



Unit 5 – Heat and Mass transfer

Quick Revision Notes

CONDUCTION

1. Introduction to Modes of Heat Transfer

Heat transfer is the movement of thermal energy from higher to lower temperature regions. There are **three primary modes**:

Conduction: Heat transfer through direct molecular contact (solids, stationary fluids). Governed by Fourier's Law.

Convection: Heat transfer due to bulk motion of fluid (liquids/gases)

Involves conduction + advection. Newton's Law of Cooling.

Radiation: Heat transfer via electromagnetic waves. No medium required. Stefan-Boltzmann Law.

Fourier's Law (Conduction): $q = -kA \frac{dT}{dx}$ where, q = heat transfer rate (W),

k = thermal conductivity (W/m·K), A = area (m²), dT/dx = temperature gradient.

Newton's Law of Cooling (Convection): $q = hA(T_s - T_\infty)$

h = convective heat transfer coefficient (W/m²·K).

Stefan-Boltzmann Law (Radiation): $q = \epsilon \sigma A(T_s^4 - T_{sur}^4) = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$,

ϵ = emissivity.

2. One-Dimensional Heat Conduction

Steady state, no heat generation, one direction only.

Plane Wall (Constant k): Temperature distribution: Linear. Heat transfer rate: $q = \frac{kA(T_1 - T_2)}{L}$;

L = thickness.

Cylindrical Wall (Hollow Cylinder/Pipe): $q = \frac{2\pi kL(T_i - T_o)}{\ln(r_o/r_i)}$

Spherical Wall: $q = \frac{4\pi k r_i r_o (T_i - T_o)}{r_o - r_i}$



Critical Radius of Insulation (Cylinder): $r_{cr} = \frac{k}{h}$ (Insulation increases heat loss below this radius.)

Thermal Conductivity (k): Ability of material to conduct heat. High for metals, low for insulators.

3. Thermal Resistance Concept & Electrical Analogy

Heat flow is analogous to electric current.

Thermal Resistance (R_{th}): Opposition to heat flow. $q = \frac{\Delta T}{R_{th}}$ (like $I = V/R$)

Conduction Resistance:

- Plane wall: $R_{cond} = \frac{L}{kA}$
- Cylinder: $R_{cond} = \frac{\ln(r_o/r_i)}{2\pi kL}$
- Sphere: $R_{cond} = \frac{r_o - r_i}{4\pi k r_i r_o}$

Convection Resistance: $R_{conv} = \frac{1}{hA}$

Radiation Resistance (linearized): $R_{rad} = \frac{1}{h_r A}$ where $h_r = \epsilon \sigma (T_s + T_{sur})(T_s^2 + T_{sur}^2)$

Composite Systems: Resistances in series or parallel. Total resistance $R_{total} = \sum R_i$ (series).

Overall Heat Transfer Coefficient U : $q = UA\Delta T$, where $\frac{1}{UA} = R_{total}$.

Contact Resistance: Additional resistance at interfaces due to imperfect contact.

4. Unsteady (Transient) Heat Conduction

Temperature changes with time. No steady state.

Lumped Capacitance Method (valid when $Bi < 0.1$):

Biot Number $Bi = \frac{hL_c}{k}$ ($L_c = V/A$, characteristic length).

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{hA_s}{\rho V c_p} t\right)$$

Time constant $\tau = \frac{\rho V c_p}{hA_s}$.

Heisler Charts / One-Term Approximation (for $Bi > 0.1$):

Used for plane wall, cylinder, sphere. Dimensionless temperature at centre: $\theta_0 = C_1 \exp(-\zeta_1^2 Fo)$



Fourier Number $Fo = \frac{\alpha t}{L^2} (\alpha = k/\rho c_p = \text{thermal diffusivity})$.

Semi-Infinite Solid: Surface suddenly exposed to new condition. Penetration depth increases with $\sqrt{\alpha t}$.

5. Fins (Extended Surfaces)

Used to increase heat transfer area when h is small (e.g., air cooling).

Fin Efficiency (η_f): Actual heat transfer / Heat transfer if entire fin at base temperature.

Fin Effectiveness (ϵ_f): Heat transfer with fin / without fin.

Common Fin Types & Formulas (Constant cross-section, steady state):

1. **Straight Rectangular Fin** (or Pin Fin): $m = \sqrt{\frac{hP}{kA_c}}$

(P = perimeter, A_c = cross-section area)

$$\text{Heat transfer: } q_f = \sqrt{hPkA_c}(T_b - T_\infty)\tanh(mL) \text{ (adiabatic tip)}$$

$$\text{For very long fin: } q_f = \sqrt{hPkA_c}(T_b - T_\infty)$$

2. **Fin with Convective Tip:** More accurate, uses corrected length $L_c = L + t/2$.

Optimum Fin Design: Balance material cost and heat transfer. Fin effectiveness > 1 for usefulness.

Fin Array: Total heat transfer = $N\eta_f q_{single}$ + heat from unfinned area.

2. One-Dimensional Steady State Heat Conduction Equation

General Heat Conduction Equation (Cartesian): $\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q}_{gen} = \rho c_p \frac{\partial T}{\partial t}$

One-Dimensional Steady State Forms (No generation, constant k):

- **Cartesian (Plane Wall):** $\frac{d^2 T}{dx^2} = 0 \Rightarrow \text{Laplace Equation}$
- **With Internal Heat Generation (Poisson Equation):** $\frac{d^2 T}{dx^2} + \frac{\dot{q}_{gen}}{k} = 0$
- **Cylindrical Coordinates (Radial):** $\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + \frac{\dot{q}_{gen}}{k} = 0$
- **Spherical Coordinates (Radial):** $\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) + \frac{\dot{q}_{gen}}{k} = 0$



Plane Wall with Heat Generation: Parabolic temperature distribution.

$$T(x) = -\frac{\dot{q}_{gen}}{2k}x^2 + C_1x + C_2$$

3. Thermal Resistance Concept & Electrical Analogy

Equivalent Resistance:

- **Series:** $R_{total} = R_1 + R_2 + R_3 + \dots$
- **Parallel:** $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

Overall Heat Transfer Coefficient (U): $q = UA\Delta T_{overall}, \frac{1}{UA} = R_{total}$

4. Composite Systems

A. Composite Plane Wall (Series)

Layers with different k and thickness L .

$$q = \frac{T_1 - T_n}{R_{total}} = \frac{(T_1 - T_n)}{\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \dots + \frac{L_n}{k_n A} + \frac{1}{h_i A} + \frac{1}{h_o A}}$$

Parallel Walls (Heat flow perpendicular to interface): $\frac{1}{R_{total}} = \frac{A_1 k_1}{L} + \frac{A_2 k_2}{L}$

B. Composite Cylinder (Multi-layer Pipe)

$$q = \frac{2\pi L(T_i - T_o)}{\frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2} + \dots + \frac{1}{h_i r_i} + \frac{1}{h_o r_o}}$$

Equivalent Resistance: $R_{total} = \sum \frac{\ln(r_{m+1}/r_m)}{2\pi k_m L} + \frac{1}{2\pi r_i L h_i} + \frac{1}{2\pi r_o L h_o}$

C. Composite Sphere (Multi-layer)

$$q = \frac{4\pi(T_i - T_o)}{\frac{1}{k_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{k_2}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \dots + \frac{1}{h_i 4\pi r_i^2} + \frac{1}{h_o 4\pi r_o^2}}$$

Critical Radius of Insulation:

- Cylinder: $r_{cr} = k/h$
- Sphere: $r_{cr} = 2k/h$



Important Definitions

- **Thermal Resistance:** Temperature difference per unit heat flow rate.
- **Overall Heat Transfer Coefficient (U):** Single coefficient representing combined conduction + convection effect.
- **Biot Number (Bi):** Ratio of internal conduction resistance to surface convection resistance. ($Bi < 0.1 \rightarrow$ Lumped system valid)
- **Fourier Number (Fo):** Dimensionless time in transient conduction.
- **Critical Insulation Radius:** Radius at which heat transfer is maximum (insulation paradox in cylinders).
- **Fin Efficiency:** Ratio of actual heat dissipated by fin to heat dissipated if entire fin were at base temperature.
- **Internal Heat Generation ($\dot{q}_{gen}$):** Heat produced per unit volume (e.g., electrical heating, nuclear fuel, chemical reaction).

CONVECTION

1. Introduction to Convection

Convection is heat transfer between a solid surface and a moving fluid (liquid or gas) due to **bulk motion** of fluid combined with conduction.

- Heat transfer occurs by **advection** (fluid motion) + **conduction** (molecular diffusion).
- Requires a **medium** (unlike radiation).
- Governed by **Newton's Law of Cooling**.

Newton's Law of Cooling: $q = hA(T_s - T_\infty)$ where h = convective heat transfer coefficient ($\text{W/m}^2\cdot\text{K}$), T_s = surface temperature, T_∞ = fluid free-stream temperature.

Classification of Convection:

1. **Forced Convection:** Fluid motion caused by external means (pump, fan, blower). Higher h .
2. **Free (Natural) Convection:** Fluid motion due to buoyancy forces from density difference caused by temperature variation. Lower h .
3. **Mixed Convection:** Both forced and free convection significant.

2. Important Dimensionless Parameters

- **Prandtl Number (Pr):**
 - $Pr \approx 0.7$ (air), $Pr \approx 7$ (water), $Pr \gg 1$ (oils), $Pr \ll 1$ (liquid metals).
 - High $Pr \rightarrow$ momentum boundary layer thicker than thermal boundary layer.

3. Boundary Layers

- **Velocity (Hydrodynamic) Boundary Layer (δ):** Region where velocity changes from 0 (no-slip) to 99% of free stream velocity u_∞ .



- **Thermal Boundary Layer (δ_t):** Region where temperature changes from T_s to 99% of T_∞ .

Relation: $\frac{\delta}{\delta_t} \approx Pr^{1/3}$ (For $Pr > 1$, $\delta > \delta_t$; for $Pr < 1$, $\delta < \delta_t$)

Boundary Layer Thickness (Flat Plate, Laminar): $\delta \approx \frac{5x}{\sqrt{Re_x}}$

4. Nusselt Number Correlations

A. Flow Over Flat Plate (External Flow)

Laminar Flow ($Re < 5 \times 10^5$):

- **Local Nusselt Number** (Constant Surface Temperature): $Nu_x = 0.332 Re_x^{1/2} Pr^{1/3}$
- **Average Nusselt Number:** $\bar{Nu}_L = 0.664 Re_L^{1/2} Pr^{1/3}$

Turbulent Flow ($Re > 5 \times 10^5$):

- **Local:** $Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3}$
- **Average:** $\bar{Nu}_L = 0.037 Re_L^{4/5} Pr^{1/3}$

Combined Laminar + Turbulent (0 to $Re = 10^8$): $\bar{Nu}_L = (0.037 Re_L^{0.8} - 871) Pr^{1/3}$

Constant Heat Flux:

- Laminar local: $Nu_x = 0.453 Re_x^{0.5} Pr^{1/3}$
- Turbulent: Slightly higher constants than isothermal case.

B. Flow Through Pipes / Tubes (Internal Flow)

Fully Developed Flow:

- **Laminar Flow ($Re < 2300$):**
Constant Surface Temperature: $Nu = 3.66$
Constant Heat Flux: $Nu = 4.36$
- **Turbulent Flow ($Re > 10^4$, $L/D > 10$):**

Dittus-Boelter Equation: $Nu = 0.023 Re^{0.8} Pr^n$ where $n = 0.4$ (heating), $n = 0.3$ (cooling)



Gnielinski Correlation (more accurate, $0.5 < Pr < 2000$): $Nu = \frac{(f/8)(Re-1000)Pr}{1+12.7(f/8)^{0.5}(Pr^{2/3}-1)}$

Entry Length Region: Higher Nu near entrance.

Transition Region ($2300 < Re < 10^4$): Use Gnielinski or other correlations.

C. Free (Natural) Convection – Important Correlations

Vertical Plate / Cylinder:

- Laminar ($10^4 < Ra < 10^9$): $\bar{Nu}_L = 0.59Ra_L^{1/4}$
- Turbulent ($Ra > 10^9$): $\bar{Nu}_L = 0.10Ra_L^{1/3}$

Horizontal Cylinder: $\bar{Nu}_D = \left\{ 0.60 + \frac{0.387Ra_D^{1/6}}{[1+(0.559/Pr)^{9/16}]^{8/27}} \right\}^2$

Horizontal Plate (Hot surface facing up / Cold surface facing down): $\bar{Nu}_L = 0.54Ra_L^{1/4}$ ($10^4 < Ra < 10^7$)

- **Nusselt Number (Nu):** Direct measure of convective heat transfer enhancement over pure conduction.
- **Thermal Boundary Layer:** Thinner for high Pr fluids $\rightarrow$ higher h .
- **Forced vs Free:** Use Re for forced; use Gr or Ra for free convection.
- **Properties:** Evaluate at **Film Temperature** $T_f = (T_s + T_\infty)/2$ (except for some free convection cases).

HEAT EXCHANGERS

1. Introduction to Heat Exchangers

Heat exchangers are devices used to transfer heat between two fluids at different temperatures without mixing them. They are widely used in boilers, condensers, radiators, refrigerators, and power plants.

Energy Balance (Neglecting losses): $\dot{m}_h c_{ph} (T_{h,i} - T_{h,o}) = \dot{m}_c c_{pc} (T_{c,o} - T_{c,i})$

Heat lost by hot fluid = Heat gained by cold fluid.

Capacity Rates: $C_h = \dot{m}_h c_{ph}$, $C_c = \dot{m}_c c_{pc}$ C_{min} = smaller capacity rate, C_{max} = larger capacity rate.

2. Types of Heat Exchangers

1. **Parallel Flow (Co-current):** Both fluids flow in same direction.
2. **Counter Flow:** Fluids flow in opposite directions (most efficient).



3. **Cross Flow:** Fluids flow perpendicular to each other (e.g., car radiator).
4. **Shell and Tube:** Most common industrial type (multi-pass possible).
5. **Plate Type, Double Pipe, Compact Heat Exchangers.**

3. Temperature – Length (T-L) Diagram

- **Parallel Flow:** Outlet temperatures approach each other; maximum temperature difference at inlet.
- **Counter Flow:** More uniform temperature difference; outlet temperature of cold fluid can exceed outlet temperature of hot fluid.
- Counter flow gives **higher LMTD** and better performance for same area.

4. LMTD Method (Log Mean Temperature Difference)

Heat Transfer Rate: $Q = UA\Delta T_{lm}$ where U = Overall heat transfer coefficient,

A = Heat transfer area.

LMTD Formula: $\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$

For Parallel Flow:

- $\Delta T_1 = T_{h,i} - T_{c,i}$
- $\Delta T_2 = T_{h,o} - T_{c,o}$

For Counter Flow:

- $\Delta T_1 = T_{h,i} - T_{c,o}$
- $\Delta T_2 = T_{h,o} - T_{c,i}$

For Cross Flow & Multi-pass: Use correction factor $FQ = UA F \Delta T_{lm, counter}$ $F = 1$ for pure counter flow, $F < 1$ for others (read from charts).

Advantages of LMTD: Simple when inlet/outlet temperatures are known.

5. NTU – Effectiveness Method

Used when outlet temperatures are unknown or when sizing the exchanger.

Number of Transfer Units (NTU): $NTU = \frac{UA}{C_{min}}$

Capacity Ratio: $C_r = \frac{C_{min}}{C_{max}}$

Effectiveness (ϵ): $\epsilon = \frac{Q_{actual}}{Q_{max}} = \frac{C_h(T_{h,i} - T_{h,o})}{C_{min}(T_{h,i} - T_{c,i})} = \frac{C_c(T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})}$



Effectiveness Relations:

1. **Counter Flow Heat Exchanger:** $\varepsilon = \frac{1 - \exp[-NTU(1 - C_r)]}{1 - C_r \exp[-NTU(1 - C_r)]}$

2. **Parallel Flow Heat Exchanger:** $\varepsilon = \frac{1 - \exp[-NTU(1 + C_r)]}{1 + C_r}$

3. **Cross Flow (Both fluids unmixed):** $\varepsilon = 1 - \exp\left[\frac{NTU^{0.22}}{C_r} \{\exp(-C_r NTU^{0.78}) - 1\}\right]$

1. Overall Heat Transfer Coefficient (U)

The **Overall Heat Transfer Coefficient (U)** represents the combined effect of all thermal resistances between the two fluids in a heat exchanger or composite wall.

Basic Equation: $Q = UA\Delta T$ where U = Overall heat transfer coefficient ($\text{W/m}^2 \cdot \text{K}$), A = Heat transfer area, ΔT = Appropriate temperature difference (LMTD or others).

Based on Reference Area (Important):

- $U_i \rightarrow$ based on inside area (A_i)
- $U_o \rightarrow$ based on outside area (A_o)

1/U Formula for Tubular Heat Exchanger (Clean Surface):

$$\frac{1}{U} = \frac{1}{h_i} + \frac{r_i \ln(r_o/r_i)}{k_{\text{wall}}} + \frac{r_i}{r_o} \cdot \frac{1}{h_o}$$

For Plane Wall:

$$\frac{1}{U} = \frac{1}{h_i} + \frac{L}{k} + \frac{1}{h_o}$$

Including Fouling (Most Important for Exams):

$$\frac{1}{U_{\text{dirty}}} = \frac{1}{h_i} + R_{f,i} + \frac{L}{k} + R_{f,o} + \frac{1}{h_o}$$

RADIATION

1. Introduction to Thermal Radiation



Thermal radiation is the transfer of heat in the form of **electromagnetic waves** due to temperature. It requires **no medium** and travels at the speed of light. All bodies above absolute zero emit radiation.

Key Features:

- Depends strongly on temperature (T^4).
- Wavelength range: 0.1 μm to 100 μm (thermal radiation).
- Governed by **Stefan-Boltzmann Law**.

2. Important Laws of Radiation

Stefan-Boltzmann Law (Black Body): $E_b = \sigma T^4$ $\sigma = 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4$ E_b = emissive power of black body.

Wien's Displacement Law: $\lambda_{max} T = 2898 \mu\text{m} \cdot \text{K}$ (λ_{max} = wavelength at which maximum radiation occurs).

Planck's Law: Gives spectral distribution of black body radiation (basis for Stefan-Boltzmann and Wien's laws).

Kirchhoff's Law (Very Important): At thermal equilibrium: $\alpha = \epsilon$ Absorptivity = Emissivity (for a given wavelength and temperature). For grey body: $\alpha = \epsilon$ = constant.

3. Black Body, Grey Body & Real Surfaces

Emissive Power of Grey Body: $E = \epsilon \sigma T^4 = \epsilon E_b$

Radiosity (J): Total radiation leaving a surface (emitted + reflected). $J = \epsilon E_b + \rho G$ where G = irradiation (incident radiation).

4. Shape Factor (View Factor / Configuration Factor)

Definition: Fraction of radiation leaving surface i that is intercepted by surface j . Notation: F_{ij}

Properties of Shape Factors:

1. **Reciprocity Relation:** $A_i F_{ij} = A_j F_{ji}$
2. **Summation Rule:** For enclosure, $\sum F_{ij} = 1$
3. $F_{ii} = 0$ for plane or convex surfaces.
4. $F_{ij} = 1$ for concave surface seeing only itself.

Important Shape Factors:

- Two infinite parallel plates: $F_{12} = 1$
- Concentric spheres/cylinders: $F_{12} = 1$ (inner to outer)
- Small object in large enclosure: $F_{12} \approx 1$



5. Radiation Network Analysis (Electrical Analogy)

Surface Resistance (Space Resistance): $R_{surface} = \frac{1-\epsilon_i}{\epsilon_i A_i}$

Space Resistance (between two surfaces): $R_{space} = \frac{1}{A_i F_{ij}}$

Net Heat Transfer between two surfaces: $Q_{net} = \frac{E_{b1}-E_{b2}}{R_{total}}$

For Two Grey Surfaces (Infinite parallel plates or large enclosure): $Q_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$

For Small Grey Object in Large Enclosure: $Q = \epsilon_1 A_1 \sigma (T_1^4 - T_2^4)$

6. Radiation Shields

Used to reduce radiation heat transfer.

Heat Transfer with One Shield: $Q_{with\ shield} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1 + \frac{1}{\epsilon_{s1}} + \frac{1}{\epsilon_{s2}} - 1}$

With **n shields**: Heat transfer reduces to $\frac{1}{n+1}$ times the value without shield.

7. Important Definitions & Equations

- **Irradiation (G)**: Incident radiation on a surface.
- **Radiosity (J)**: Total radiation leaving the surface.
- **Emissivity (ϵ)**: Ability of surface to emit radiation compared to black body.
- **Solar Absorptivity**: Important for solar radiation (different from thermal emissivity).
- **Greenhouse Effect**: Short wavelength passes, long wavelength trapped.

1. Introduction to Thermal Radiation

Thermal radiation is electromagnetic wave emission due to temperature. No medium required.

2. Important Laws

- **Stefan-Boltzmann Law**: $E_b = \sigma T^4$, $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$
- **Wien's Displacement Law**: $\lambda_{max} T = 2898 \mu\text{m} \cdot \text{K}$
- **Kirchhoff's Law**: $\alpha = \epsilon$ (at thermal equilibrium)
- **Planck's Law**: Spectral emissive power distribution

3. Black, Grey & Real Surfaces

- **Black Body**: $\epsilon = 1$, $\alpha = 1$
- **Grey Body**: $\epsilon = \alpha = \text{constant} (< 1)$



- **Radiosity:** $J = \varepsilon\sigma T^4 + \rho G$

4. Shape Factor (View Factor)

- F_{ij} = fraction of radiation leaving i intercepted by j
- Reciprocity: $A_i F_{ij} = A_j F_{ji}$
- Summation: $\sum_j F_{ij} = 1$

5. Radiative Heat Exchange Between Parallel Plates

Case 1: Two Infinite Black Parallel Plates

- $F_{12} = 1$
- Net heat flux: $\frac{Q}{A} = \sigma(T_1^4 - T_2^4)$

Case 2: Two Infinite Grey Parallel Plates

- Most common case in exams.

Net Heat Flux:

$$\frac{Q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

Special Cases:

- If $\varepsilon_1 = \varepsilon_2 = \varepsilon$: $\frac{Q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{2}{\varepsilon} - 1}$
- If one surface is black ($\varepsilon_1 = 1$): $\frac{Q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_2}} = \varepsilon_2 \sigma(T_1^4 - T_2^4)$

Case 3: Radiation Shields Between Parallel Plates

Radiation shields reduce net heat transfer.

With One Shield (emissivity ε_s on both sides):

$$\frac{Q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 + \frac{2}{\varepsilon_s} - 1} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2}{\varepsilon_s} - 2}$$

With N Shields (all shields have same emissivity ε_s):



$$\frac{Q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 + (N)\left(\frac{2}{\varepsilon_s} - 1\right)}$$

→ Heat transfer reduces to $1/(N+1)$ times the unshielded value (if all emissivities are equal).

6. Radiation Network Analysis

- **Surface Resistance:** $\frac{1-\varepsilon}{\varepsilon A}$
- **Space Resistance:** $\frac{1}{A_i F_{ij}}$
- Net heat transfer: $Q = \frac{E_{b1} - E_{b2}}{R_{total}}$

For Two Grey Parallel Plates: Total resistance = Surface resistance 1 + Space resistance + Surface resistance 2 (This leads to the same formula shown above).

MASS TRANSFER

1. Basic Concepts of Mass Transfer

Mass transfer is the movement of a chemical species from a region of higher concentration to lower concentration. It occurs due to **concentration gradient**.

Modes of Mass Transfer:

- **Diffusion Mass Transfer** (Molecular): Due to random molecular motion.
- **Convective Mass Transfer:** Due to bulk motion of fluid.

Analogy with Heat Transfer:

- Temperature difference → Concentration difference
- Heat flux (q) → Mass flux (N)
- Thermal conductivity (k) → Diffusivity (D)
- Heat transfer coefficient (h) → Mass transfer coefficient (h_m)

Concentration Units:

- Mass concentration (ρ_A), Molar concentration (C_A), Mole fraction (x_A or y_A)

2. Diffusion Mass Transfer & Fick's Law

Fick's First Law of Diffusion (Most Important): $J_A = -D_{AB} \frac{dC_A}{dx}$ where,

- J_A = Diffusive mass flux of species A (kmol/m²·s or kg/m²·s)
- D_{AB} = Binary diffusion coefficient / Mass diffusivity (m²/s)



- $\frac{dC_A}{dx}$ = Concentration gradient

Negative sign indicates diffusion from high to low concentration.

Fick's Second Law (Unsteady diffusion): $\frac{\partial C_A}{\partial t} = D_{AB} \frac{\partial^2 C_A}{\partial x^2}$

3. Steady State Molecular Diffusion

A. Diffusion through a Plane Membrane / Stagnant Film (Equimolar Counter Diffusion): $N_A = \frac{D_{AB}}{L} (C_{A1} - C_{A2})$

B. Diffusion through Stagnant Gas (Stefan Diffusion): One component is stagnant (e.g., evaporation of water into air). $N_A = \frac{D_{AB} P}{RT L (p_{B,lm})} (p_{A1} - p_{A2})$ where $p_{B,lm}$ = Log mean partial pressure of stagnant component B.

C. Diffusion in Cylindrical Coordinates (e.g., through tube wall): Similar to conduction in cylinder.

4. Convective Mass Transfer

Mass transfer between a surface and a moving fluid due to both diffusion and bulk motion.

Convective Mass Transfer Coefficient (h_m or k_c): $N_A = h_m (C_{A,s} - C_{A,\infty})$

or $N_A = k_c (\rho_{A,s} - \rho_{A,\infty})$

5. Momentum, Heat and Mass Transfer Analogy

Reynolds Analogy (Simplest): $St = St_m = \frac{f}{2}$ (St = Stanton number for heat, St_m for mass)

Chilton-Colburn Analogy (Most Important & Widely Used): $j_H = j_M = \frac{f}{2}$ where $j_H = St \cdot Pr^{2/3}$
 $Pr^{2/3} j_M = St_m \cdot Sc^{2/3}$

6. Momentum, Heat and Mass Transfer Analogy

Chilton-Colburn Analogy (Most Important):

$$\frac{h}{\rho u c_p} Pr^{2/3} = \frac{h_m}{u} Sc^{2/3}$$

Simplified Form (Reynolds Analogy modified):



$$j_H = j_M = \frac{f}{2}$$

Where:

- j_H = Colburn j -factor for heat transfer = $St \cdot Pr^{2/3}$
- j_M = Colburn j -factor for mass transfer = $St_m \cdot Sc^{2/3}$
- St = Stanton number for heat = $\frac{h}{\rho u c_p}$
- St_m = Stanton number for mass = $\frac{h_m}{u}$

Key Significance: This analogy allows us to calculate the **mass transfer coefficient** (h_m) using known heat transfer data or friction factor (f).

7. Convective Mass Transfer Correlations

A. Flow over Flat Plate (External Flow)

Laminar Flow:

- **Local Sherwood Number:** $Sh_x = 0.332 Re_x^{1/2} Sc^{1/3}$
- **Average Sherwood Number:** $\bar{Sh}_L = 0.664 Re_L^{1/2} Sc^{1/3}$

B. Flow inside Tubes (Internal Flow - Turbulent)

Dittus-Boelter type correlation:

$$Sh = 0.023 Re^{0.8} Sc^{1/3}$$

(Valid for turbulent flow, $Re > 10^4$, $0.6 < Sc < 3000$)

C. Natural (Free) Convection

$$Sh = C (Gr \cdot Sc)^n$$

Where C and n are constants depending on Rayleigh number range (analogous to natural convection heat transfer correlations).

D. For Evaporation / Dissolution Problems

Use Sherwood number correlations similar to Nusselt number correlations by replacing:



- $Nu \rightarrow Sh$
- $Pr \rightarrow Sc$

Important Definitions:

- **Sherwood Number (Sh):** $Sh = \frac{h_m L}{D_{AB}}$ → Ratio of convective mass transfer to diffusive mass transfer.
- **Schmidt Number (Sc):** $Sc = \frac{\nu}{D_{AB}}$ → Momentum diffusivity / Mass diffusivity.