Polar Plot and its Analysis - GATE Study Material in PDF

Previously, in our earlier GATE Study Material, we learned about the basic introduction and concepts in Frequency Response of the Control System Analysis. To analyze the properties and design the system based on frequency response, various kinds of plots are used. These are as follows

1. Polar Plot

2. Nyquist Plot

3. Bode Plot

In these free GATE Notes, we can learn about Polar Plots and its Analysis. The Polar Plot and its Analysis forms an important topic in Frequency Response Analysis of Control Systems. This GATE preparation notes are also useful for both streams GATE EE, and GATE EC. Use these notes to ace GATE 2017, BARC, BSNL, IES, DRDO.

Recommended Reading –

Frequency Response for Control System Analysis

Root Locus Diagram

Time Response of Second Order Systems

Routh Hurwitz Stability Criteria

Stability of Control Systems

Polar Plot and its Analysis

Earlier, we have seen the system having sinusoidal input with gain G(s) or G(jω).

This transfer function can be written as

$G(j\omega) = M\angle\phi$
Where $M = |G(J\omega)|, \quad \phi = \angle G(j\omega)$

Here both magnitude and phase are functions of frequency. Here ω is varied from 0 to ∞ and M & φ are plotted as a phasor.

The locus of tip of the phasor with frequency is called the polar plot. Hence the polar plot contains entire frequency response characteristics in a single plot. It can determine closed loop stability using open loop transfer function. But it cannot determine the number of roots causing instability to the system.

Polar plot is created using following steps

1. Put s = jω in the transfer function.

2. Get expression for $|G(j\omega)|$ and $\angle G(j\omega)$

3. For ω = 0 and ω = ∞, calculate both $|G(j\omega)|$ and $\angle G(j\omega)$

Taking an example for open loop transfer function, $G(s) = \frac{k}{1+sT}$

Putting $= j\omega, G(j\omega) = \frac{k}{1+j\omega T}$

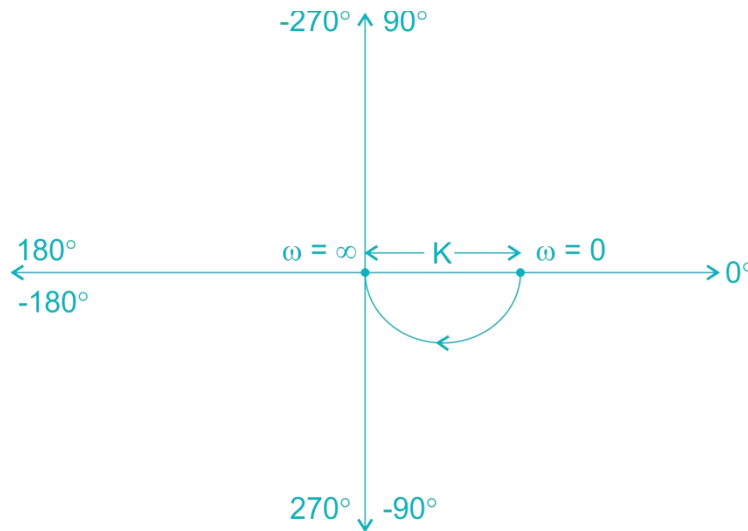$|G(j\omega)| = \frac{k}{\sqrt{1+\omega^2 T^2}}, \angle G(j\omega) = -\tan^{-1}(\omega T)$

At $\omega = 0, |G(j\omega)| = k, \angle G(J\omega) = 0°$

$\omega = \frac{1}{T}, |G(j\omega)| = \frac{k}{\sqrt{2}}, \angle G(j\omega) = -45°$

$\omega = \infty, |G(j\omega)| = 0, \angle G(j\omega) = -90°$

Now, while creating polar plots, angles in anti-clockwise direction are considered positive and angles in clockwise direction are considered negative. Polar plot for previously mentioned transfer function will be given as

The direction of polar plot will be from $\omega = 0$ to $\omega = \infty$. Between these points, there may be points on real and/or imaginary axis which polar plot cuts. There are various methods to find out these points.

1) For the polar plot to cut real axis, put imaginary part of $G(j\omega)$ equal to zero and find a frequency ω such that $0 < \omega < \infty$

2) For a polar plot to cut imaginary axis, put real part of $G(j\omega)$ equal to zero and find a frequency ω such that $0 < \omega < \infty$

3) For the polar plot to cut negative real axis put $\angle G(j\omega)= -180°$ and solve for frequency such that $0 < \omega < \infty$

4) For the polar plot to cut $-90°$ axis put $\angle G(j\omega) = -90°$ and solve for ω such that $0 < \omega < \infty$

The points (3) & (4) are applicable to only "all pole transfer functions"

We will take another example. Here suppose

$$G(s) = \frac{K}{(1+sT_1)(1+sT_2)}$$

Putting s = jω, we get

$$G(j\omega) = \frac{K}{(1+j\omega T_1)(1+j\omega T_2)}$$

$$|G(j\omega)| = \frac{K}{\sqrt{1+\omega^2 T_1^2}\sqrt{1+\omega^2 T_2^2}}$$

$$\angle G(j\omega) = -\tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

For $\omega = 0, |G(j\omega)| = K, \angle G(j\omega) = 0°$

For $\omega = \infty, |G(j\omega)| = 0, \angle G(j\omega) = -180°$

To find out the points on which the plot cuts real and/or imaginary axis, we derive

$$G(j\omega) = \frac{K}{(1+j\omega T_1)(1+j\omega T_2)} \frac{(1-j\omega T_1)(1-j\omega T_2)}{(1-j\omega T_1)(1-j\omega T_2)}$$

$$= \frac{K(1-j\omega T_1-j\omega T_2-\omega^2(T_1 T_2))}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}$$

$$= \frac{K(1-\omega^2 T_1 T_2)}{(1+\omega^2 T_1^2)(1+\omega^2)T_2^2} + j\frac{(-K\omega(T_1+T_2))}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}$$

If we put real part to zero, then

$$1 - \omega^2 T_1 T_2 = 0$$

$$\omega = \frac{1}{\sqrt{T_1 T_2}}$$

Putting this value in G(jω), we get negative imaginary value, hence the plot is cutting on negative imaginary axis.
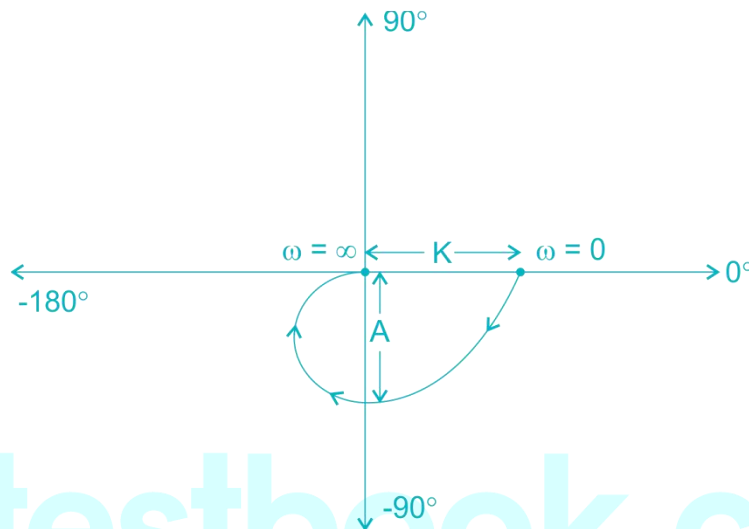
Putting imaginary part to zero, we get = 0 ; hence the plot does not cuts real axis.

Value at which plot cuts negative imaginary axis,

$$A = |G(j\omega)|_{\omega=\frac{1}{\sqrt{T_1 T_2}}} = \frac{K\sqrt{T_1 T_2}}{T_1+T_2}$$ (derive it yourself)

The system in question will have polar plot as,

Stability using Polar Plot

The polar plots use open loop transfer function, hence the reference point for determining stability is shifted to (−1, 0)

For closed loop, q(s) = 1 + G(s) H(s) → (0, 0)

For polar plot, G(s) H(s) → (−1, 0)

The stability using polar plot is determined as follows

1) If (−1,0) is left of the polar plot or (−1,0) is not enclosed by it then closed loop system is stable. Here closed loop system is corresponding to the open loop transfer function for which the polar plot is sketched.

2) If (−1,0) is on right side of the polar plot or (−1,0) is enclosed by polar plot then closed loop system is unstable.

3) If (−1,0) is on the polar plot then the closed loop system is marginally stable.

Now, to determine the left or right side of polar plot arrow direction is used given as follows

This concludes the description and use of the polar plot. We will continue with other plots in further articles.

Shortcut to Draw Polar Plot for All Pole Systems

Nyquist Plot and Stability Criteria

Relative Stability and Bode Plot