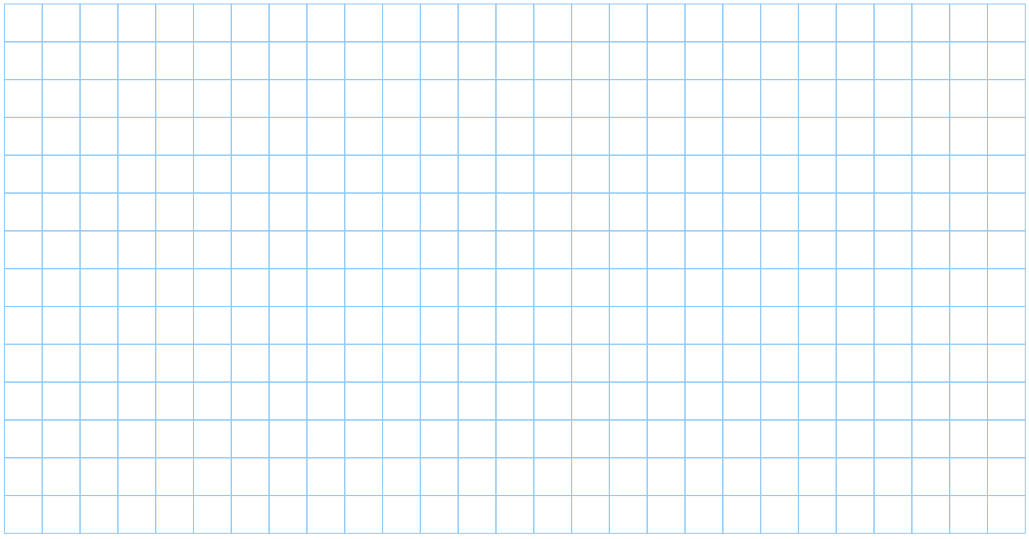
(b) $\sqrt{6} \cdot \sqrt[3]{12} \cdot \sqrt[6]{96}$

(c) $\sqrt{10} \cdot \sqrt[3]{5} \cdot \sqrt[4]{2} \cdot \sqrt[12]{200}$

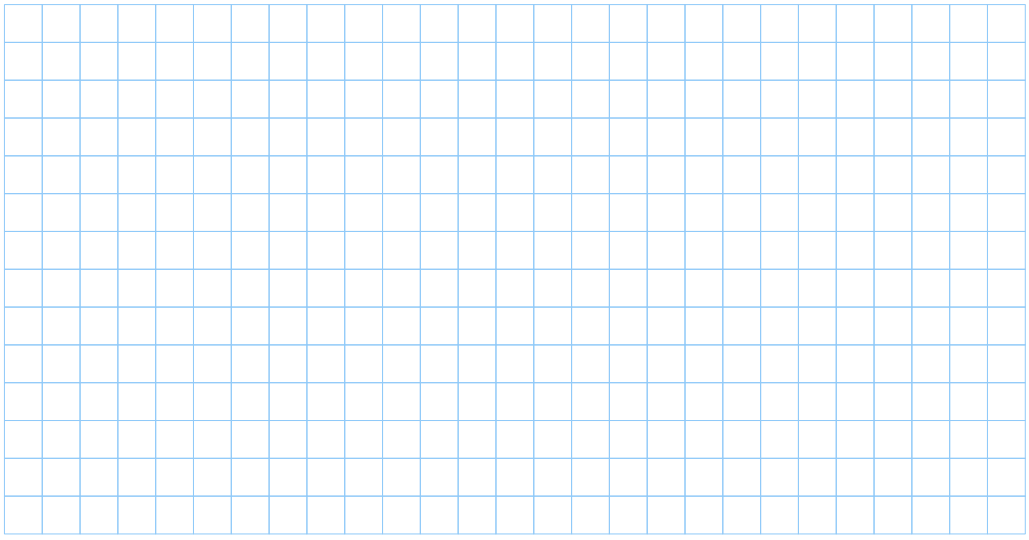
(d) $\sqrt{2} \cdot \sqrt[6]{2} - \sqrt[3]{4}$



(e) $\left(\sqrt[3]{54} - \sqrt[3]{16}\right) \cdot \sqrt[6]{16}$

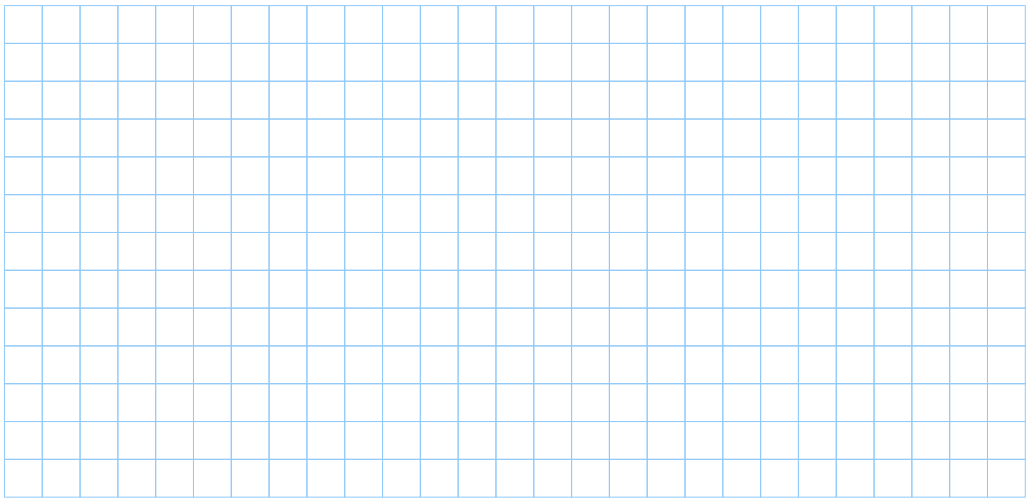


(f) $\left(\sqrt[4]{512} - \sqrt[4]{32} - \sqrt[4]{2}\right) \cdot \sqrt[4]{8}$

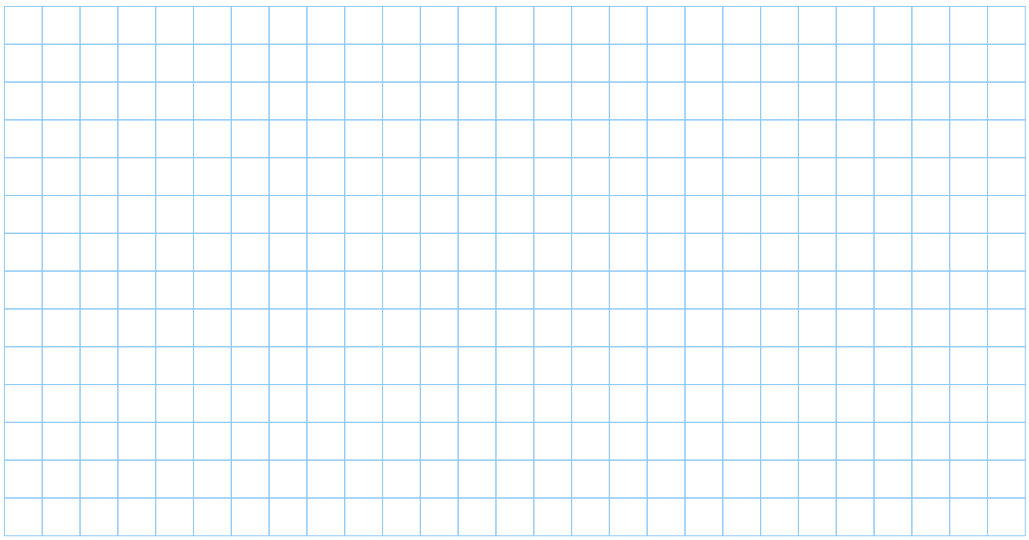


4. Vereinfachen Sie und stellen Sie das Resultat sowohl in Potenz- als auch in Wurzelschreibweise dar.

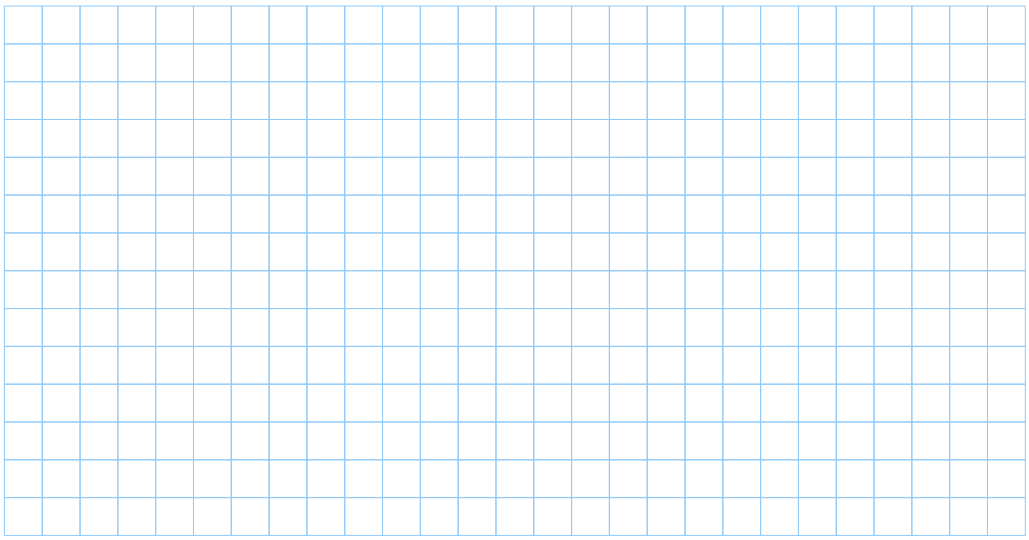
(a) $3^{0.5} \cdot 9^{0.75}$



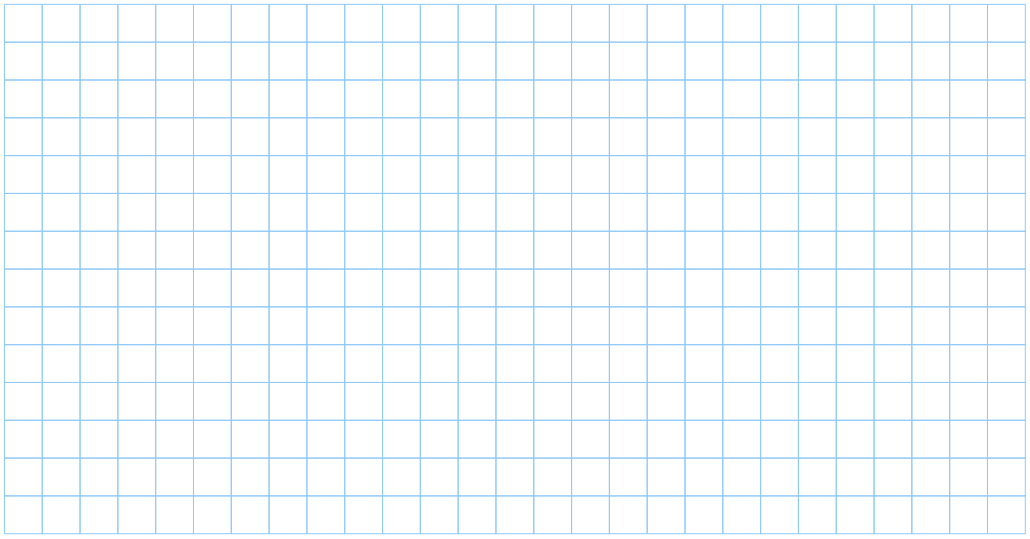
(b) $4^{\frac{1}{5}} \cdot 2^{\frac{1}{10}} : 8^{\frac{1}{4}}$



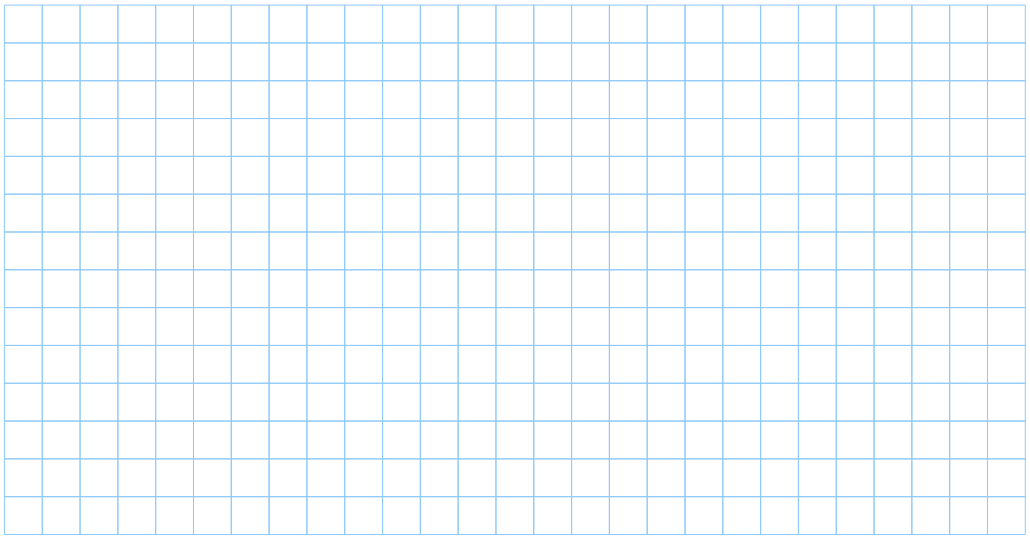
(c) $\sqrt[4]{27} \cdot \sqrt[5]{9} \cdot \sqrt[20]{27}$



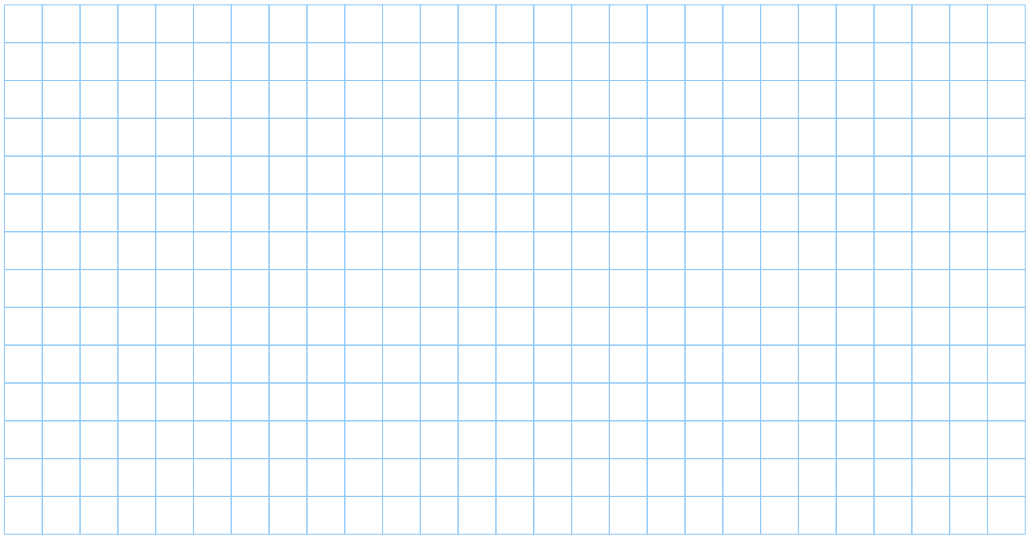
(d) $\sqrt[3]{4} : \sqrt[3]{36}$



(e) $\sqrt[3]{\frac{4}{5}} : \sqrt[6]{25}$



(f) $\sqrt[5]{\frac{8}{9}} : \sqrt[5]{\frac{27}{128}}$



5. Vereinfachen Sie so weit wie möglich und stellen Sie in Potenz- und Wurzelschreibweise dar.

Beispiel. $\left(\sqrt[4]{16}\right)^3 = 16^{\frac{3}{4}} = \left(2^4\right)^{\frac{3}{4}} = 2^{4\cdot\frac{3}{4}} = 2^3 = \boxed{8}$

(a) $\sqrt[4]{3^{16}}$

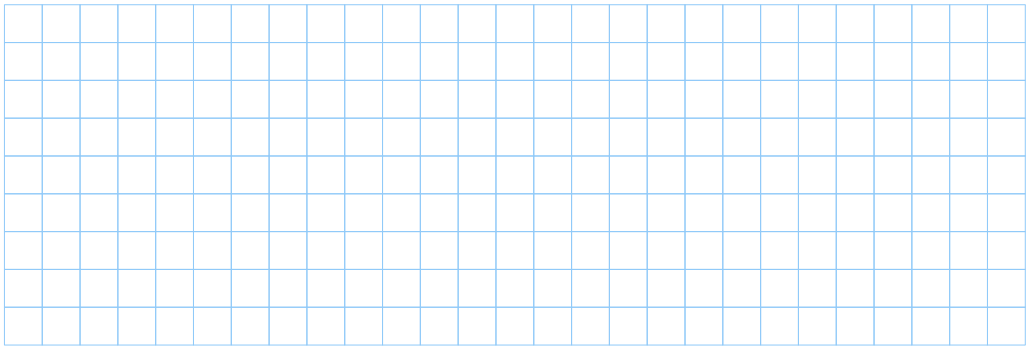
(b) $\sqrt{\sqrt[3]{2}}$

(c) $\sqrt[3]{\sqrt[4]{2}}$

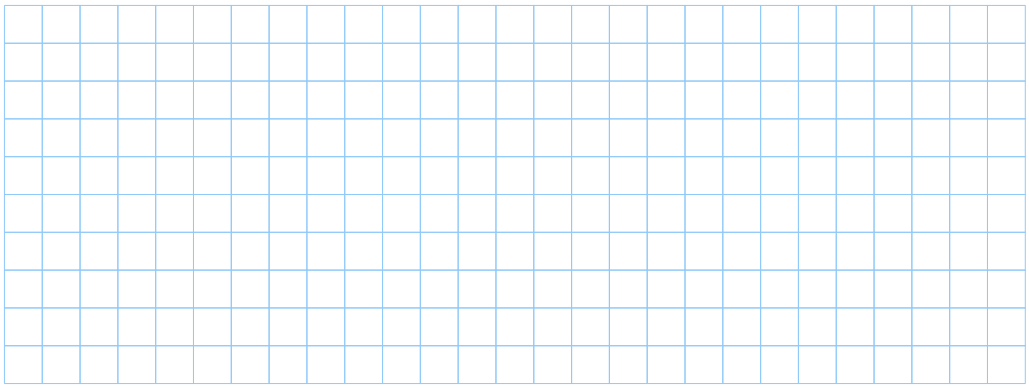
(d) $\sqrt{\sqrt[3]{32}}$

(e) $\sqrt[3]{\sqrt[5]{\sqrt[4]{16}}}$

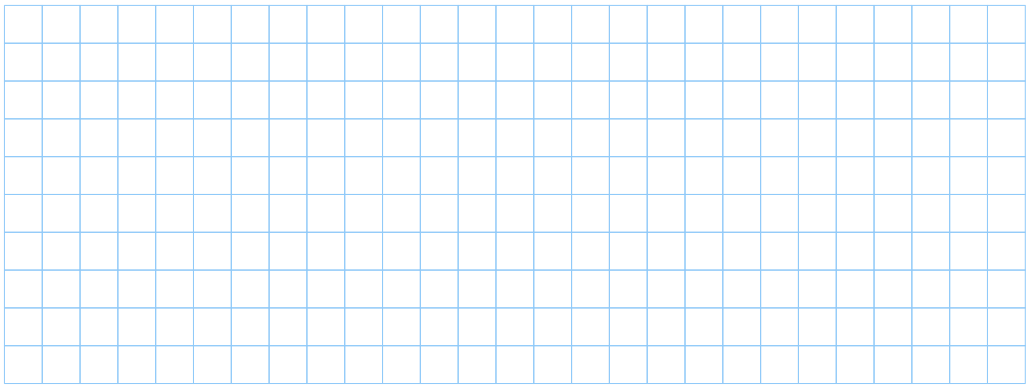
(f) $\sqrt{\sqrt{625}}$



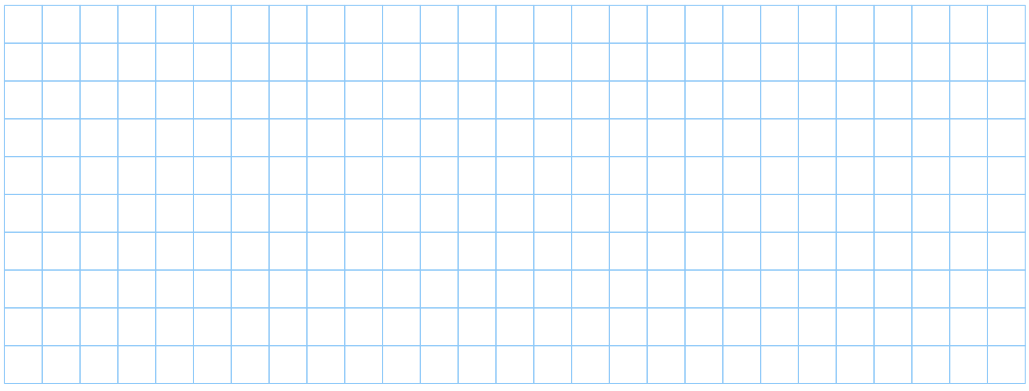
(g) $\sqrt[3]{4 \cdot \sqrt[4]{8}}$



(h) $\sqrt[5]{9 \cdot \sqrt[3]{3 \cdot \sqrt{27}}}$

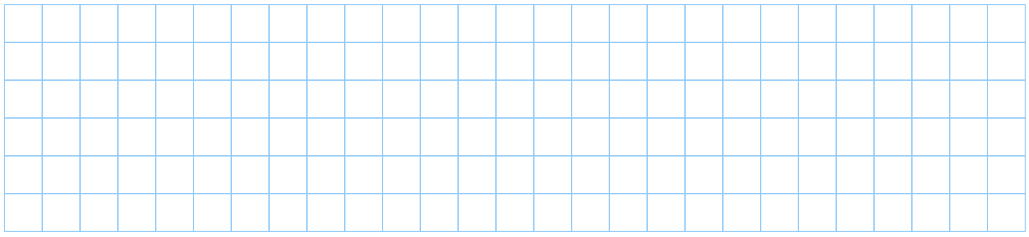


(i) $\sqrt[6]{6 \cdot \sqrt[4]{6 \cdot \sqrt[3]{6}}}$

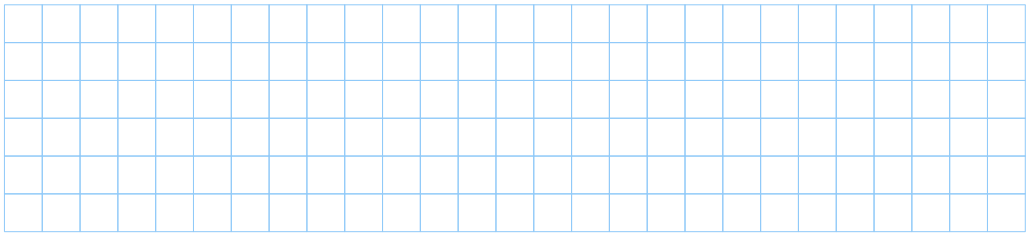


6. Wurzelterme. Vereinfachen Sie.

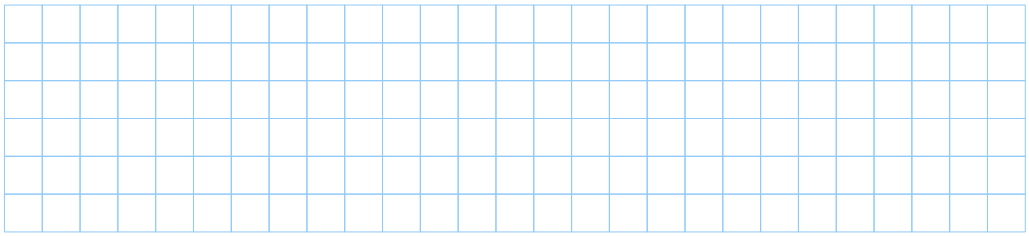
(a) $a^2 \cdot \sqrt{a}$



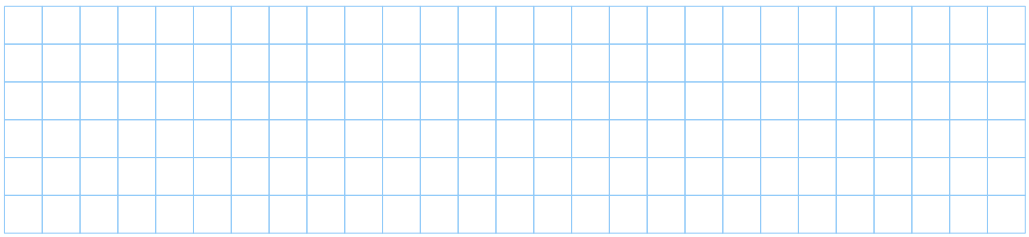
(b) $c \cdot \sqrt[3]{c}$



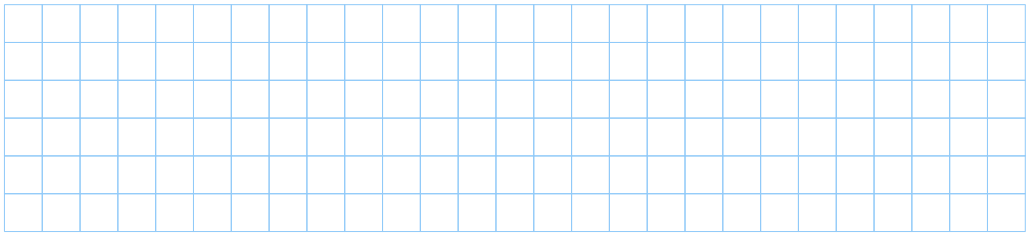
(c) $\sqrt{k} \cdot \sqrt[3]{k}$



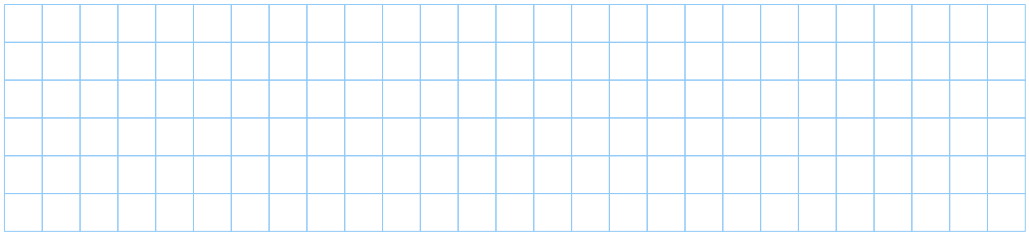
(d) $\sqrt[5]{s^3} \cdot \sqrt[5]{s^2}$



(e) $\dfrac{e}{\sqrt{e}}$



(f) $\dfrac{b^2}{\sqrt[4]{b}}$



(g) $\dfrac{\sqrt[3]{d}}{\sqrt[4]{d}}$

(h) $\dfrac{\sqrt[3]{y^2}}{y}$

(i) $\left(\sqrt[4]{a}\right)^2$

(j) $\left(\sqrt[3]{x}\right)^9$

(k) $\sqrt[3]{\sqrt{u}}$

(l) $\sqrt{\sqrt[5]{z^4}}$

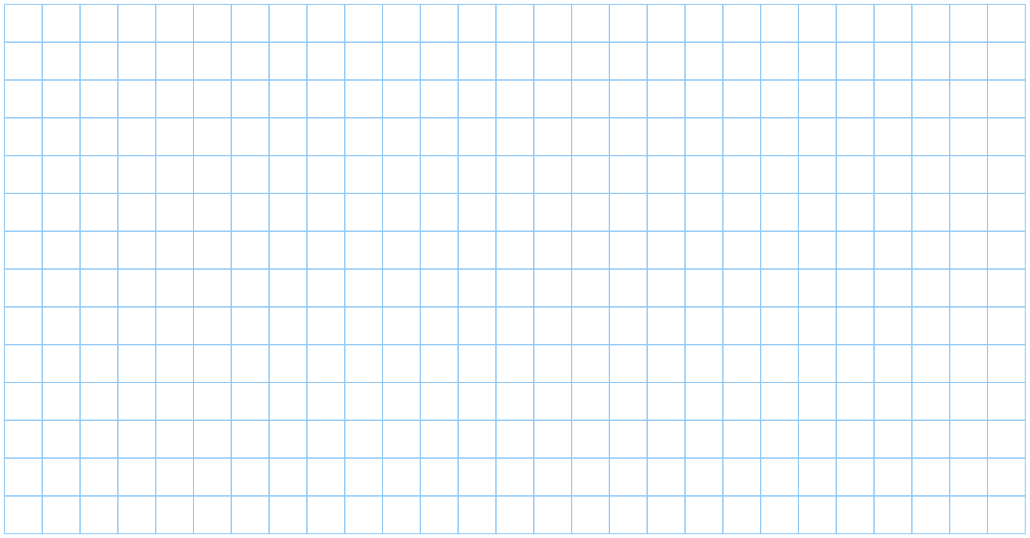
7. Vereinfachen Sie so weit wie möglich und stellen Sie als Wurzel dar.

(a) $a^{\frac{3}{4}} : \left(a^{\frac{2}{3}} : a\right)$

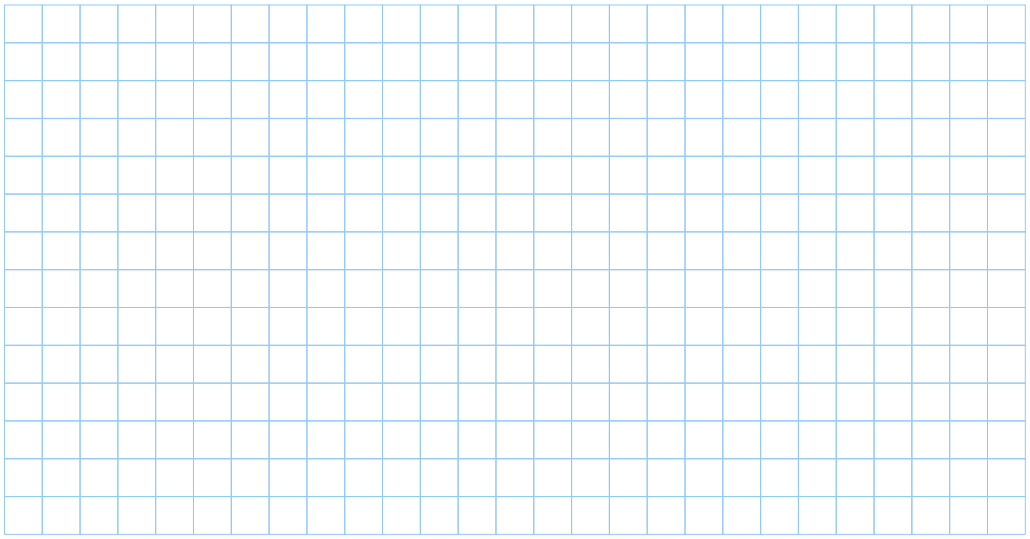
(b) $\left(b^{\frac{1}{4}} : b^{\frac{1}{5}}\right) \cdot b^{\frac{1}{10}}$

(c) $c^{\frac{2}{3}} \cdot c^{\frac{3}{2}}$

(d) $d^{1.4} \cdot d^{1.5} \cdot d^{0.6}$



(e) $\sqrt[4]{e^9} \cdot \sqrt[5]{e^9}$



(f) $(2f)^{0.25} \cdot \left(8f^2\right)^{0.25} \cdot f^{0.25}$

