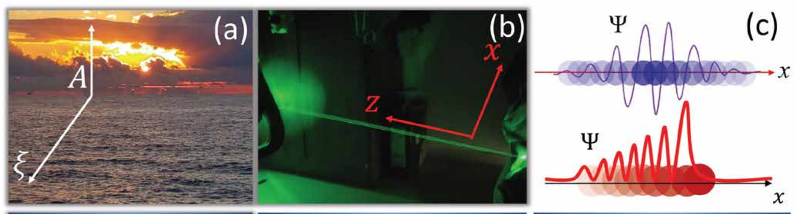
Temporal Refraction, Reflection and Impedance Matching of Surface Gravity Water Waves

Georgi Gary Rozenman, Yakir Hadad, Lev Shemer and Ady Arie.
School of Physics, Department of Electrical Engineering, Tel Aviv University, Israel

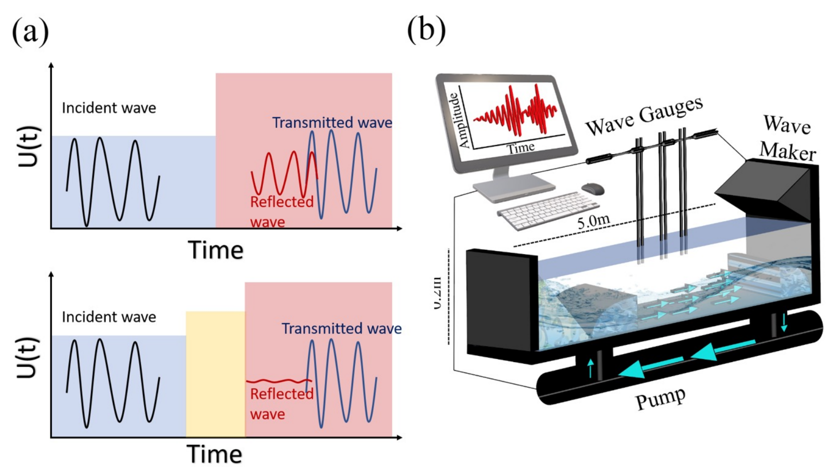
The time evolution of a wave function in quantum mechanics is analogous to that of surface gravity water wave pulses[1] . We have utilized this analogy and studied the propagation of surface gravity water waves in an effective linear potential[2], a long-standing problem presented by Kennard in 1927, realized by means of a time-dependent homogeneous and well-controlled water flow. We have measured the cubic phase[2], for the first time for both Gaussian and Airy wave packets. This study also led to the observation of accelerating solitons[3] and projectile motion of wave packets[4]. Currently, we extend this analogies to Talbot carpets[5], diffractive guiding[6], dark focusing[7], Bohm trajectories, decoherence, black holes and *temporal refractions*.

Analogies



Surface Gravity Waves	Optics	Quantum mechanics
$i\frac{\partial A}{\partial \xi} - \frac{\partial^2 A}{\partial \tau^2} = 0$ Linearly paraxial wave equation describing evolution of surface gravity water waves.	$i\frac{\partial E}{\partial z} + \frac{1}{2k}\frac{\partial^2 E}{\partial x^2} = 0$ Paraxial Helmholtz equation describing wave propagation dynamics in optics, plasmonics, electronics, acoustics, etc.	$i\hbar\frac{\partial \Psi}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} = 0$ Schrödinger equation describing the evolution of a free particle

Experimental Setup



(a) Schematic representation of the temporal refraction and reflection owing to a step change in the water flow (top) and a quarter-wavelength impedancetransformer having an intermediate step in the water flow. (b) Experimental setup for generating surface gravity water wave packets moving in a time-dependent flow created by a water pump and are measured by capacitance type wavegauges.

Motivation

In recent years, there is a renewed interest in wave propagation in time varying media, with many new proposals to study new effects. Specifically, it was proposed to study reflection and refraction through temporal boundaries, and to extend the concept of impedance matching, antireflection coating and wide band absorption, which are well known in the spatial domain, to the temporal domain.

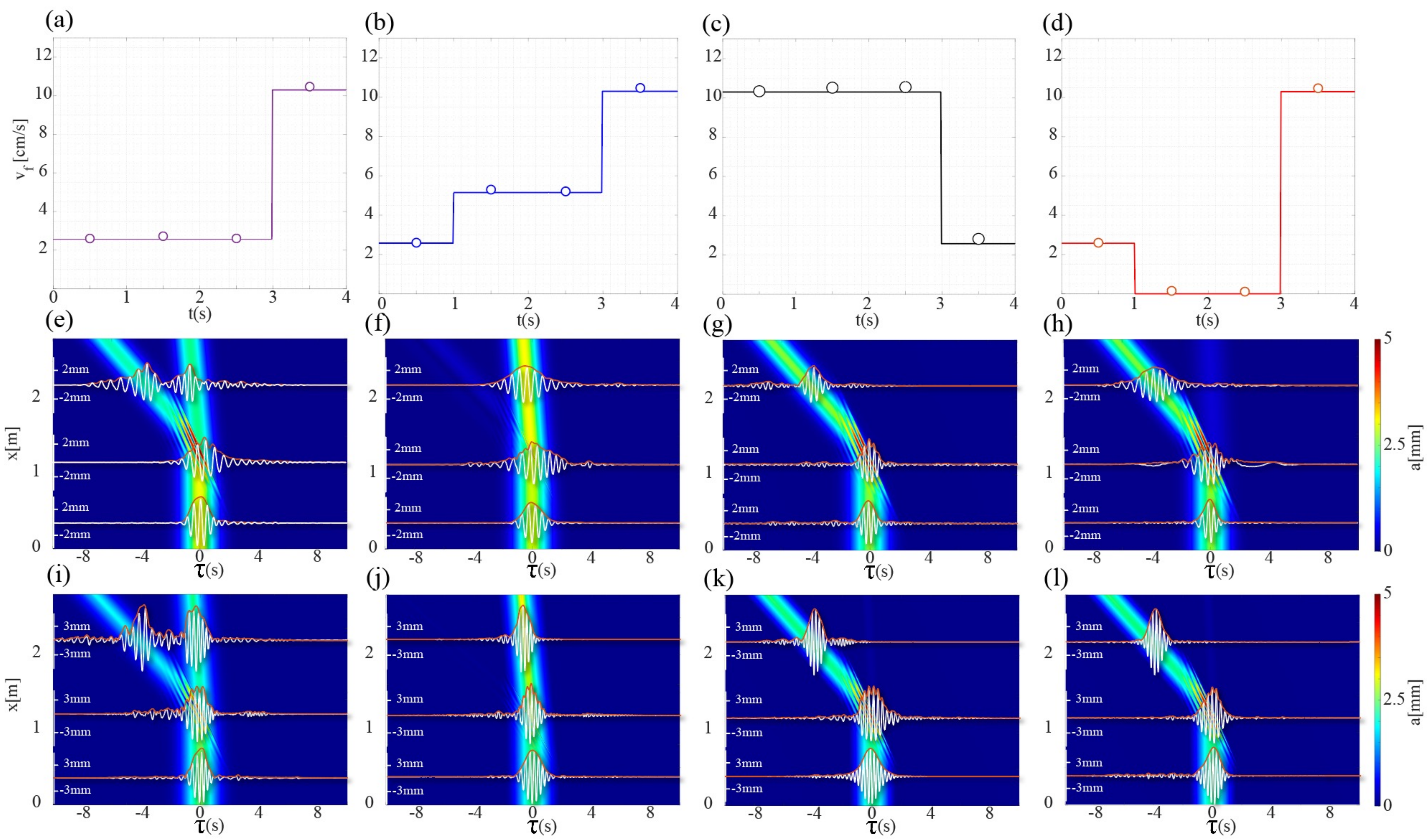
Theory

The wave equation for deep wave surface gravity waves for the normalized amplitude envelope in the moving frame with the group velocity c_g is:

$$i\frac{\partial A}{\partial \xi} = \frac{\partial^2 A}{\partial \tau^2} + V(\tau)A$$

The scaled dimensionless variables ξ and τ are related to the propagation coordinate x and the time t by $\xi \equiv \varepsilon^2 k_0 x$ and $\tau \equiv \varepsilon\omega_0\left(\frac{x}{c_g} - t\right)$

Results



(a-d) water pump input as a function of time t. τ is the temporal coordinate in the moving frame and t is in the lab frame. The input is shown as a solid line and is also measured, shown with circles. (e) transmitted and reflect waves without impedance matching (f) transmitted wave with impedance matching (g) reflected wave with impedance matching without a second rise in flow (h) reflected wave with impedance matching. The white lines represent measurements at $x = 0.3,1.2,2.2[m]$, and the continuous curves represent simulations.

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References:
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