Duality of Networks - GATE Study Material in PDF

In these free **GATE preparation notes**, we will look at the **Concept of Duality of Networks**. The dual of a relationship is formed by interchanging voltage and current in an expression. The dual expression thus produced is of the same form, and the reason that the dual is always a valid statement can be traced to the duality of electricity and magnetism.

Before you start with this module, you are advised to go through previous **GATE notes**. This GATE study material on Duality of Networks is useful for GATE EE, GATE EC, IES, BARC, BSNL, DRDO and other exams. These GATE Study Notes may be downloaded as PDF to help ensure that your exam preparation is made easy.

Recommended Reading –

Basic Network Theory Concepts

Source Transformation & Reciprocity Theorem

Kirchhoff's Laws, Node and Mesh Analysis (for EE)

KCL and KVL in Electrical Networks (for EC)

Nodal and Mesh Analysis (for EC)

Voltage Division in Series Circuit (for EE)

Voltage and Current Division, Star to Delta Conversion (for EC)

Thevenin Norton Equivalencies (for EE)

Thevenin, Norton and Tellegen Theorems (for EC)

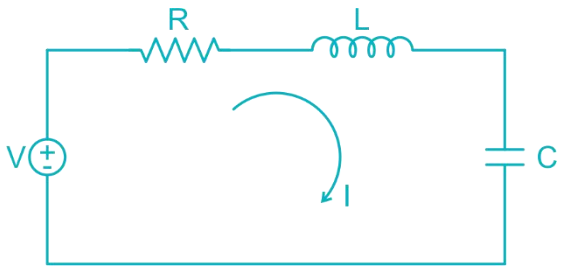
Superposition Theorem

Maximum Power Transfer Theorem

Duality of Networks

Identical behaviour patterns observed between voltages and currents in two circuits, illustrate the principal of duality.

Series RLC Circuit

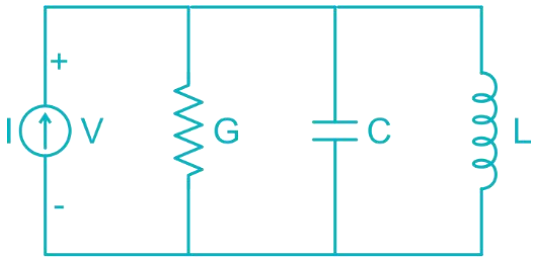


Mesh equation -

$$-V + IR + L\frac{dI}{dt} + \frac{1}{C}\int I\,dt = 0$$

$$V = IR + L\frac{dI}{dt} + \frac{1}{C}\int I\,dt \rightarrow (1)$$

Parallel GCL Circuit



Node equation

$$-I + VG + C\frac{dV}{dt} + \frac{1}{L}\int Vdt = 0$$

$$I = VG + C\frac{dV}{dt} + \frac{1}{L}\int Vdt \rightarrow (2)$$

The role of current and voltage in the two networks have been interchanged. The solution of one equation is also the solution of the other. This similarity is termed the principle of duality.

An inspection of the terms of equations (1) and (2) shows that the following are the analogues quantities.

- RI and GV
- $\frac{1}{c}\int Idt$ and $\frac{1}{L}\int Vdt$
- $L\frac{dI}{dt}$ and $C\frac{dV}{dt}$

By looking into above two equations we can say that the resistance, inductances and capacitances in the original network appear in dual network as conductance, capacitances and inductances respectively with the same numerical values.

Evidently, the following pairs are dual quantities.

- R ↔ G	- Mesh ↔ Node
- L ↔ C	- Open circuit ↔ Short circuit
- Z ↔ Y	- Switch open ↔ Switch close
- Y ↔ Δ	- Link ↔ Twig
- π ↔ T	- Voltage Source ↔ Current Source
- $\int Idt \leftrightarrow \int Vdt$	

The following procedure may be followed in constructing the dual of a given network.

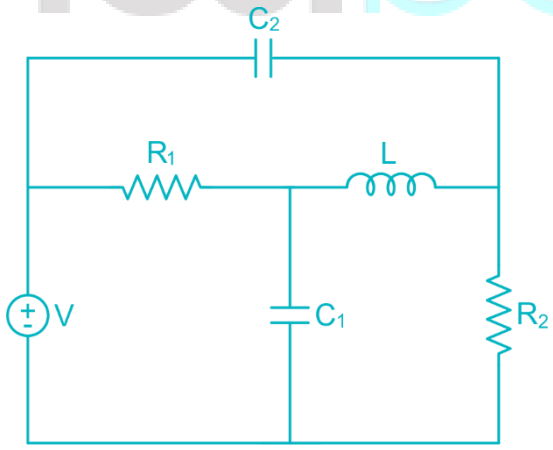
1. Inside each loop place a node and give it a number for all loops for convenience, place an extra node, the reference node, external to the network say 'O' node. Place an extra the same numbered nodes on a separate space on the paper for the construction of the dual.

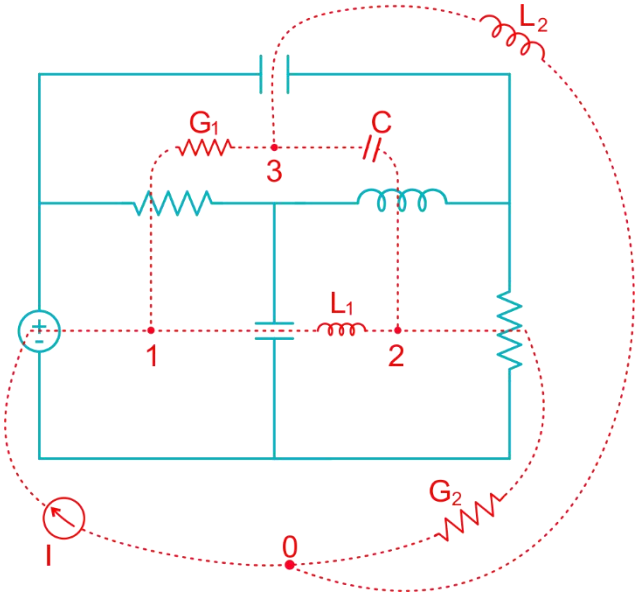
2. Draw a line between two nodes of the original network traversing only one element at a time. Thus draw Lines from node to node through the elements in the original network. For each element traversed in the original network, connect the dual elements from the Listing just given on dual network being constructed.

3. Continue this process unit the number of possible paths through single elements is exhausted. If we go through a connecting leads assumed to be a short, the dual elements is an open circuit.

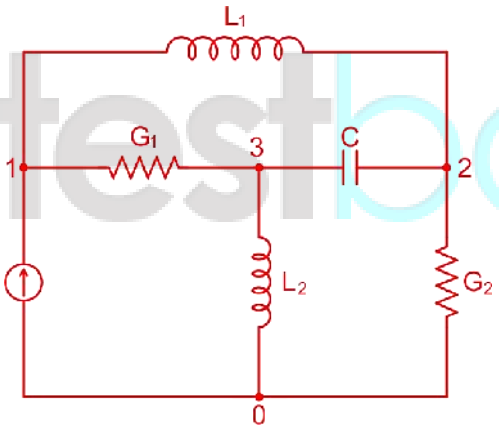
4. The network so constructed is a dual network.

Example 1:





The dual network is



The voltage source in the original network are replace by the current sources in the dual network and vice – versa.

The most important task is to decide the orientation of these sources in the dual network, it is determined as follows.

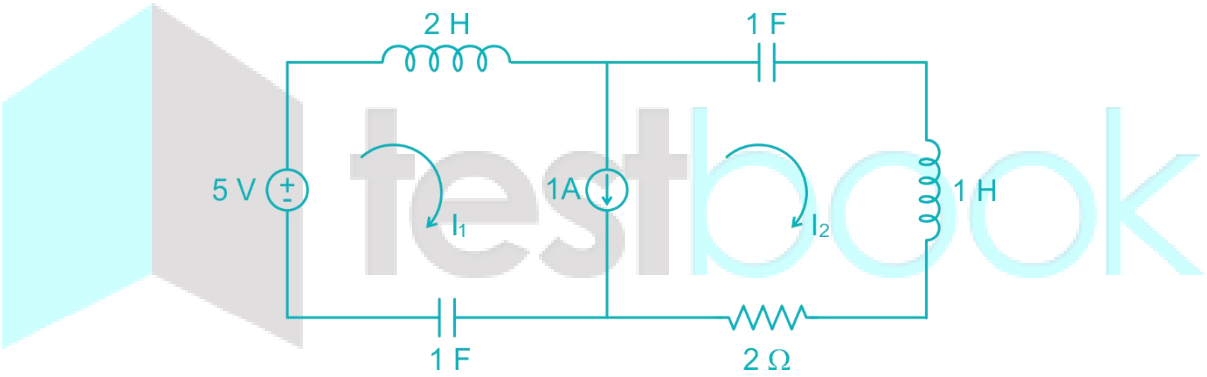
1. First of all, we assume a clock wise direction for all mesh currents in original network N_1. Each mesh current enclosing a dot (node) of the dual network say N_2.

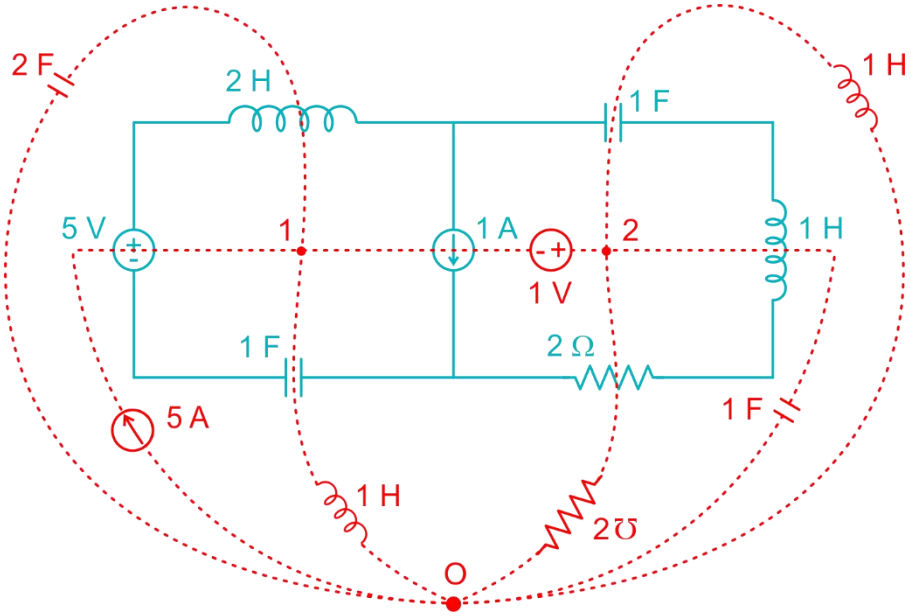
2. It is important to note that the orientation of the voltage source is identified with the direction of the mesh current whereas the orientation of the current source is identified with the node which is enclosed by mesh current.

3. If the direction of the voltage source and clockwise mesh current is same in the network N_1, then the orientation of the current source in the dual network N_2 is towards the node which is enclosed by the said mesh current in N_1.

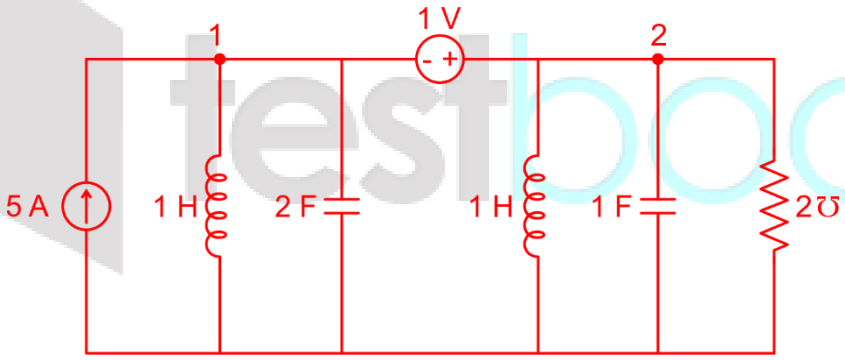
4. Conversely if the directions of the voltage source and the clock wise mesh current are opposite then the current of the current source in N_2 is directed away from the node which is enclosed by the said mesh current in N_1.

Example 2:





The dual network is



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