CONTROL SYSTEMS

Presented

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Unit IV: Control Systems (15 Questions)

Mathematical Modelling of Physical Systems – Transfer Function - Block Diagrams and Signal Flow Graphs and their Reduction using Mason's Rule –Time Domain and Frequency Domain Analysis of Linear Time Invariant (LTI) System – Errors for Different Type of Inputs and Stability Criteria for Feedback Systems –Stability Analysis Using Routh – Hurwitz Array – Nyquist Plot and Bode Plot – Root Locus–Gain and Phase Margin – Basic Concepts of Compensator Design –PI, PD and PID Controllers – State Variable formulation – state transition matrix –Eigen values and Eigen vectors – free and forced responses of Time Invariant systems –controllability and observability.

What is a Control System?

- A Control system is a system or a set of devices that manages command and directs the behavior of other devices or systems.
- It works on the principle of the input-process-output cycle. since the output is controlled by varying input.
- They are widely used in electronics, automation, and engineering.
- It is made up of three components mainly: sensor, controller, and actuator. Here the sensor senses physical characteristics such as pressure, and temperature and converts them into an electrical signal and it generates an output signal which is used to control the actuator.

Examples of Control Systems

Control systems has various examples and it is widely used in day to day applications. Some examples are listed below:

- Elevators - In order to control or to regulate acceleration and braking mechanisms, Control systems are used.
- Smart Coffee Machine - For to control water temperature, coffee strength, and brewing rate control systems are used and all these properties are balanced to give desired flavor.
- Home Security and Automation - For to control and monitor wide aspects of home security such as cameras, sensors, motion detectors.
- Traffic light control systems - When an input processes through control system the output of three lights turned ON for some particular duration of time. Here input signal determines the output and It works based on timely basis.

Classification of Control Systems

Control system are classified as mentioned below by using suitable some parameters:

Control systems can be classified based on the type of signal we used and operate

- **Continuous signal** - Control signals are continuous with respect to time.
- **Discrete signal** - Consists of more discrete signals.

Control System can be classified on the **Number of input and Output Present**

- **SISO** - Single Input Single Output which consists of one input and one output.
- **MIMO** - Multiple Input Multiple Output which consists of two or more inputs to give outputs.

Classification of Control Systems

Control System can be classified based on the Feedback Path

- **Open loop control system** - Here control action is not dependent of desired output and it requires human efforts to operate.
- **Closed loop control system** - Here control action is dependent of output and input is applied to controller processes it to give signal.

Time-Variant vs Time-Invariant

- Time-Invariant: The system's parameters do not change with time.
- Time-Variant: The system's characteristics vary with time.

  This affects how we model and analyse the system (constant coefficients vs time-varying).

Feedback vs Feedforward

- **Feedback Control:** output is measured and used to adjust input (closed-loop). Feedback improves disturbance rejection.
- **Feedforward** Control: predetermined adjustment based on expected disturbances/input without measuring output. Often used in combination with feedback.

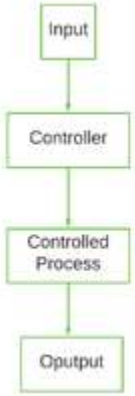
CONTROL SYSTEMS

Types of Control Systems

There are basically two types of control systems:

❖ **Open-loop control system**

❖ **Closed-loop control system**



Open Loop Control System

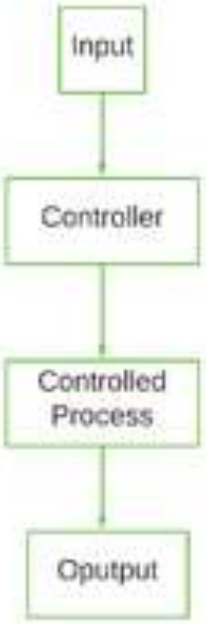
Open-Loop Control System

❖ In an **open-loop controlled system** a control signal which is considered input is sent to a system (Source), and the system responds without considering the output.

❖ The disadvantage of this system is it doesn't continuously monitor the system's behavior. The block diagram for this system is represented above..

Examples of open-loop controlled system are:

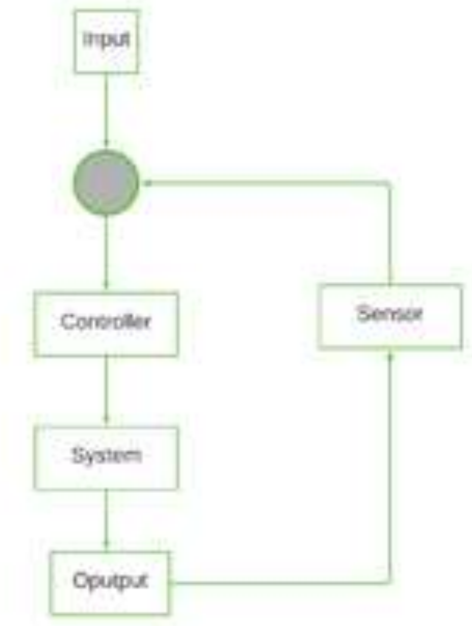
- **Washing machine** - Here the operations of soaking, drying, rinsing done on a timely basis.
- **Bread toaster and Microwave oven** - These machine run on the basis of preset time period (specific time) irrespective of an output.
- **Electric bulb** - when an electric supply is available, by switching ON and OFF electric bulb works without the parameters such as temperature, intensity of the light.



Open Loop Control System

Closed-Loop Control System

- In an **Closed-loop control system** sensors used to provide system performance and then adjust the control input accordingly.
- This system is also called **feedback control system**. well comparing to OLCS it continuously monitor the output and make some changes based on the difference between desired output and actual output.



Closed Loop Control System

Examples of Closed-loop control system are:

- **Thermostat heater** - It maintains and control the temperature.
- **Voltage stabilizer** - It measures the fluctuations in voltage and reduces & maintains the voltage to a desired level.
- **Human on any action** - When any person looks for any consequences and changes the position based on it. It is also an example of closed-loop control system.
- It also contains home appliances such as **AC, Refrigerator, and in industry appliances too.**

Difference Between Open Loop and Closed Loop Control System

Open Loop Control System	Closed Loop Control System
It is economical and inaccuracy.	It is expensive and accurate.
It does not have an feedback path.	It has an feedback path.
Control action is not dependent on output.	Control action is dependent on output.
It is designed easily.	It is hard to design.
It is called Non feedback control system	It is called Feedback control system.

Advantages of Control Systems

- Control systems having the capability to respond something faster than humans and control systems are very reliable.
- It can automate various tasks by reducing the human intervention which leads to increased productivity and decrease labor costs.
- Control systems are adaptable in any conditions and in any requirements and they can be reprogrammed to accommodate different operating functions.
- Many control systems are controlled and monitored remotely which is valuable in applications like space science and remote infrastructure.
- They are used to optimize complex processes and find the best operations to achieve the goal such as increased throughput.

Disadvantages of Control Systems

- Designing and implementation of control systems are more expensive.
- They need regular maintenance and updates for their functions in any operations.
- They highly depend on electricity which means they need continuous power supply and in case of power failure can disrupt any operation.
- They are very sensitive to environmental constraints such as light, temperature, and pressure.
- Control systems need high specialized knowledge which is complex to operate and handle.

Applications of Control Systems

Some Applications of Control System are:

- **Medical Equipment -** It is used to control various types of medical machines such as dialysis machine, X-ray machines.
- **Farming and Agriculture** - Control systems are used to automate and optimize various types of tasks in agricultural processes such as crop harvesting, fertilization etc.
- **Robotics -** Here the control systems mainly used to control and automate the movements of robots for any operations.
- **Power and Energy Systems** - It optimizes power generation, consumption and distribution improving the operational efficiency of power plants.

Applications of Control Systems

- **Environmental Control** - HVAC systems used to regulate physical or chemical characteristics.
- **Transportation -** Control systems control various aspects of transportation such as traffic control systems, air-traffic controller, etc.
- **Industrial Automation -** They optimize production processes in mills, factories, and other manufacturing industries.
- **IoT and Home automation -** IoT and home automation used to control and automate various systems in home or building such as air conditioning, heating, and security.

Mathematical Models of Physical Systems

- A mathematical model of a physical system in a control system is a set of equations, usually differential equations, that represent the dynamic behavior (input-output relationship) of a system.
- These models enable analysis of stability and performance, typically for Linear Time-Invariant (LTI) systems, using differential equations, transfer functions, or state-space representations.

Key Types of Mathematical Models

- **Differential Equation Models:** These describe the system in the time domain, applying fundamental physical laws (like Kirchhoff's voltage law for electrical systems or Newton's laws for mechanical systems).
- **Transfer Function Models:** Used for LTI systems, this is the ratio of the Laplace transform of the output to the input, assuming initial conditions are zero.
- **State-Space Models:** A set of first-order differential equations that represent the system using input, output, and state variables, allowing for detailed analysis of controllability, observability, and stability.

Steps in Mathematical Modelling

- **Identify the need:** Determine the goal of the model
- **Define parameters:** List the physical constants (e.g., mass M ,resistance, R,Capacitance C, Stiffness, k)
- **Apply Physical Laws:** Use laws like d'Alembert's principle for mechanical systems (Force = Mass X Acceleration) or Kirchhoff's laws for electrical circuits.
- **Derive Equations:** Formulate the differential equations relating input and output.
- **Obtain Transfer Function/State-Space:** Convert the model into s-domain (transfer function) or state-space form for design and analysis.

Examples of Physical Systems

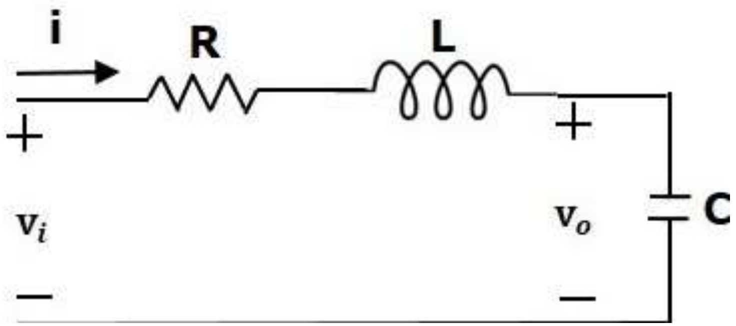
Mechanical Systems: Modeling involves force **(F),** displacement **(x) and Torque (τ)**, using elements like mass, dampers, and springs.

Electrical Systems: Modeling involves voltage (V),current(I), and charge (Q), using resistors, inductors, and capacitors.

Analogies: Mechanical systems are often modeled using electrical analogies (e.g., Force-Voltage analogy).

Example

Consider the following electrical system as shown in the following figure. This circuit consists of resistor, inductor and capacitor. All these electrical elements are connected in **series**. The input voltage applied to this circuit is V_i and the voltage across the capacitor is the output voltage V_o



Mesh equation for this circuit is

$$v_i = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + v_o$$

Substitute, the current passing through capacitor $i = c\frac{\mathrm{d}v_o}{\mathrm{d}t}$ in the above equation.

$$\Rightarrow v_i = RC\frac{\mathrm{d}v_o}{\mathrm{d}t} + LC\frac{\mathrm{d}^2 v_o}{\mathrm{d}t^2} + v_o$$

$$\Rightarrow \frac{\mathrm{d}^2 v_o}{\mathrm{d}t^2} + \left(\frac{R}{L}\right)\frac{\mathrm{d}v_o}{\mathrm{d}t} + \left(\frac{1}{LC}\right)v_o = \left(\frac{1}{LC}\right)v_i$$

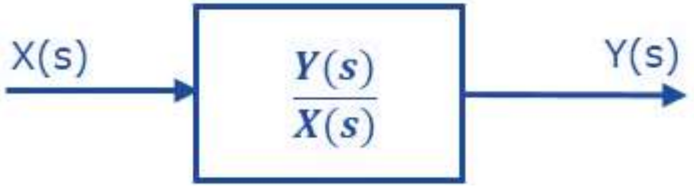
The above equation is a second order **differential equation**.

Transfer Function Model

- Transfer function model is an s-domain mathematical model of control systems.
- The **Transfer function** of a Linear Time Invariant (LTI) system is defined as the ratio of Laplace transform of output and Laplace transform of input by assuming all the initial conditions are zero.
- **I fx(t) and y(t) are t**he input and output of an LTI system, then the corresponding Laplace transforms are **X(s) and Y(s).**
- Therefore, the transfer function of LTI system is equal to the ratio of **Y(s) and X(s)**

$$i.e.,\ Transfer\ Function = \frac{Y(s)}{X(s)}$$

The transfer function model of an LTI system is shown in the following figure.



Example

Previously, we got the differential equation of an electrical system as

$$\frac{d^2v_o}{dt^2}+\left(\frac{R}{L}\right)\frac{dv_o}{dt}+\left(\frac{1}{LC}\right)v_o=\left(\frac{1}{LC}\right)v_i$$

Apply Laplace transform on both sides.

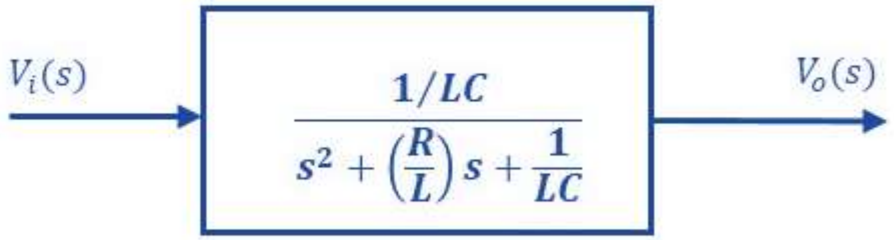
$$s^2V_o(s)+\left(\frac{sR}{L}\right)V_o(s)+\left(\frac{1}{LC}\right)V_o(s)=\left(\frac{1}{LC}\right)V_i(s)$$

$$\Rightarrow\left\{s^2+\left(\frac{R}{L}\right)s+\frac{1}{LC}\right\}V_o(s)=\left(\frac{1}{LC}\right)V_i(s)$$

$$\Rightarrow\frac{V_o(s)}{V_i(s)}=\frac{\frac{1}{LC}}{s^2+\left(\frac{R}{L}\right)s+\frac{1}{LC}}$$

- $v_i(s)$ is the Laplace transform of the input voltage v_i
- $v_o(s)$ is the Laplace transform of the output voltage v_o

The above equation is a **transfer function** of the second order electrical system. The transfer function model of this system is shown below.



Here, we show a second order electrical system with a block having the transfer function inside it. And this block has an input $V_i(s)$ and an output $V_o(s)$

Modelling of Mechanical Systems

There are two types of mechanical systems based on the type of motion.

- Translational mechanical systems
- Rotational mechanical systems

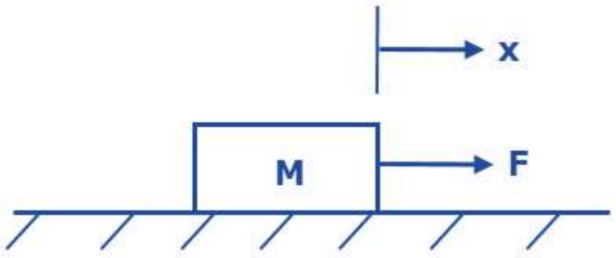
Modeling of Translational Mechanical Systems

- Translational mechanical systems move along a **straight line**. These systems mainly consist of three basic elements. Those are mass, spring and dashpot or damper.
- If a force is applied to a translational mechanical system, then it is opposed by opposing forces due to mass, elasticity and friction of the system.
- Since the applied force and the opposing forces are in opposite directions, the algebraic sum of the forces acting on the system is zero.

Let us now see the force opposed by these three elements individually.

Mass

Mass is the property of a body, which stores **kinetic energy**. If a force is applied on a body having mass **M**, then it is opposed by an opposing force due to mass. This opposing force is proportional to the acceleration of the body. Assume elasticity and friction are negligible.



$$F_m \propto a$$

$$\Rightarrow F_m = Ma = M\frac{d^2x}{dt^2}$$

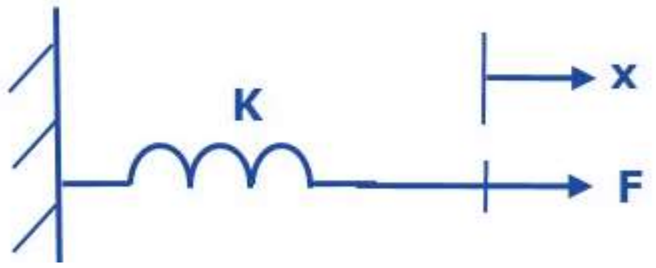
$$F = F_m = M\frac{d^2x}{dt^2}$$

Where,

- **F** is the applied force
- $\mathbf{F_m}$ is the opposing force due to mass
- **M** is mass
- **a** is acceleration
- **x** is displacement

Spring

- Spring is an element, which stores potential energy. If a force is applied on spring K, then it is opposed by an opposing force due to elasticity of spring.
- This opposing force is proportional to the displacement of the spring. Assume mass and friction are negligible.



$$F \propto x$$

$$\Rightarrow F_k = Kx$$

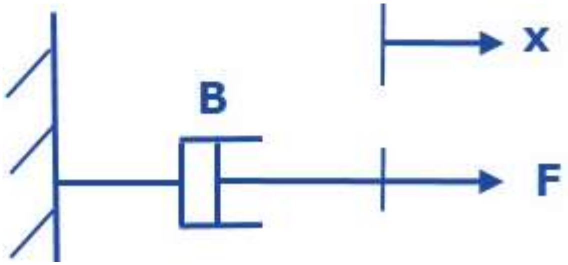
$$F = F_k = Kx$$

Where,

- F is the applied force
- F_k is the opposing force due to elasticity of spring
- K is spring constant
- x is displacement

Dashpot

- If a force is applied on dashpot B, then it is opposed by an opposing force due to friction of the dashpot.
- This opposing force is proportional to the velocity of the body. Assume mass and elasticity are negligible.



$$F_b \propto \nu$$

$$\Rightarrow F_b = B\nu = B\frac{dx}{dt}$$

$$F = F_b = B\frac{dx}{dt}$$

Where,

- Fb is the opposing force due to friction of dashpot
- B is the frictional coefficient
- v is velocity
- x is displacement

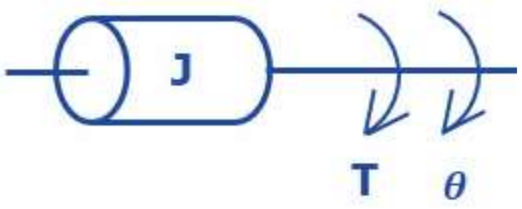
Modeling of Rotational Mechanical Systems

- Rotational mechanical systems move about a fixed axis. These systems mainly consist of three basic elements. Those are **moment of inertia, torsional spring** and **dashpot**.
- If a torque is applied to a rotational mechanical system, then it is opposed by opposing torques due to moment of inertia, elasticity and friction of the system.
- Since the applied torque and the opposing torques are in opposite directions, the algebraic sum of torques acting on the system is zero.

Let us now see the torque opposed by these three elements individually.

Moment of Inertia

- In translational mechanical system, mass stores kinetic energy. Similarly, in rotational mechanical system, moment of inertia stores kinetic energy.
- If a torque is applied on a body having moment of inertia J, then it is opposed by an opposing torque due to the moment of inertia.
- This opposing torque is proportional to angular acceleration of the body. Assume elasticity and friction are negligible.



$$T_j \propto \alpha$$

$$\Rightarrow T_j = J\alpha = J\frac{d^2\theta}{dt^2}$$

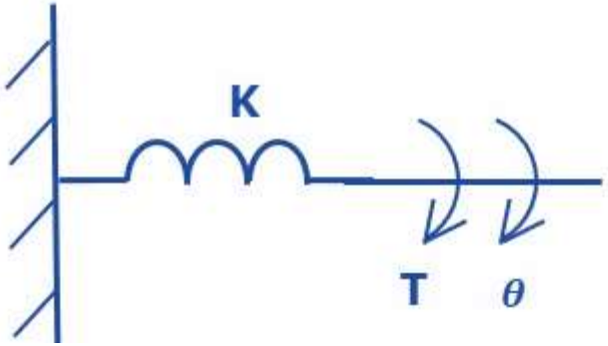
$$T = T_j = J\frac{d^2\theta}{dt^2}$$

Where,

- **T** is the applied torque
- **T_j** is the opposing torque due to moment of inertia
- **J** is moment of inertia
- **α** is angular acceleration
- **θ** is angular displacement

Torsional Spring

- In translational mechanical system, spring stores potential energy. Similarly, in rotational mechanical system, torsional spring stores potential energy.
- If a torque is applied on torsional spring K, then it is opposed by an opposing torque due to the elasticity of torsional spring. This opposing torque is proportional to the angular displacement of the torsional spring. Assume that the moment of inertia and friction are negligible.



$$T_k \propto \theta$$

$$\Rightarrow T_k = K\theta$$

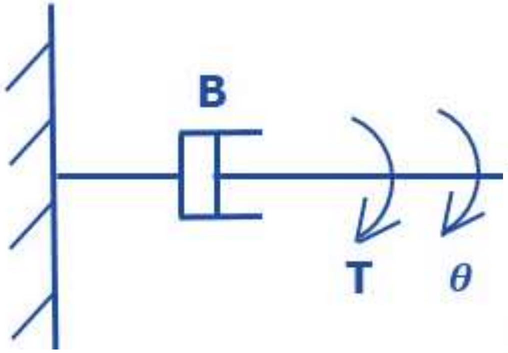
$$T = T_k = K\theta$$

Where,

- T is the applied torque
- T_k is the opposing torque due to elasticity of torsional spring
- K is the torsional spring constant
- θ is angular displacement

Dashpot

- ❖ If a torque is applied on dashpot B, then it is opposed by an opposing torque due to the rotational friction of the dashpot.
- ❖ This opposing torque is proportional to the angular velocity of the body. Assume the moment of inertia and elasticity are negligible.



$$T_b \propto \omega$$

$$\Rightarrow T_b = B\omega = B\frac{d\theta}{dt}$$

$$T = T_b = B\frac{d\theta}{dt}$$

Where,

- **T_b** is the opposing torque due to the rotational friction of the dashpot
- **B** is the rotational friction coefficient
- **ω** is the angular velocity
- **θ** is the angular displacement