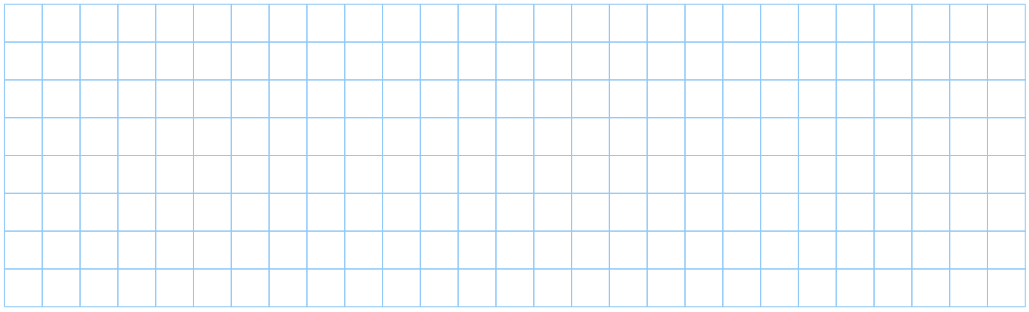
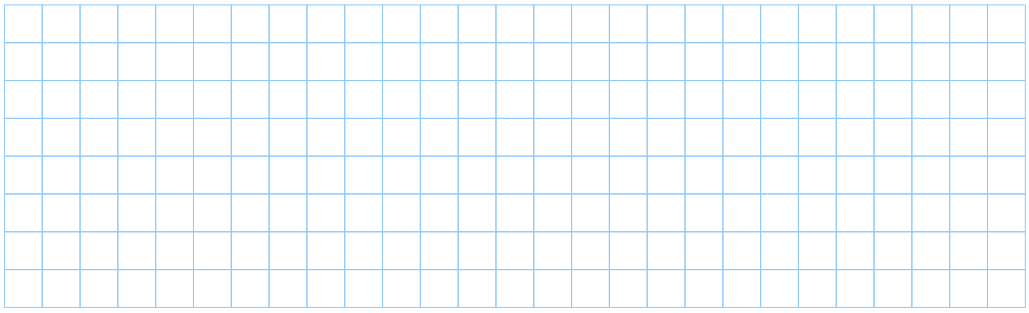
Logarithmen

1. Zinsrechnen.

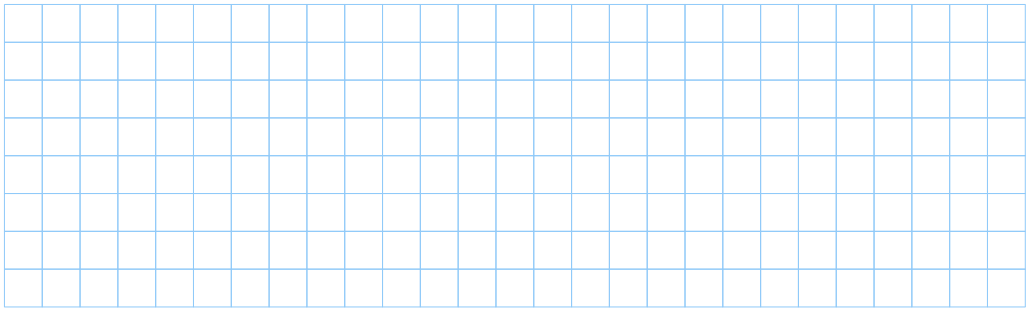
(a) Ein Kapital von 500 Franken wird zu 5 % verzinst. Der Zins wird jeweils zum Kapital geschlagen. Auf welchen Betrag wächst das Kapital in 10 Jahren?



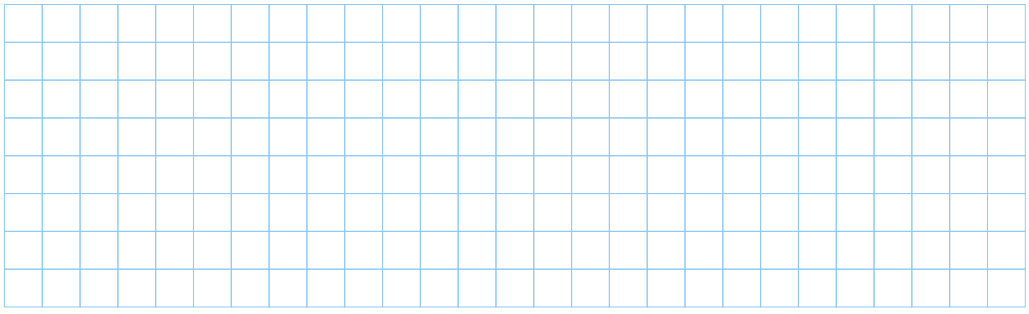
(b) Wie viel Franken muss man auf ein Sparkonto einzahlen, wenn es innert 20 Jahren durch Zinseszins auf 2000 Franken anwachsen soll? Der jährliche Zinssatz beträgt 5 %.



(c) Ein Kapital von 3600 Franken wächst in 12 Jahren auf 5000 Franken. Zu wie viel Prozent wurde es verzinst?



(d) Wie viele Jahre dauert es, bis sich ein Kapital von 1000 Franken mit einem Zinssatz von 4.5 % verdoppelt hat?

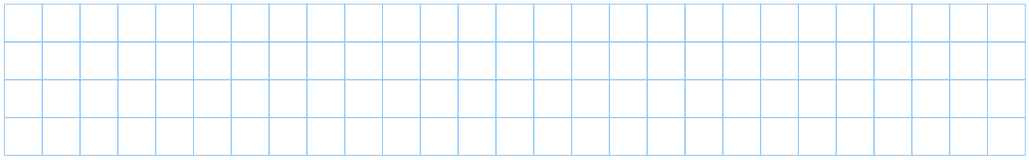


2. **Schreiben Sie als Logarithmus.** Beispiel: $10^2 = 100 \iff \log_{10}(100) = 2$
   (a) $2^5 = 32$
   (b) $5^3 = 125$
   (c) $2^6 = 64$
   (d) $2^{-3} = \frac{1}{8}$
   (e) $10^{-4} = 0.0001$
   (f) $3^4 = 81$

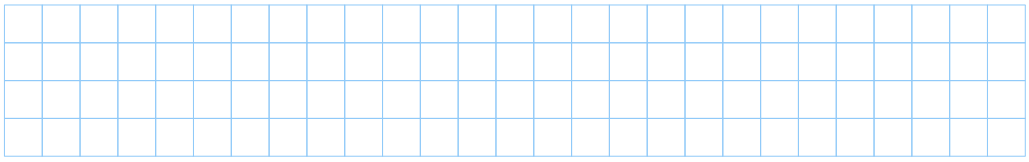
3. **Schreiben Sie als Potenz.** Beispiel: $\log_{10}(100) = 2 \iff 10^2 = 100$
   (a) $\log_4(16) = 2$
   (b) $\log_5(625) = 4$
   (c) $\log_2(32) = 5$
   (d) $\log_8(64) = 2$
   (e) $\log_2(64) = 6$
   (f) $\log_{10}(10000) = 4$

4. Logarithmus bestimmen. Bestimmen Sie x ohne Taschenrechner.

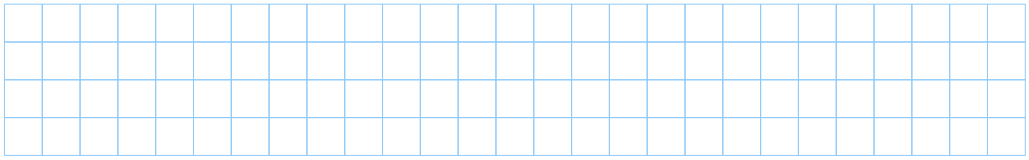
(a) $\log_4(64) = x$



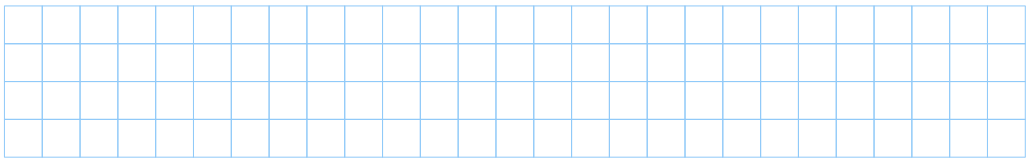
(b) $\log(100) = x$



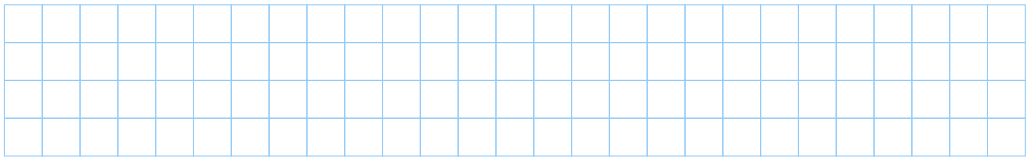
(c) $\log_4(2) = x$



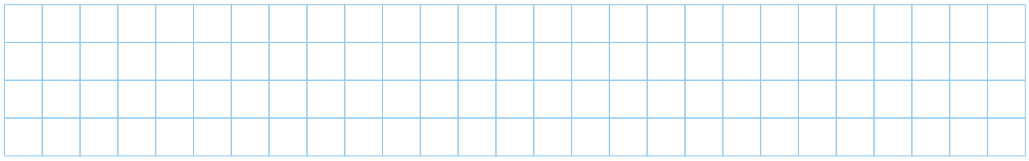
(d) $\log_3\left(\frac{1}{27}\right) = x$



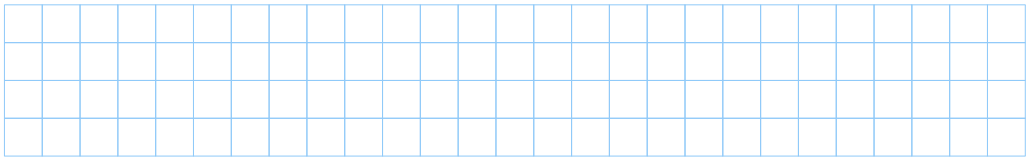
(e) $\log(1000) = x$



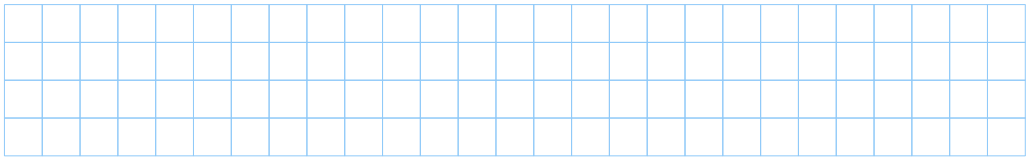
(f) $\log(10) = x$



(g) $\log_5(\sqrt{5}) = x$

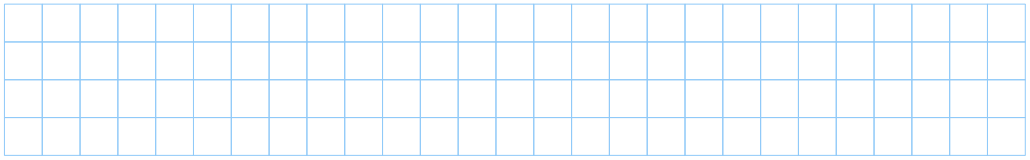


(h) $\log_3\left(\frac{1}{9}\right) = x$

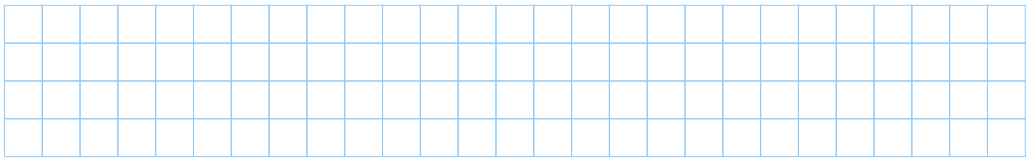


5. Numerus bestimmen. Bestimmen Sie x ohne Taschenrechner.

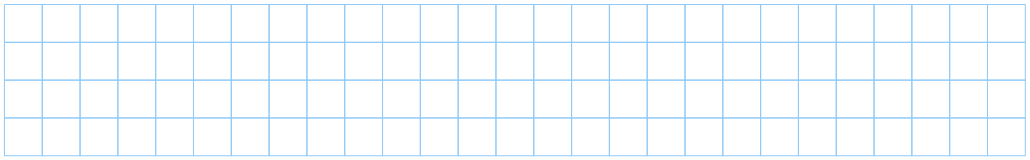
(a) $\log_8(x) = 2$



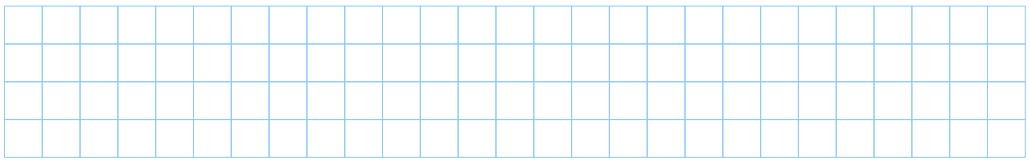
(b) $\log_4(x) = 4$



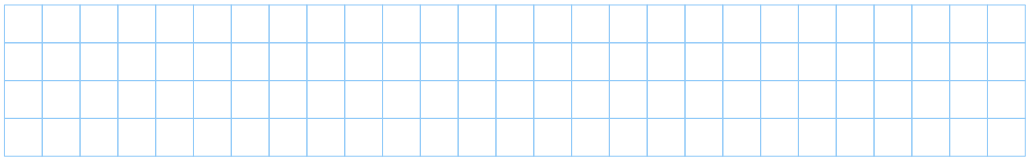
(c) $\log_2(x) = -3$



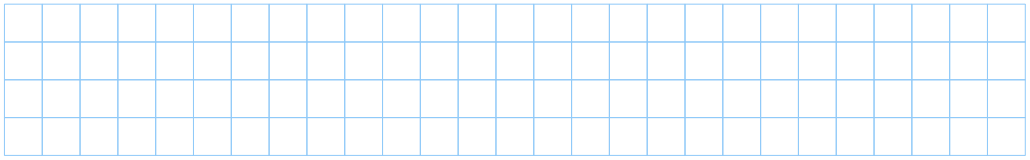
(d) $\log_8(x) = \frac{2}{3}$



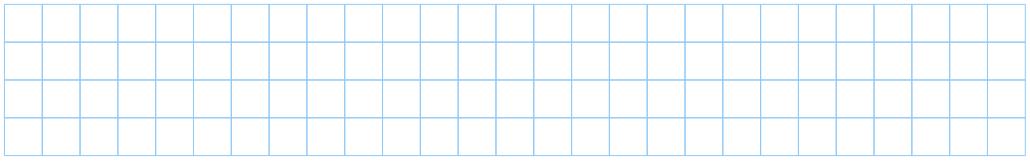
(e) $\log(x) = 2$



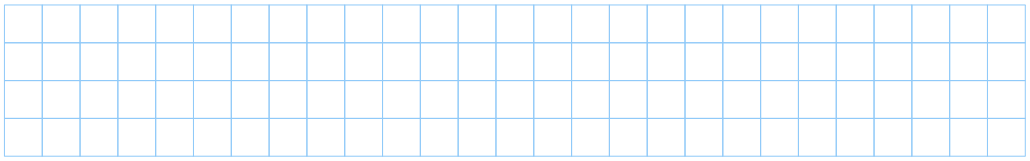
(f) $\log_4(x) = 0$



(g) $\log_4(x) = 0.5$

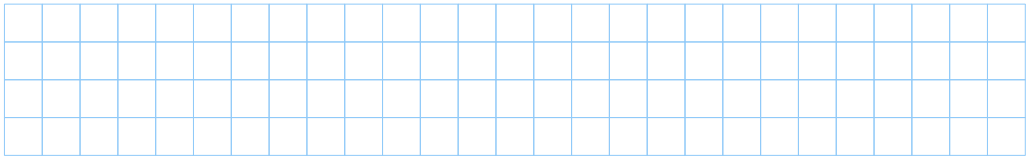


(h) $\log_{16}(x) = \frac{1}{2}$

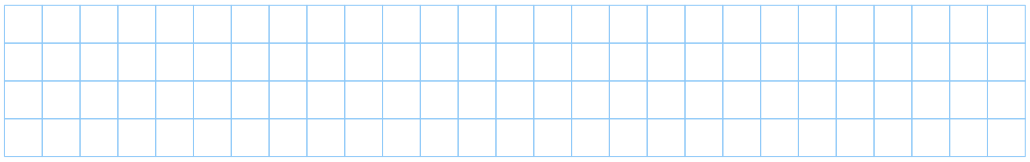


6. Basis bestimmen. Berechnen Sie x ohne Taschenrechner.

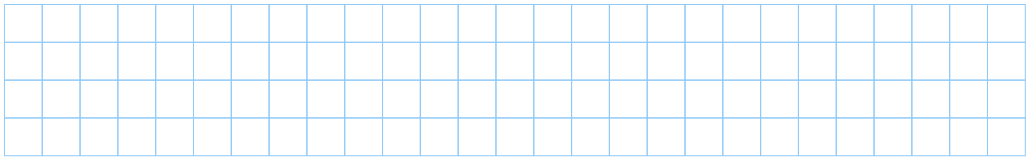
(a) $\log_x(9) = 2$



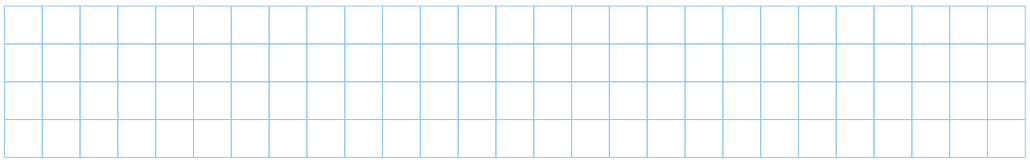
(b) $\log_x(81) = 4$



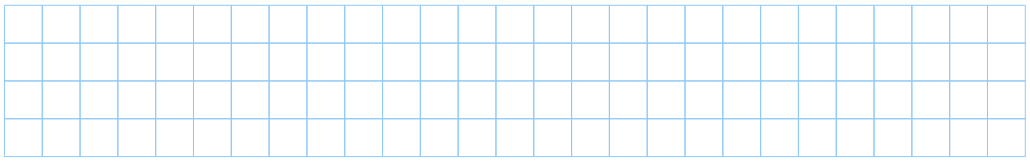
(c) $\log_x(81) = -2$



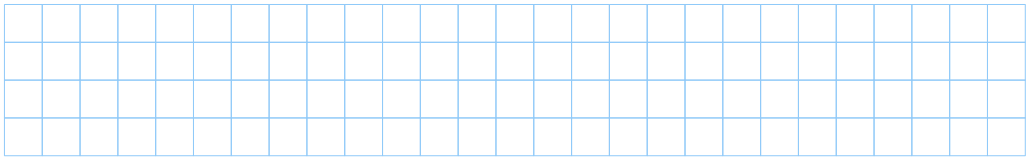
(d) $\log_x(\sqrt{27}) = \frac{3}{2}$



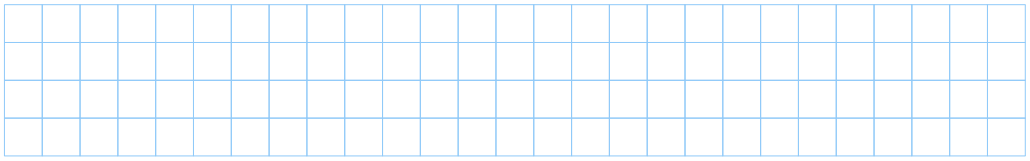
(e) $\log_x(49) = 2$



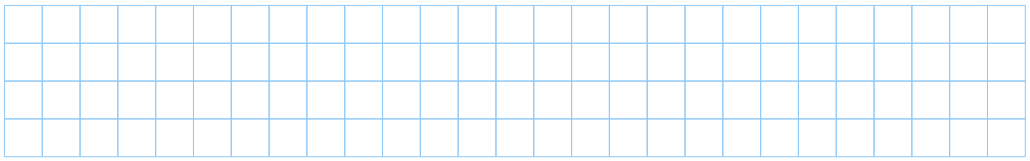
(f) $\log_x(7) = 1$



(g) $\log_x(3) = -1$



(h) $\log_x\left(\frac{1}{32}\right) = -5$



7. Vereinfachen.

(a) $\log_a(a) =$

(b) $\log_a(1) =$

(c) $\log_a\left(\frac{1}{a}\right) =$

(d) $\log_a\left(a^n\right) =$

(e) $\log_a\left(\frac{1}{a^n}\right) =$

8. Umformen. Schreiben Sie als Summe oder Produkt einfacher Logarithmen. D.h. in den Klammern des Logarithmus soll nur eine Zahl oder eine Variable stehen, z.B. $\log_{10}(a)$.

(a) $\lg(3x) =$

(b) $\log_a(abc) =$

(c) $\lg\left(u^2\right) =$

(d) $\log_a\left(2ab^2\right) =$

(e) $\log_a\left(\frac{5e}{f}\right) =$

(f) $\log_a\left(\frac{uv}{w}\right) =$

(g) $\lg\left(\sqrt{x}\right) =$

(h) $\log_a\left(\sqrt[4]{b}\right) =$

(i) $\log_a\left(\frac{x^2y^3}{u^2v^3}\right) =$

(j) $\log_a\left(\frac{1}{a^2b^4c^7}\right) =$

(k) $\mathrm{lb}\left(\frac{r^2st^4}{u^3v}\right) =$

(l) $\log_a\left(\sqrt{a^{11}b^3c^5}\right) =$

9. Exponentialgleichungen. Lösen Sie die Gleichungen nach x auf. Geben Sie die Lösung auf vier Dezimalstellen genau an. Beispiel:

$$
\begin{array}{rrl|l}
 & 1.2^{4x-7} &= 9 & \log \\
\iff & \log\left(1.2^{4x-7}\right) &= \log(9) & \text{Logarithmusgesetz} \\
\iff & (4x-7)\cdot\log(1.2) &= \log(9) & :\log(1.2) \\
\iff & 4x-7 &= \frac{\log(9)}{\log(1.2)} & \text{Taschenrechner} \\
\iff & 4x-7 &\approx 12.0514 & +7 \\
\iff & 4x &\approx 19.0514 & :4 \\
\iff & x &\approx 4.7628 &
\end{array}
$$

(a) $4^x = 12$

(b) $2.4^x = 3.9$

(c) $1.14^y = 0.7$

(d) $0.45^z = 1.9$

(e) $3.7^{2x} = 5$

(f) $1.46^{3x} = 0.8$

(g) $8.2^{-x} = 4.9$

(h) $5.6^{-2x} = 1.4$

(i) $2 \cdot 3^{x} = 1.4$

(j) $6 \cdot 1.5^{y} = 2.3$

(k) $1.3 \cdot 5^{-x} = 2.8$

(l) $0.9 \cdot 1.4^{t} = 3.2$

10. Exponentialgleichungen. Lösen Sie die Gleichungen nach x auf. Geben Sie die Lösung auf vier Dezimalstellen genau an.

(a) $10^{x-1} = 6$

(b) $6^{x+1} = 108$

(c) $5^{1-2x} = 17$

(d) $10^{5x+1} = 2$

(e) $3 \cdot 8^{-x-2} = 25$

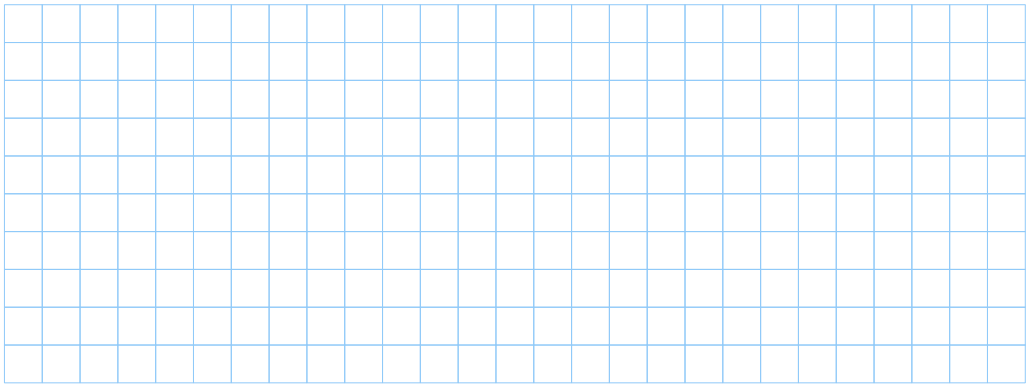
(f) $\left(7^{2x-1}\right)^2 = 36$

(g) $5 \cdot 2^{-3x+4} = 1$

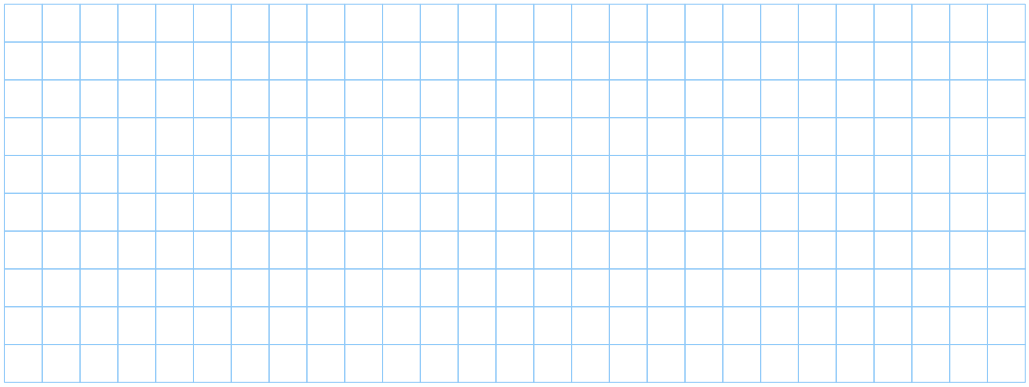
(h) $3 \cdot 4^{5-x} = 1$

11. Exponentialgleichungen. Lösen Sie die Gleichungen durch **Vergleichen der Exponenten**. Beispiel: $2^{x-2} = 2^3 \iff x - 2 = 3 \iff x = 5$

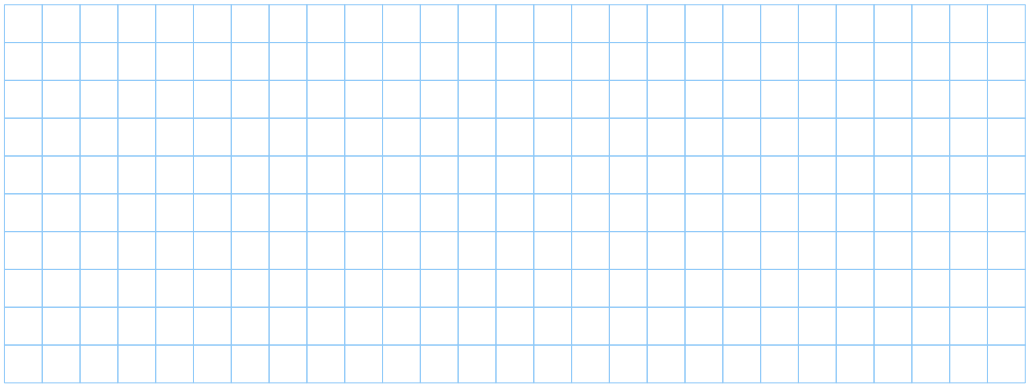
(a) $5^{2x-3} = 5$



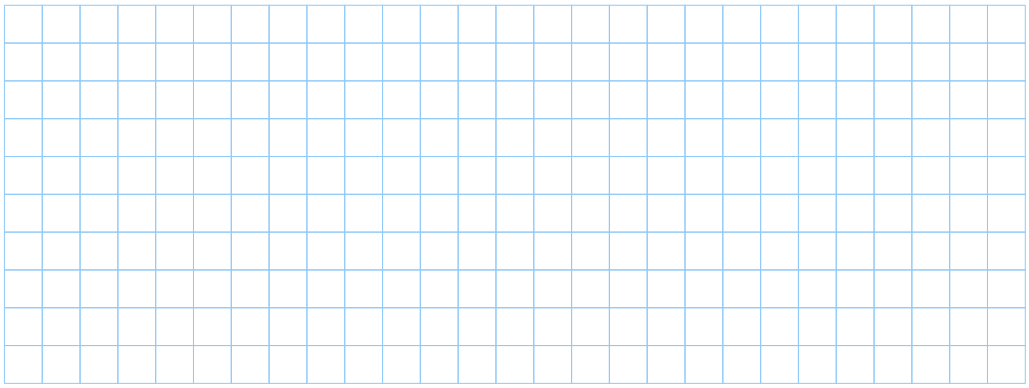
(b) $6^{4x-5} = 216$



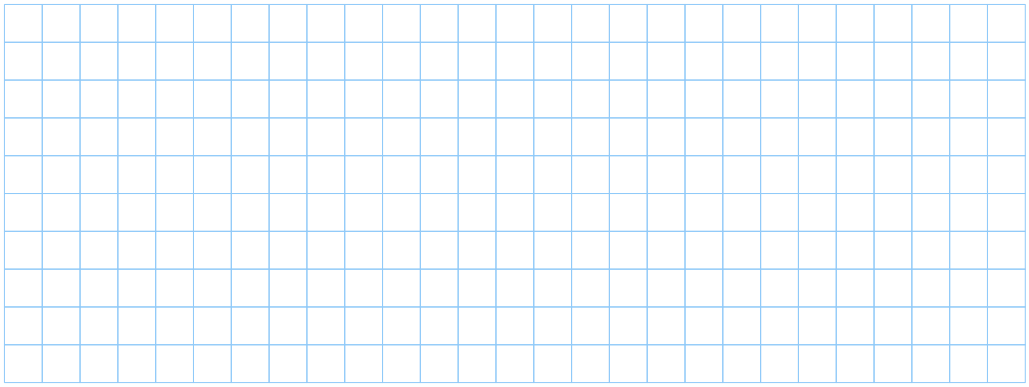
(c) $4^{2x-1} = 64$



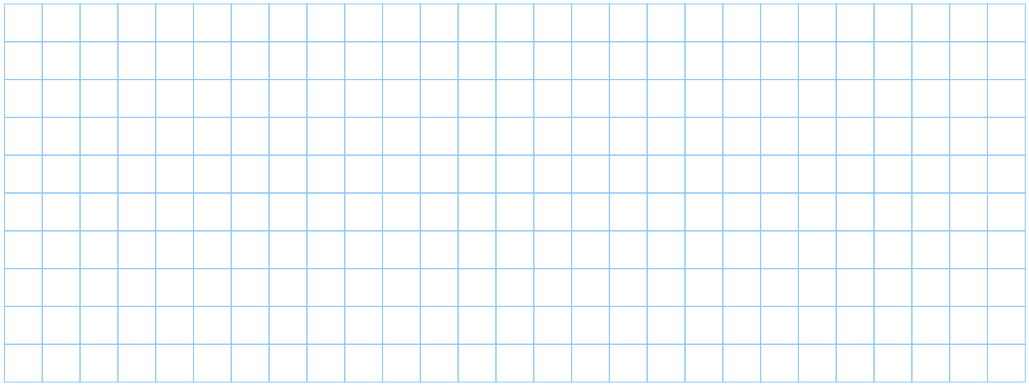
(d) $3^{2x+1} = 3^{x+2}$



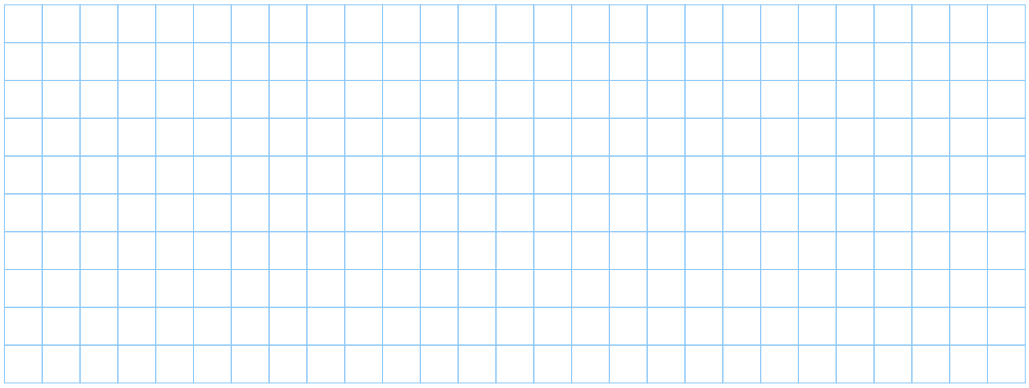
(e) $3^x = \frac{1}{81}$



(f) $25^{x+1} = \frac{1}{5}$



(g) $7^{x-2} = \sqrt{7}$



(h) $3^{5x} = \sqrt[3]{3^2}$

