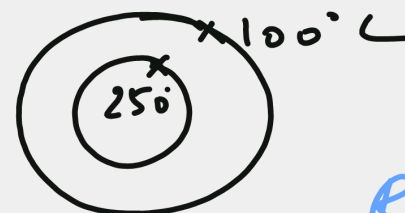
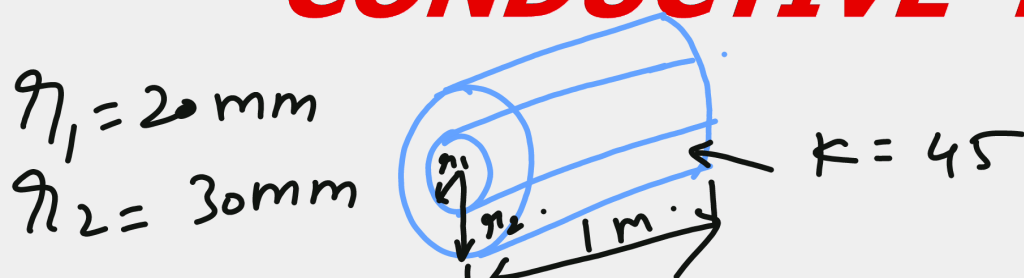


CONDUCTIVE HEAT TRANSFER



$$Q = \frac{\Delta T}{R}$$

$$R_{\text{con}} = \frac{1}{2\pi k L} \ln\left(\frac{r_2}{r_1}\right)$$

Simple Hollow Cylinder (Calculating Heat Transfer per Meter)

Problem: A steam pipe ($k = 45 \text{ W/m}\cdot\text{K}$) has an inside diameter of 40 mm and an outside diameter of 60 mm. The inside surface is at 250°C and the outside surface is at 100°C . Calculate the heat loss per meter length of the pipe.

Solution:

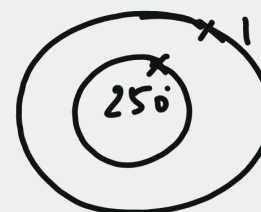
Given: $r_1 = 0.02 \text{ m}$, $r_2 = 0.03 \text{ m}$, $k = 45 \text{ W/m}\cdot\text{K}$, $T_1 = 250^\circ\text{C}$, $T_2 = 100^\circ\text{C}$.

Formula: $\frac{Q}{L} = \frac{2\pi k(T_1 - T_2)}{\ln(r_2/r_1)}$

Calculation: $\frac{Q}{L} = \frac{2 \times \pi \times 45 \times (250 - 100)}{\ln(0.03/0.02)} = 104590.6 \text{ W/m (or } 104.59 \text{ kW/m)}$

CONDUCTIVE HEAT TRANSFER

$$Q = \frac{\Delta T}{\frac{1}{2\pi k L} \ln\left(\frac{r_2}{r_1}\right)} = \frac{\Delta T \times 2\pi k L}{\ln\left(\frac{r_2}{r_1}\right)}$$



$$0.4055$$

$$Q = \frac{\Delta T}{R} \quad R_{m} = \frac{1}{2\pi k L} \ln\left(\frac{r_2}{r_1}\right)$$

Simple Hollow Cylinder (Calculating Heat Transfer per Meter)

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Given: $r_1 = 0.02 \text{ m}$, $r_2 = 0.03 \text{ m}$, $k = 45 \text{ W/m}\cdot\text{K}$, $T_1 = 250^\circ\text{C}$, $T_2 = 100^\circ\text{C}$.

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CONDUCTIVE HEAT TRANSFER

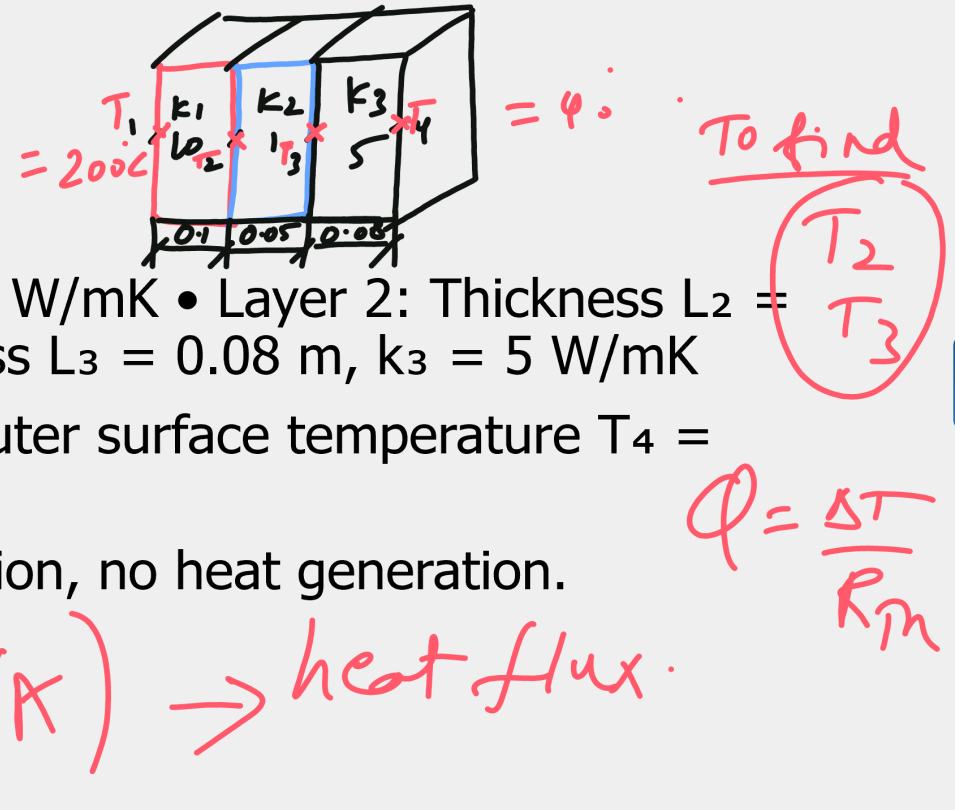
• Problem Statement

- A composite wall consists of three layers:
 - Layer 1: Thickness $L_1 = 0.1$ m, $k_1 = 10$ W/mK
 - Layer 2: Thickness $L_2 = 0.05$ m, $k_2 = 1$ W/mK
 - Layer 3: Thickness $L_3 = 0.08$ m, $k_3 = 5$ W/mK
- Inner surface temperature $T_1 = 200^\circ\text{C}$ Outer surface temperature $T_4 = 40^\circ\text{C}$

- **Assumptions:** Steady-state, 1D conduction, no heat generation.

• Find:

- Heat transfer rate per unit area (q)
- Interface temperatures (T_2 and T_3)



CONDUCTIVE HEAT TRANSFER

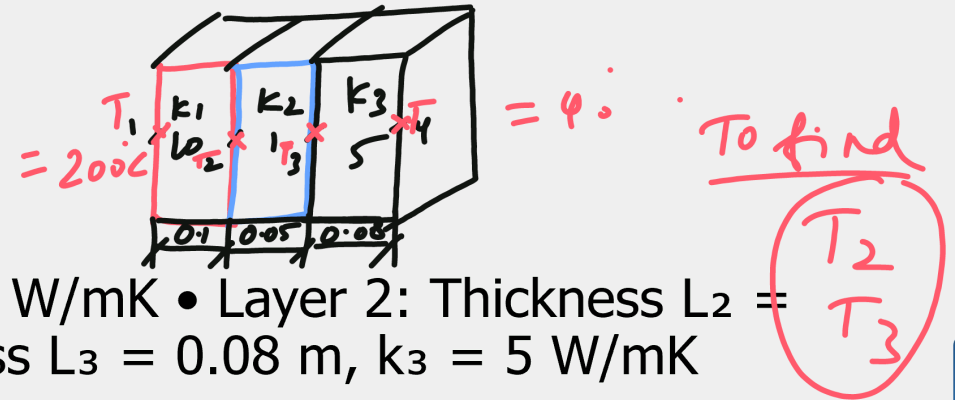
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- **Assumptions:** Steady-state, 1D conduction, no heat generation.

• Find:

- Heat transfer rate per unit area (q)
- Interface temperatures (T_2 and T_3)



To find
 T_2
 T_3

$$Q = \frac{\Delta T}{R_{\text{th}}}$$

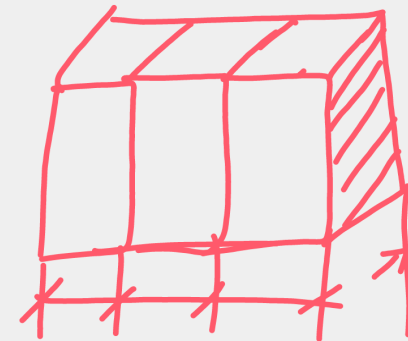
$(Q/A) \rightarrow$ heat flux.

$$R_{\text{th series}} = R_{\text{eq}} = R_1 + R_2 + R_3.$$

CONDUCTIVE HEAT TRANSFER

$$R_{eq \text{ series}} = R_1 + R_2 + R_3$$

$$= \frac{L_1}{k_1 A_1} + \frac{L_2}{k_2 A_2} + \frac{L_3}{k_3 A_3}$$



$$A_1 = A_2 = A_3$$

$$Q = \frac{\Delta T}{R}$$

$$Q = \frac{T_1 - T_4}{\frac{1}{A} (0.228)}$$

$$\frac{Q}{A} = \frac{200 - 40}{0.228} = 707.76 \text{ W/m}^2$$

$$= \frac{1}{A} \left[\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} \right] = \frac{1}{A} \left[\frac{0.1}{10} + \frac{0.05}{1} + \frac{0.08}{5} \right]$$

$$= \frac{1}{A} (0.228)$$



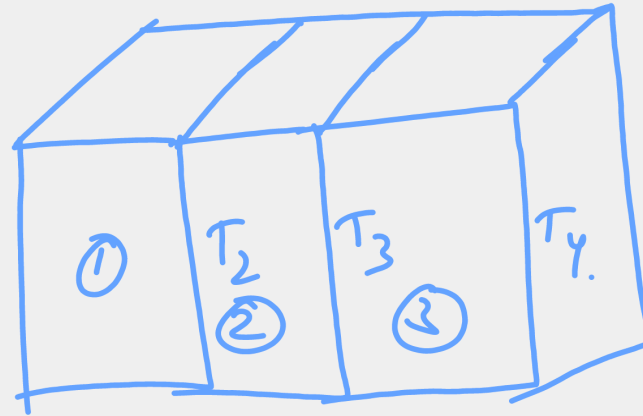
CONDUCTIVE HEAT TRANSFER

$$R_{m1} = \frac{L_1}{k_1 A_1}$$

$$Q = 2105.2 = \frac{200}{T_1 - T_2}$$

$$\frac{\frac{L_1}{k_1 A_1}}{A_1}$$

$$2105.2 = \frac{200 - T_2}{\left(\frac{0.1}{10}\right)}$$



$$\Rightarrow T_2 = 178.95$$

$$Q_2 = \frac{T_2 - T_3}{\frac{L_2}{k_2 A_2}}$$

$$2105.2 = \frac{178.95 - T_3}{0.05}$$

$$T_3 = 73.7^\circ\text{C}$$

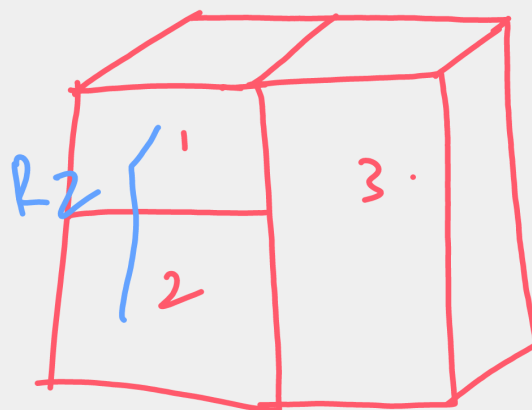
CONDUCTIVE HEAT TRANSFER

- **Step 1: Thermal Resistances** $R_1 = 0.1/10 = 0.01 \text{ m}^2\text{K/W}$
- $R_2 = 0.05/1 = 0.05 \text{ m}^2\text{K/W}$
- $R_3 = 0.08/5 = 0.016 \text{ m}^2\text{K/W}$
- $R_{\text{total}} = 0.01 + 0.05 + 0.016 = 0.076 \text{ m}^2\text{K/W}$
- **Step 2: Heat Flux** $q = (200 - 40) / 0.076 = 2105.26 \text{ W/m}^2$
- **Step 3: Interface Temperatures**
- Temperature drop across each layer: $\Delta T_1 = 2105.26 \times 0.01 = 21.05^\circ\text{C}$
 $\Delta T_2 = 2105.26 \times 0.05 = 105.26^\circ\text{C}$ $\Delta T_3 = 2105.26 \times 0.016 = 33.68^\circ\text{C}$
- Interface temperatures: $T_2 = 200 - 21.05 = \underline{178.95^\circ\text{C}}$ $T_3 = 178.95 - 105.26 = \underline{73.69^\circ\text{C}}$
- $T_4 \approx 40^\circ\text{C}$ (checks satisfied)

CONDUCTIVE HEAT TRANSFER

p

Composite wall in parallel Config.



To find
 $R_{eq\ 2,3}$
 $= R_2 + R_3.$

$$Q = \frac{\Delta T}{R} = \frac{\Delta T}{R_{eq\ 2,3}}.$$

$R_{eq\ 1,2}$ for 1 & 2

$$R_{eq\ 1,2} \Rightarrow \frac{1}{R_{eq\ 1,2}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= R_2 \text{ (say)}$$



$$R_{B1} = \frac{0.2}{4 \times 1} = 0.05, \quad R_{B2} = \frac{0.2}{6} = 0.033$$

$\frac{2}{2} = 0.025$

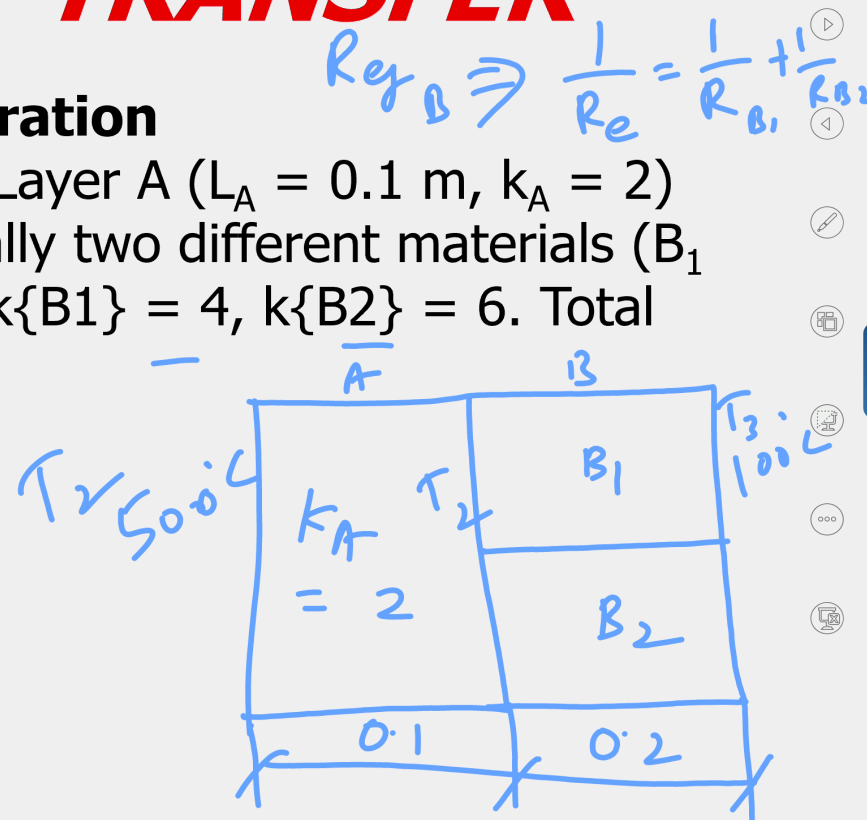
CONDUCTIVE HEAT TRANSFER

Composite Wall: Series and Parallel Configuration

Problem: A wall consists of two layers in series: Layer A ($L_A = 0.1$ m, $k_A = 2$) and Layer B ($L_B = 0.2$ m, $k_B = 5$). Layer B is actually two different materials (B_1 and B_2) in parallel, each occupying half the area. $k\{B1\} = 4$, $k\{B2\} = 6$. Total Area $A = 1$ m². $T_{in} = 500^\circ\text{C}$, $T_{out} = 100^\circ\text{C}$. Find Q .

Solution:

- Resistance A: $R_A = \frac{L_A}{k_A A} = 0.05$ K/W
- Resistance B1 (Parallel): $R_{B1} = 0.1$ K/W
- Resistance B2 (Parallel): $R_{B2} = 0.0667$ K/W
- Equivalent Parallel Resistance: $R_{B,eq} = 0.04$ K/W
- Total Resistance: $R_{total} = 0.09$ K/W
- Heat Transfer: $Q = \frac{\Delta T}{R_{total}} = \frac{500-100}{0.09} = 4444.4$ W



CONDUCTIVE HEAT TRANSFER

$$T_m = T_s + \frac{q_g}{8kL}$$

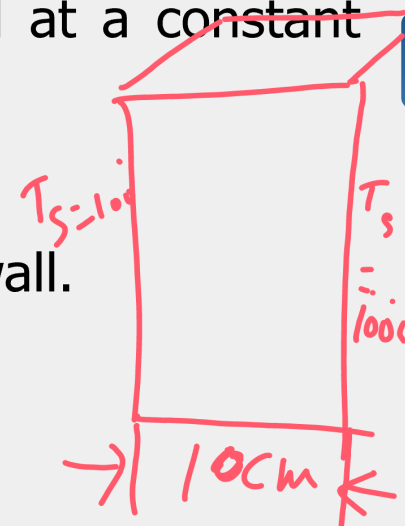
Problem: A large plate of thickness $L = 10\text{cm}$ (0.1 m) supports a uniform internal heat generation of $\dot{q}_g = 5 \times 10^5 \text{W/m}^3$. The thermal conductivity of the wall is $k = 25 \text{W/m}\cdot\text{K}$. Both surfaces of the wall are maintained at a constant temperature of $T_s = 100^\circ\text{C}$.

Calculate:

The maximum temperature $T_{\max}$ in the wall.

The temperature at a distance of 2.5 cm from the center of the wall.

The heat flux leaving the surface of the wall.



CONDUCTIVE HEAT TRANSFER

Solution:

1. Calculate the Maximum Temperature ($T_{\max}$)

For a wall with internal heat generation and equal surface temperatures on both sides, the maximum temperature occurs at the **mid-plane (centreline)**.

Formula:

$$T_{\max} = T_s + \frac{\dot{q}_g L^2}{8k}$$

$$T_s + \frac{\dot{q}_g L^2}{8k} = T_{\max}$$

Note: L is the total thickness. If you use the half-thickness $b = L/2$, the formula is $T_s + \frac{\dot{q}_g b^2}{2k}$

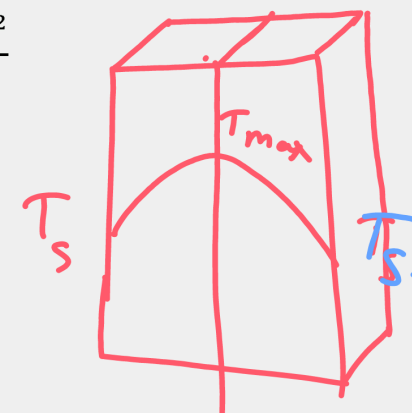
Calculation:

1. Maximum Temperature $T_{\max}$

$$T_{\max} = 100 + \frac{(5 \times 10^5) \times (0.1)^2}{8 \times 25}$$

$$T_{\max} = 100 + \frac{5000}{200}$$

$$T_{\max} = 100 + 25 = 125^\circ \text{C}$$



CONDUCTIVE HEAT TRANSFER

2. Temperature at $x = 2.5\text{cm}$ from the Center

The temperature distribution relative to the center ($x = 0$ at the mid-plane) is:

$$T(x) = T_{\max} - \left(\frac{\dot{q}_g}{2k} \right) x^2$$

from L

Given: $x = 0.025\text{m}$

Calculation:

$$T(x) = 125 - \left(\frac{5 \times 10^5}{2 \times 25} \right) (0.025)^2$$
$$T(x) = 125 - (10,000) \times (0.000625)$$
$$T(x) = 125 - 6.25 = 118.75^\circ\text{C}$$

CONDUCTIVE HEAT TRANSFER

A steam pipe of 20 mm outside diameter is to be covered with an insulating material having a thermal conductivity $k = 0.5 \text{ W/m}\cdot\text{K}$. The surface heat transfer coefficient to the surrounding air is $10 \text{ W/m}^2\cdot\text{K}$.

1. Determine the critical radius of insulation.
2. If the pipe is covered with this critical thickness, calculate the percentage change in the critical radius required if the external convection coefficient increases to $25 \text{ W/m}^2\cdot\text{K}$.

Solution:

Step 1: Critical Radius r_c for Cylinder

$$r_c = \frac{k}{h} = \frac{0.5}{10} = 0.05 \text{ m} = 50 \text{ mm}$$

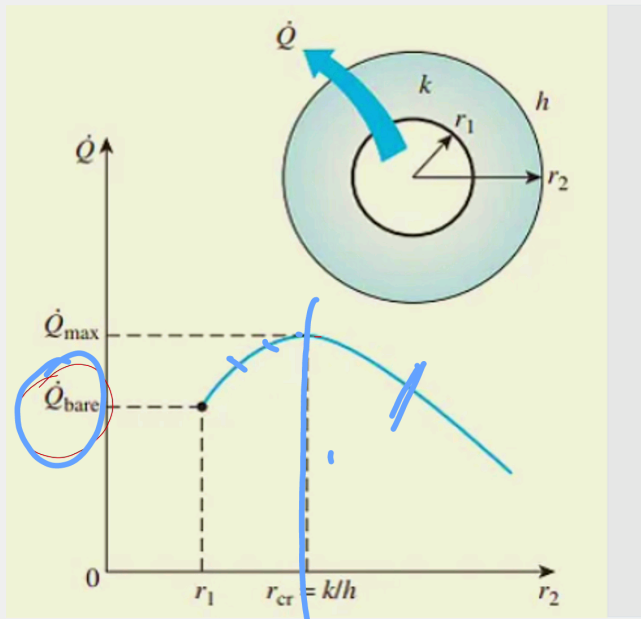
Step 2: New Critical Radius $r_{c,new}$

$$r_{c,new} = \frac{0.5}{25} = 0.02 \text{ m} = 20 \text{ mm}$$



$$r_c = \frac{k_{ins}}{h_{air}}$$

CRITICAL RADIUS OF INSULATION



The critical radius of insulation for a cylindrical body:

$$r_{cr, cylinder} = \frac{k}{h} \quad (m)$$

The critical radius of insulation for a spherical shell:

$$r_{cr, sphere} = \frac{2k}{h}$$

The **critical radius of insulation** is the outer radius at which the total thermal resistance is minimized and heat transfer is maximized for cylindrical or spherical objects, such as pipes or cables. Adding insulation below this radius *increases* heat loss; adding it above *decreases* heat loss.

CONDUCTIVE HEAT TRANSFER

$$\frac{T - T_{\infty}}{T_b - T_{\infty}} = e^{-hx} \cos m x$$

Extended Surfaces (Fins)

Purpose To increase the surface area and enhance the heat transfer rate from a surface to the surrounding fluid.

$$\uparrow Q = -kA \frac{dT}{dx}$$
$$\uparrow Q = hA \Delta T$$

Common Types

Rectangular/Uniform Fin: Constant cross-section.

Pin Fin: Cylindrical rods.

Triangular/Tapered Fin: Non-uniform cross-section (saves material).

Fin Performance Metrics • Fin Effectiveness (ϵ): Ratio of fin heat transfer to the heat transfer without the fin. $\epsilon > 2$ is usually desired.

• **Fin Efficiency (η):** Ratio of actual heat transfer to the maximum possible heat transfer (if the entire fin were at the base temperature).

CONDUCTIVE HEAT TRANSFER

Convection

$$Q = hA \Delta T \checkmark$$

$$= \frac{\Delta T}{\left(\frac{1}{hA}\right)}$$

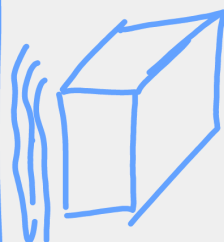
$$Q = \frac{\Delta T}{R_m}$$

$$R_m = \frac{1}{hA}$$

Conduction

$$Q = KA \frac{dT}{dx} \text{ (or) } Q = \frac{\Delta T}{R}$$

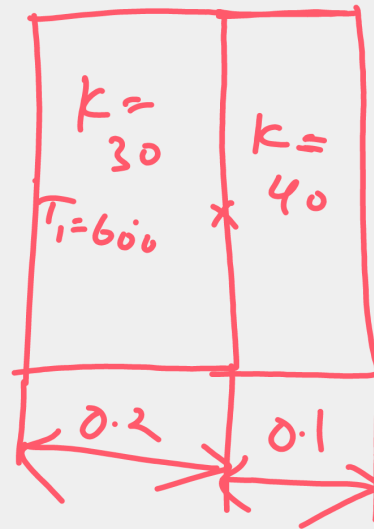
$$= \frac{dT}{\frac{dx}{kA}} = \frac{\Delta T}{\left(\frac{L}{kA}\right)}$$



$$\Rightarrow R_m = \frac{L}{kA}$$

CONDUCTIVE HEAT TRANSFER

$T_1 = 800^\circ$
 $h = 300 \frac{W}{m^2 K}$
 $Q = h A \Delta T$
 $= 800 \times (800 - 600)$
 $= 1,60,000 \text{ W}$



$h = 2 \frac{W}{m^2 K}$
 $T = 200^\circ C$

$Q = ?$
 $Q = \frac{600}{\frac{1}{h_1} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_2}}$
 $= \frac{600}{\frac{1}{300} + \frac{0.2}{30} + \frac{0.1}{40} + \frac{1}{2}}$
 $T_2 =$