Parallel Resonance - GATE Study Material in PDF

In the previous articles we have seen the behavior of sinusoidal circuits in steady state, in AC & DC Circuits as well as power relations in AC Circuits. We also learnt more about Series Resonance. In these **free** GATE 2018 **Notes** we will mainly discuss **Parallel Resonance in Circuits**. This primarily means we will discuss resonance in parallel circuits.

These GATE Study Notes are useful for GATE EC, GATE EE, IES, BARC, DRDO, BSNL and other exams. You can also have these GATE Notes **downloaded in PDF** to have your preparation made easy and ace your GATE Exam.

Before you read further though, make sure you have gone through the previous articles whose concepts you will require.

Recommended Reading –

Parameters of Periodic Wave Forms

Sinusoidal Response of Parallel Circuits

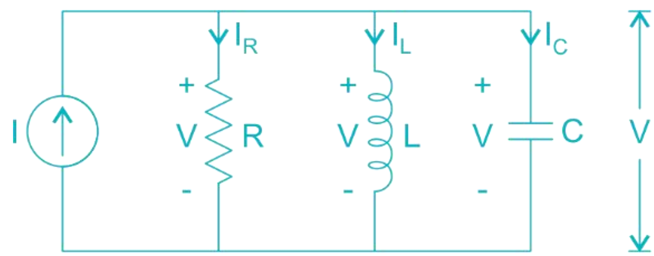
Power Relations in AC Circuits

What is Parallel Resonance?

Resonance in electric circuits is because of the presence of energy storing elements called capacitor and inductor.

At a fixed frequency f_0, the elements L and C will exchange their energy freely as a function of time which results in sinusoidal oscillations either across inductor (or) capacitor.

Consider a parallel RLC circuit at resonance.



$Y = Y_R + Y_L + Y_C$

$$= \frac{1}{R} + \frac{1}{j\omega L} + j\omega C = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

Current I = Y.V

$$\text{At } \omega = \omega_0 \Rightarrow \omega_0 C = \frac{1}{\omega_0 L} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\therefore Y(j\omega_0) = \frac{1}{R}$$

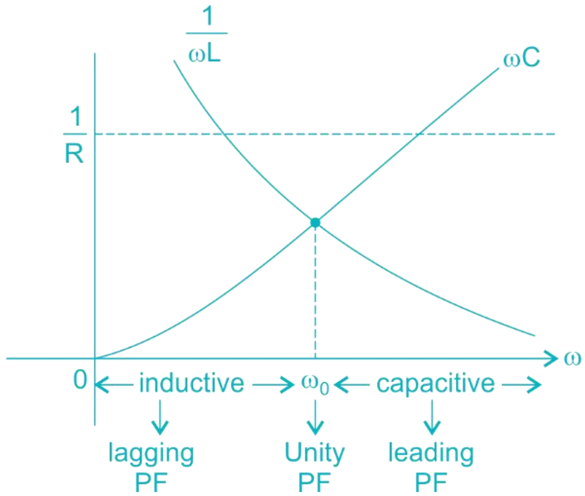
$$\text{At } \omega = \omega_0, \quad I_R = \frac{V}{R} = \frac{IR}{R} = I$$

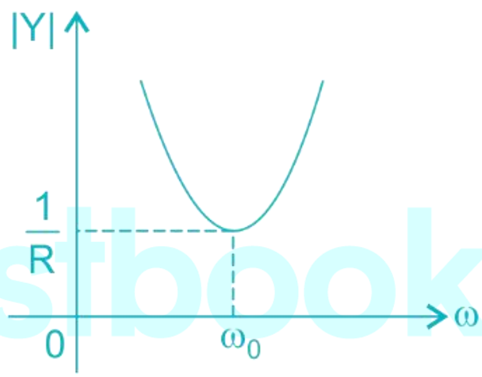
$$I_L = \frac{V}{j\omega L} = \frac{IR}{j\omega_0 L} = Q.I\angle -90° \text{ where } Q = \frac{R}{\omega_0 L}$$

$I_C = V.j\omega C = I.R.j\omega C = Q.I\angle 90°$ where $Q=\omega_0 CR$

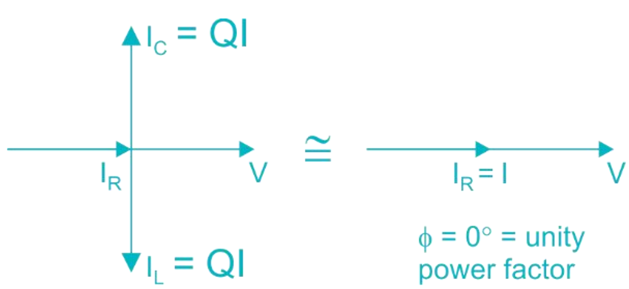
$$\therefore Q = \frac{R}{\omega_0 L} = \omega_0 CR = R\sqrt{\frac{C}{L}}$$

The behavior of parallel RLC circuit is given by,





Phasor Diagram:



Note:

i. The LC combination in a parallel RLC circuit acts like an open circuit at resonance.

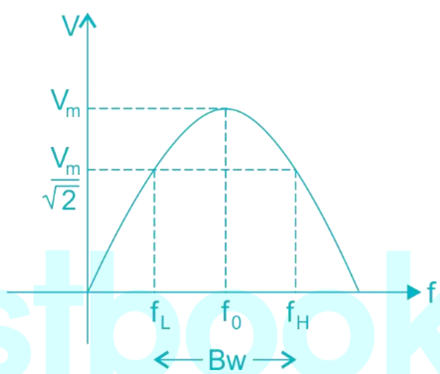
ii. At resonance the AC circuit behaves like DC circuit.

iii. Generally frequency response of parallel RLC circuit is also similar to Band pass filter response.

Frequency Response:

Voltage, $V = \frac{I}{Y}$

$$|V| = \frac{I}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$



Bandwidth, $BW = f_H - f_L = \frac{f_0}{Q}$

where, $Q = \frac{R}{\omega_0 L} = \omega_0 CR = R\sqrt{\frac{C}{L}}$

Also, $\sqrt{f_H f_L} = f_0$

Note:

i. At resonance, $Y \rightarrow$ minimum $\Rightarrow$ Z is maximum $\Rightarrow$ I is minimum. Hence it is called rejecter circuit.

ii. Since current through L and C elements are magnified by Q times. Hence parallel RLC circuit at resonance is also called as current magnification circuit.

iii. $Q = \frac{R}{\omega_0 L} = \omega_0 CR = R\sqrt{\frac{C}{L}} = \omega_0 \times \frac{\text{maximum energy stored in L and C at resonance}}{\text{average power dissipated at resonance}}$

iv. For the physical existence of the circuit a maximum R is to be maintained (for maximum Q) and this resistance is known as critical resistance of the circuit, where damping ratio is 1.

$$\therefore \xi = \frac{1}{2Q} = 1$$

$$\therefore Q = \frac{1}{2} = R\sqrt{\frac{C}{L}}$$

$$\therefore R_{max} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

Example 1:

If all the components present in a parallel RLC circuit are doubled then the value of new Q-factor at resonance is

Solution:

For parallel RLC circuit,

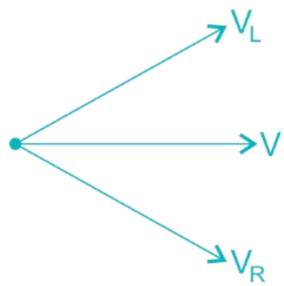
$$Q = R\sqrt{\frac{C}{L}}$$

$$\text{Now, } Q' = (2R)\sqrt{\frac{2C}{2L}}$$

$$Q' = 2.R\sqrt{\frac{C}{L}} = 2Q.$$

Example 2:

The Phasor diagram of a parallel RLC circuit at a frequency is shown below.



The operating frequency of circuit is

(a) f = 0

(b) $f < f_0$

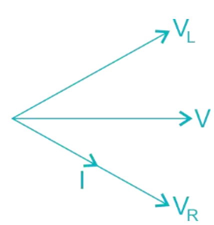
(c) $f > f_0$

(d) $f = f_0$

Solution:

Always we know that I and V_R are in phase.

Hence Phasor diagram becomes



From the figure current I lag Voltage V.

So circuit has inductive nature.

Since it is a parallel RLC Circuit, the given circuit operates at f < f_0.

In the next article we will learn all about Two Port Networks and Parameters for Standard Networks.

Did you like this article on Series Resonance? Let us know in the comments. You may also enjoy reading the following articles–

Two Port Networks

Sinusoidal Response of Series Circuits

Network Theory Revision Test 1

Magnetically Coupled Circuits

Control Systems Revision Test 1

Laplace Transforms Formula List

Control Systems Sensitivity

Conversion of Grey Code to Binary & Vice Versa

Mathematical Representation of Signals