The Andrew and Erna Viterbi Faculty of ELECTRICAL & COMPUTER ENGINEERING

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Phase Retrieval of Vortices in Bose-Einstein Condensates

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Intro - Bose-Einstein Condensates

Bose-Einstein condensation (BEC) is a quantum state of matter where particles are trapped and cooled until they form a macroscopic population in a single quantum wavefunction. Measurements of such states are performed by imaging the particles in-situ or in the "far-field" by opening the trap and recording time-of-flight (TOF) images. In these measurements, one records the atoms' density (similar to the intensity of optical beams). Alas, as this is a pure quantum state, it also has a well-defined phase structure [1], which cannot be resolved by these measurements alone. Thus, to recover the phase structure of the BEC wavefunction, interference measurements have been suggested, which poses a major experimental challenge. This raises a natural question: Can the phase structure be recovered without atomic interference? As we explain below, this question is related to the well-known phase retrieval problem from optics, but with some important differences.

Here, we propose and demonstrate in numerical simulations a simple scheme for the complete characterization of a Bose-Einstein condensation wavefunction by applying a phase retrieval algorithm on TOF measurements. Our method can resolve ambiguities and facilitates the recovery of single vortices and vortex arrays, including flow directionality, which are highly difficult tasks that have not been demonstrated without atomic interference.

Phase Retrieval

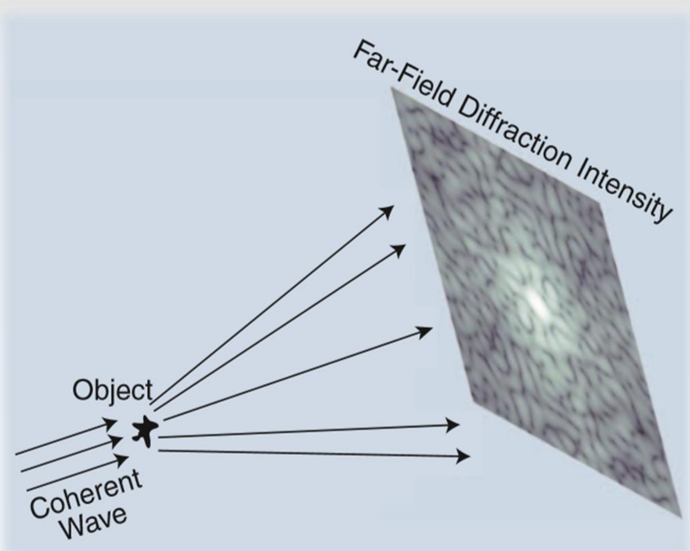
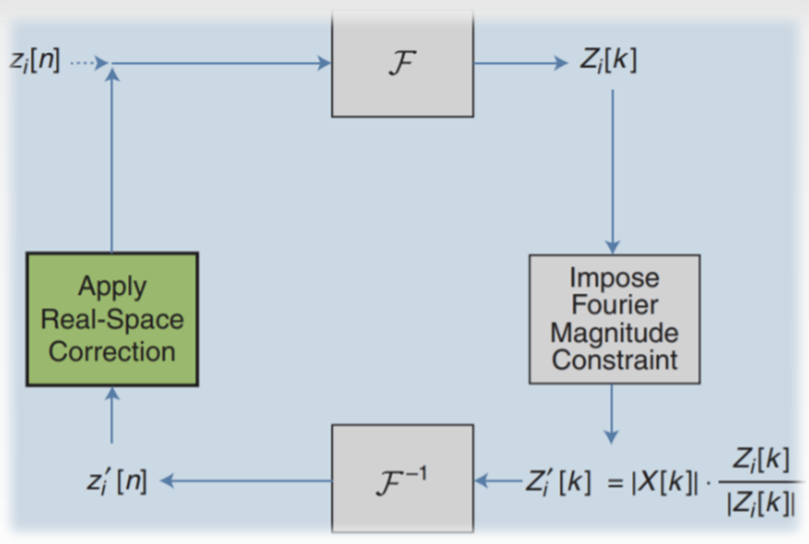


Image from [2]

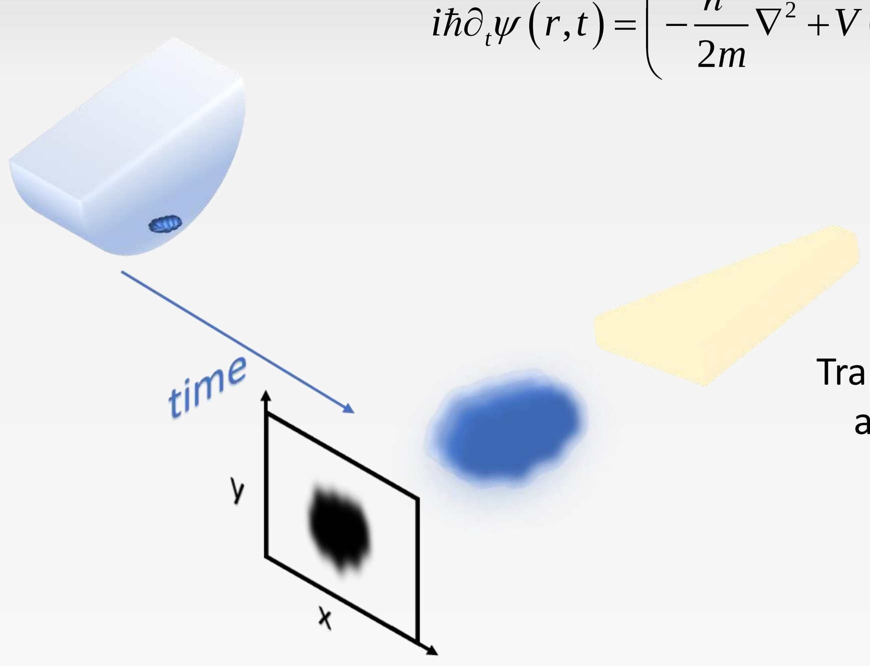


Fienup's algorithm[3]

Phase Retrieval - BEC

In BEC dynamics are no longer given by the Fourier transform. They are described by the Gross-Pitaevskii Equation(GPE):

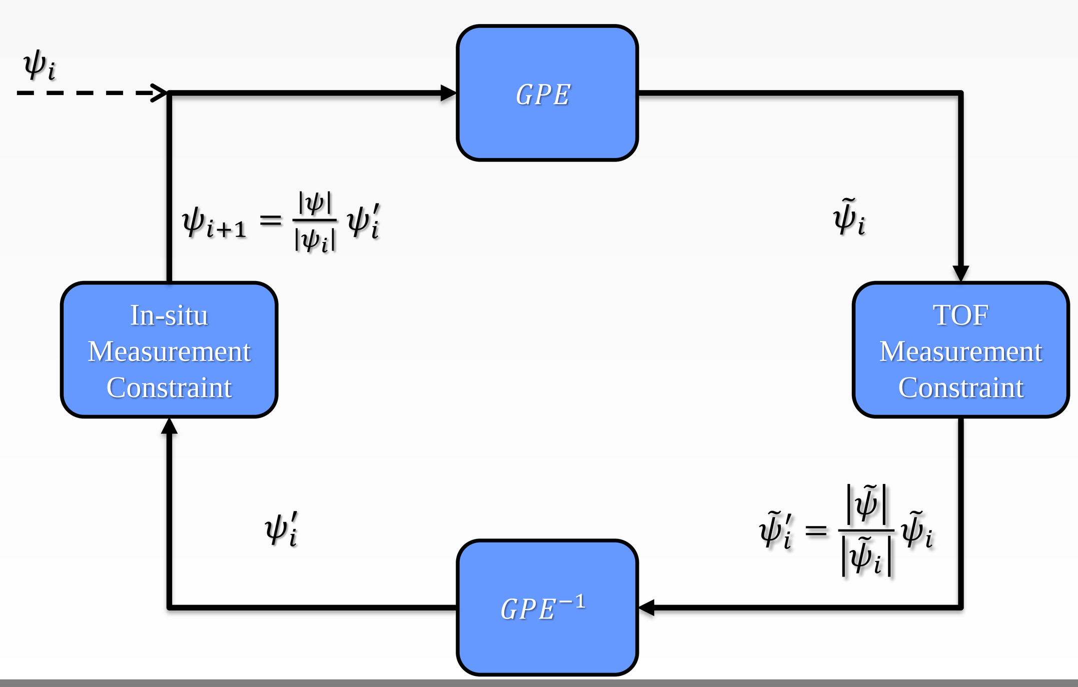
$$i\hbar\partial_t\psi(r,t)=\left(-\frac{\hbar^2}{2m}\nabla^2+V(r)+U_0|\psi(r,t)|^2\right)\psi(r,t)$$



$$i\hbar\partial_t\psi(r,t)=\left(-\frac{\hbar^2}{2m}\nabla^2+V(r)+U_0|\psi(r,t)|^2\right)\psi(r,t)$$

Trap is opened, cloud expands and measured after some propagation time

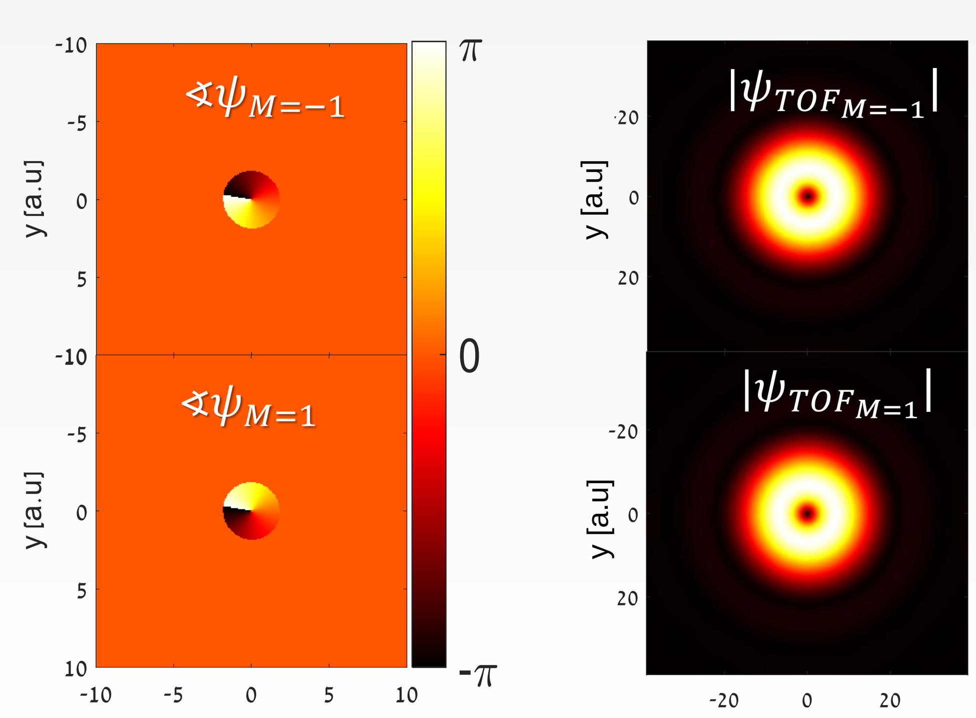
$$i\hbar\partial_t\psi(r,t)=\left(-\frac{\hbar^2}{2m}\nabla^2+U_0|\psi(r,t)|^2\right)\psi(r,t)$$



Vortices

Vortices imprinted on BEC's quantum wavefunctions are extremely interesting[4,5]: they inherently possess a quantum number (the topological charge). They are sometimes naturally occurring when they indicate phase-transitions (e.g., the Kosterlitz-Thouless phase transition[6]; Physics Nobel 2016), and they carry quantum information.

The concept of transferring the phase retrieval problem to nonlinear propagation has been proposed for optics [7] and also for BEC TOF measurements [8]. However, all methods proposed thus far, with either linear or nonlinear evolution, are not suitable for reconstructing vortex states since the topological charge is conserved in propagation; therefore the charge will manifest in the phase of the "far-field".



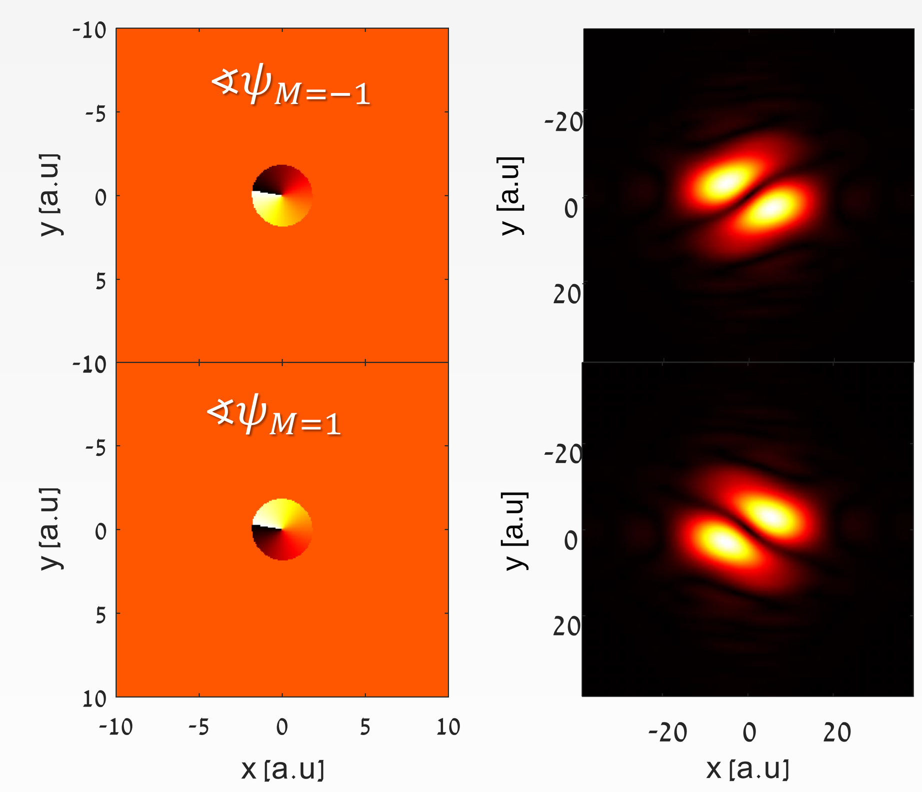
Asymmetric propagation – Lift the ambiguity!

$$i\hbar\partial_t\psi(x,y,t)=\left(-\frac{\hbar^2}{2m}\nabla^2+V(x,y)+U_0|\psi(x,y,t)|^2\right)\psi(x,y,t)$$

$$i\hbar\partial_t\psi(x,y,t)=\left(-\frac{\hbar^2}{2m}\nabla^2+V(x)+U_0|\psi(x,y,t)|^2\right)\psi(x,y,t)$$

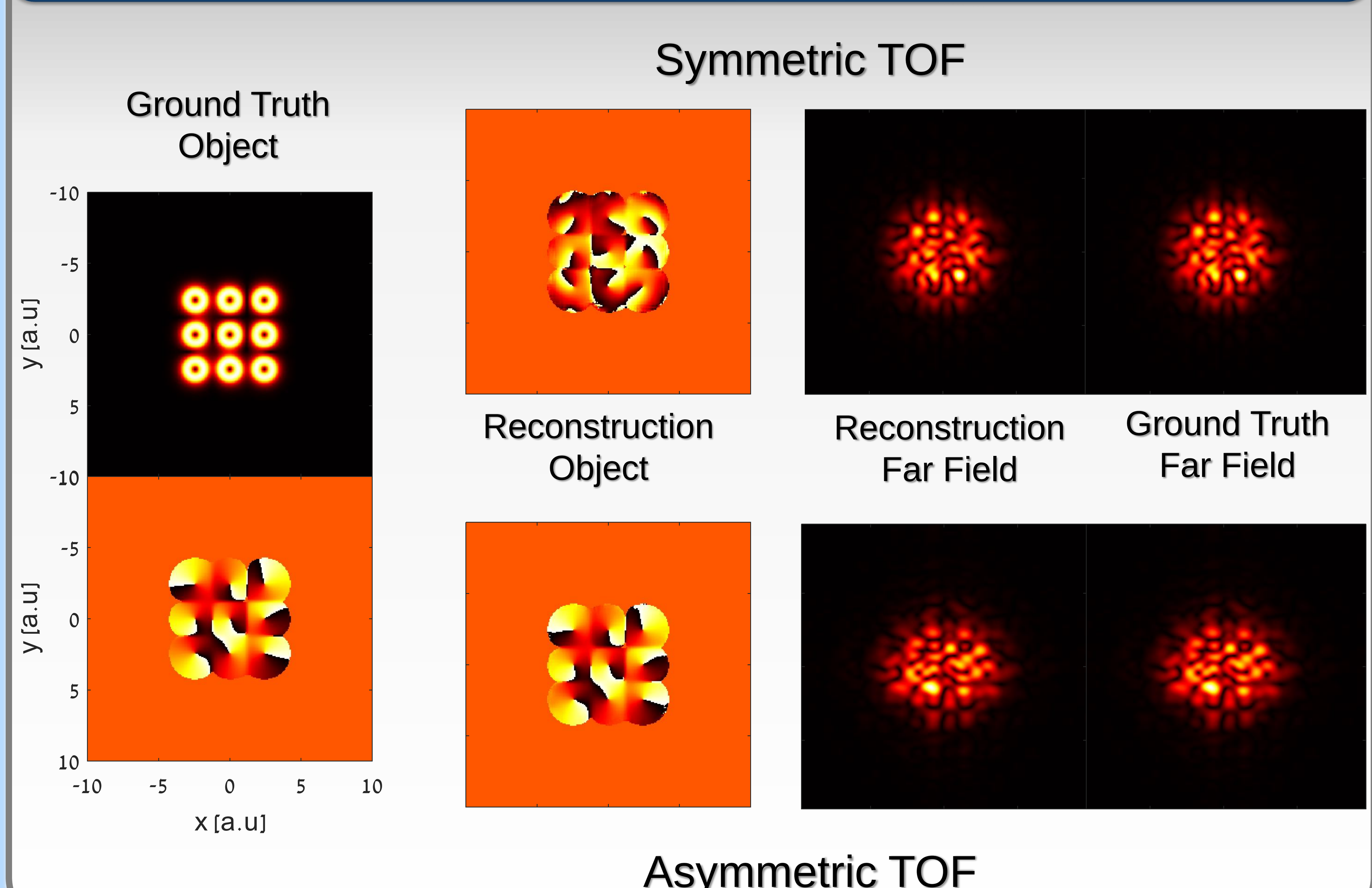
Break symmetry | Expansion in one axis

$$i\hbar\partial_t\psi(x,y,t)=\left(-\frac{\hbar^2}{2m}\nabla^2+U_0|\psi(x,y,t)|^2\right)\psi(x,y,t)$$



$$\max_{T_1}\left\|\left|\psi_1(x,y,T_1+T_2)\right|^2-\left|\psi_{-1}(x,y,T_1+T_2)\right|^2\right\|_2^2\approx\max_{T_1}\frac{1}{\sqrt{T_2(T_1+T_2)}}\left|\sin\left(\frac{\Delta E T_1}{\hbar}\right)\right|$$

Lattice reconstruction



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