



Lines of Flow:

- **Path Line** → The path followed by a single fluid particle
- **Streak Line** → Path followed by more than one (many) fluid particle
- **Stream Line** → An Imaginary line in fluid flow, the tangent at any point on stream line will give the direction of velocity at that point
- **Stream Surface** → Several stream line put together forms stream surface
- **Stream Tube** → If stream surface is closed, it forms the stream tube.

- Velocity Potential Function (ϕ): ϕ

The velocity potential function (or) potential function is defined as “a scalar function of space and time, such that its negative derivative with respect to any direction gives velocity in that direction”.

(ie) For steady flow

$$u = -\frac{\partial \phi}{\partial x}, \quad v = -\frac{\partial \phi}{\partial y}, \quad w = -\frac{\partial \phi}{\partial z}$$

For 3-dimensional continuity eqn

$$\rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad w = -\frac{\partial \phi}{\partial z}$$

$$\frac{\partial}{\partial x} \left(-\frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left(-\frac{\partial \phi}{\partial y} \right) + \frac{\partial}{\partial z} \left(-\frac{\partial \phi}{\partial z} \right) = 0$$

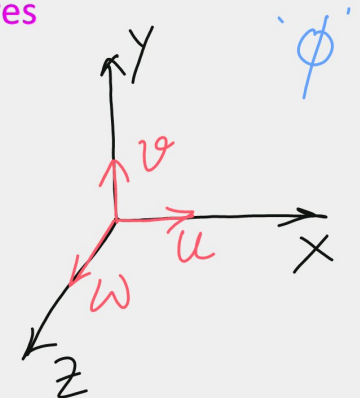
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \rightarrow \boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0}$$

Laplace eqn for ~~stream~~ function (ϕ)

Potential

For 2-dimensional eqn

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$





Properties of velocity potential function:

- a) If velocity potential exist the flow should be irrotational .
- b) If velocity potential function satisfies the laplace eqn, it represents the possible steady, incompressible and irrotational flow.



STREAM FUNCTION (Ψ)

It is defined as “a scalar function of space & time such as its partial derivative with respect to any direction gives velocity component at right angles that direction”. (ie) its differentiation with respect to x gives velocity in y-direction at its differentiation with respect to y gives velocity in x-direction

For steady flow

$$u = -\frac{\partial \Psi}{\partial y}, v = \frac{\partial \Psi}{\partial x}$$

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For continuity eqn in 2-dimensional eqn

$$\rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

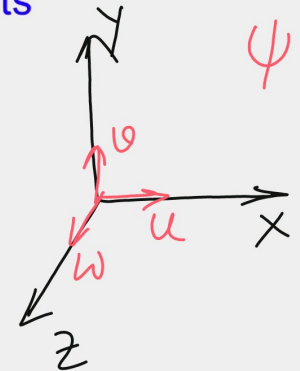
Substituting u & v in above eqn

$$u = -\frac{\partial \Psi}{\partial y}, v = \frac{\partial \Psi}{\partial x}$$

For irrotational flow

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = 0 \rightarrow \text{laplace eqn for stream function}(\Psi)$$

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = 0$$





Properties of stream function:

- a) If stream function exist then it is a case of possible fluid flow.
- b) If stream function satisfies laplace eqn then it is a case of irrotational flow

Incompressible

Relation between stream function(Ψ) & velocity potential function (ϕ)

Velocity potential function $u = -\frac{\partial \phi}{\partial x}$, $v = -\frac{\partial \phi}{\partial y}$

Stream function $u = -\frac{\partial \Psi}{\partial y}$

$$v = \frac{\partial \Psi}{\partial x}$$

Equating; $-\frac{\partial \phi}{\partial x} = -\frac{\partial \Psi}{\partial y}$;

$$\frac{\partial \phi}{\partial x} = \frac{\partial \Psi}{\partial y}$$

$$-\frac{\partial \phi}{\partial y} = \frac{\partial \Psi}{\partial x}$$

$$\frac{\partial \phi}{\partial y} = -\frac{\partial \Psi}{\partial x}$$

Relationship between ϕ & Ψ

ϕ

↓

$$u = -\frac{\partial \phi}{\partial x}$$

$$v = -\frac{\partial \phi}{\partial y}$$

Ψ

↓

$$u = -\frac{\partial \Psi}{\partial y}$$

$$v = \frac{\partial \Psi}{\partial x}$$

$$\rightarrow \frac{\partial \phi}{\partial x} = \frac{\partial \Psi}{\partial y}$$

$$\rightarrow -\frac{\partial \phi}{\partial y} = \frac{\partial \Psi}{\partial x}$$



State if the flow is represented by $u = 3x - 4y$; $v = 2x + 3y$ is rotational or irrotational?

$$u = 3x - 4y \rightarrow \frac{\partial u}{\partial x} = 3$$

$$v = 2x + 3y \rightarrow \frac{\partial v}{\partial y} = 3$$

$$\text{Continuity Eqn} \rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$3 + 3 \neq 0$$

↓
The flow is not irrotational
↓
rotational

A stream function for 3-dimensional flow is given by $\psi = 8xy$. Calculate velocity at point (4,5).
 $x \ y$

$$\psi = 8xy$$

$$\text{velocity } V_R = \sqrt{u^2 + v^2}$$

$$\downarrow$$
$$u = -\frac{\partial \psi}{\partial y} = -8x \rightarrow u_{(4,5)} = -8(4) = -32$$

$$v = \frac{\partial \psi}{\partial x} = 8y \rightarrow v_{(4,5)} = 8(5) = 40$$

$$\therefore V_R = \sqrt{(-32)^2 + (40)^2}$$

$$V_R = 51.22 \text{ unit}$$

The velocity component of a possible incompressible flow field $u=2xy$, $v=a^2+x^2-y^2$. (i) Do they satisfy continuity equation (ii) is the flow irrotational?

$$u = 2xy \rightarrow \frac{\partial u}{\partial x} = 2y$$

$$v = a^2 + x^2 - y^2 \rightarrow \frac{\partial v}{\partial y} = -2y$$

$$\text{Continuity Eqn} \rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rightarrow 2y - 2y = 0 \rightarrow \text{It satisfies continuity Eqn.}$$

$\therefore$ The flow is **IRROTATIONAL**

A 2-dimensional flow is described by $v=y/b^2$, $u=x/a^2$. Verify whether the flow is irrotational.

$$u = \frac{x}{a^2} \rightarrow \frac{\partial u}{\partial x} = \frac{1}{a^2}$$

$$v = \frac{y}{b^2} \rightarrow \frac{\partial v}{\partial y} = \frac{1}{b^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{1}{a^2} + \frac{1}{b^2} \neq 0 \rightarrow \text{continuity Eqn is not satisfied}$$

$\therefore$ The flow is not Irrotational

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Flow is Rotational



Flow Net:

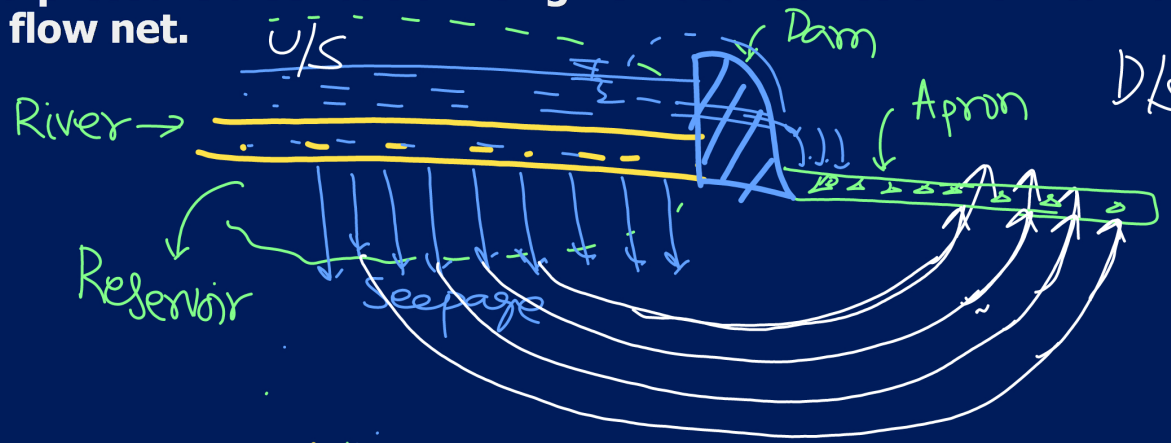
A grid obtained by a series of stream lines and equipotential lines is called as **Flow Net**.

(Equipotential lines pass through points of equal pressure.)

A streamline is a path traced out by a fluid particle as it moves with the flow)

Uses of Flow net:

- Flow nets are useful to determine the **quantity of seepage** and **uplift pressure** below hydraulic structures.
- The velocity and pressure distribution for given boundaries of flow can be determined with the use of flow net.





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