Relative Stability and Bode Plot - GATE Study Material in PDF

Earlier, in our previous GATE Preparation Notes, we discussed Nyquist plot and its usage in determining the stability of the system. Apart from absolute stability, Nyquist Criteria can also be used for finding relative stability. Relative stability is calculated in terms of gain margin and phase margin. It is always carried out between stable systems. In these free GATE notes, we discuss Relative Stability and Bode Plot.

The concepts of Relative Stability and Bode Plot form an important part of Frequency Response Analysis chapter of Control Systems. These GATE study notes will be helpful whether you are preparing for GATE EE or GATE EC. Also useful for other exams like BARC, BSNL, DRDO, IES etc.

Before you begin with these notes though, it is advised that you go through the recommended reading material listed below.

Recommended Reading –

Nyquist Plot & Its Stability Criteria

Polar Plot and its Analysis

Frequency Response for Control System Analysis

Root Locus Diagram

Time Response of Second Order Systems

Routh Hurwitz Stability Criteria

Stability of Control Systems

Relative Stability and Bode Plot

In polar plot, we saw that the absolute stability can be determined by non-encirclement of (1+j0) by the plot. As the polar plot gets close to this point the system tends toward instability.

Gain Margin

It is the factor by which the system gain can be increased before the system reaches to the verge of instability.

Gain margin is always calculated at the frequency at which the phase of the system is -180, also known as phase cross over frequency.

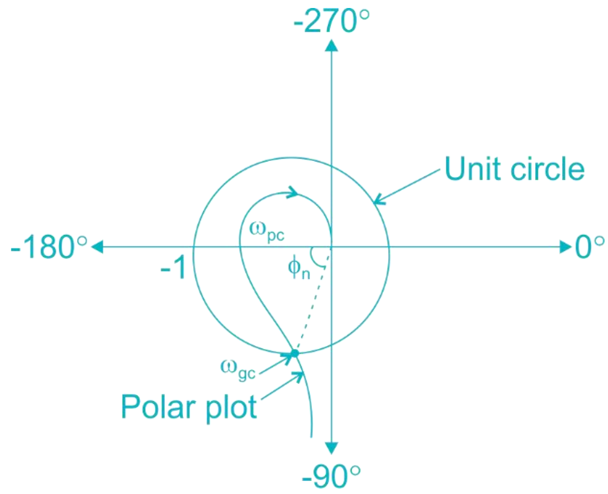
Phase Margin

It is the additional amount of phase lag which can be added to the system before the system reaches to the verge of instability.

Phase margin is always calculated at the frequency at which the gain of the system is 1 or 0 dB, also known as gain cross over frequency.

Relative Stability in Polar Plot

Suppose for a given system, polar plot is as follows



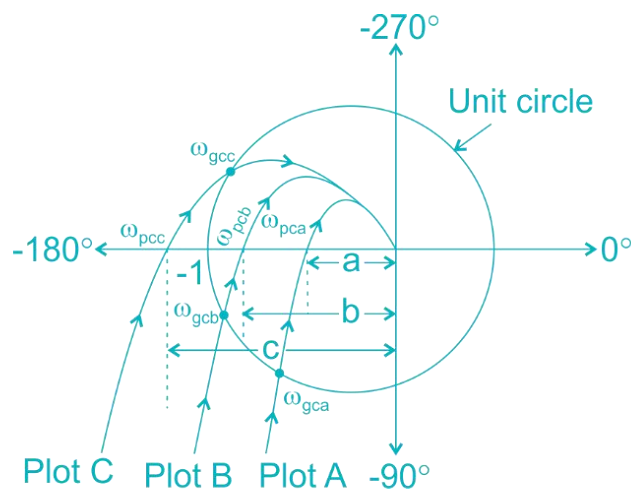
ω_{pc} $\rightarrow$ Phase cross over frequency

ω_{gc} $\rightarrow$ Gain cross over frequency

Gain margin is calculated by, $GM = \frac{1}{|GH(j\omega)|_{\omega=\omega_{pc}}}$

Similarly, phase margin $= 180° + \angle GH(j\omega)_{\omega=\omega_{gc}}$

Now, if we look for stability using gain margin; consider different types of polar plots



In the above figure three are polar plots of three systems A, B and C having gains at their respective phase cross over frequencies being a, b and c respectively, As seen, a, b < 1 & c > 1

Also $\frac{1}{a} > \frac{1}{b} > \frac{1}{c}$

Or $GM_A > GM_B > GM_c$.

As seen in the polar plots, A and B does not encircle (-1,0) while C does. Hence system A and B are stable while system C is unstable

Also $GM_A, GM_B > 1, GM_c < 1$

∴, if calculated in dB $GM_A GM_B > 0$ while $GM_C < 0$

Thus, for stable system, gain margin is greater than 1 or positive in dB and for an unstable system, gain margin is less than 1 or negative in dB

Now, in terms of phase margin

For system A & B, $\angle GH(j\omega)_{\omega=\omega_{gc}} > -180°$

While for system C, $\angle GH(j\omega)_{\omega=\omega_{gc}} < -180°$

∴ For A & B phase margin is positive while for C phase margin is negative,

as phase margin $= 180° + \angle GH(j\omega)_{\omega=\omega_{gc}}$

Also, we observed that

For system A and B, $\omega_{pc} > \omega_{gc}$ which are stable systems

While for system C, $\omega_{gc} > \omega_{pc}$ which is an unstable system.

We have also seen, $GM_A > GM_B$ and $PM_A > PM_B$

Hence system A is more stable than system B.

∴ GM, PM α stability.

Bode Plot

It is a graphical representation from frequency response to assess the stability of a transfer function. It also determines the stability through open loop transfer function.

Bode plot consists of two plots. These are as follows:

1. Magnitude plot: It is plotted as $20\log_{10}|GH(j\omega)|$ v/s $\log\omega$

2. Phase plot: It is plotted as ϕ v/s $\log_{10}\omega$.

These plots can also be drawn v/s ω if x – axis is on decade scale instead of linear.

Now, suppose an open loop transfer function a system is given as

$$G(s)H(s) = \frac{K(1+sT_1)(1+sT_2).(1+sT_3)--------}{s^r(1+sP_1)(1+sP_2)(1+sP_3)--------}$$

$$GH(j\omega) = \frac{K(1+j\omega T_1)(1+j\omega T_2).---------}{(j\omega)^r(1+j\omega P_1)(1+j\omega P_2)-------}$$

$$|GH(j\omega)| = \frac{K\sqrt{1+\omega^2T_1^2}\sqrt{1+\omega^2T_2^2}-----}{\omega^r\sqrt{1+\omega^2P_1^2}\sqrt{1+\omega^2P_2^2}-----}$$

In dB form, it is calculated as

$$20\log_{10}|GH(j\omega)| = 20\log_{10}K + 20\log_{10}\sqrt{1+\omega^2T_1^2} + 20\log_{10}\sqrt{1+\omega^2T_2^2} + \dots\dots\dots. - \left[20\times r\log_{10}\omega + 20\log_{10}\sqrt{1+\omega^2P_1^2} + 20\log_{10}\sqrt{1+\omega^2P_2^2} + ---\right]$$

The phase can be calculated as

$$\angle GH(j\omega) = \tan^{-1}(\omega T_1) + \tan^{-1}(\omega T_2) + \cdots\dots\dots\dots. - r\times 90^\circ - [\tan^{-1}(\omega P_1) + \tan^{-1}(\omega P_2) + \cdots\dots\dots]$$

For constant K, phase angle is zero.

Each point of pole and zero acts as break point for change of slope.

At every zero, change in slope is + 20 dB/decade □r' where r' is the order of that zero.

At every pole, change in slope is -20 dB/decade □r" where r" is the order of that pole.

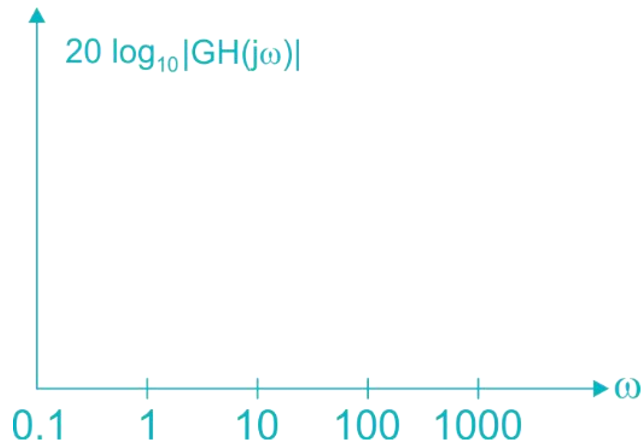
Starting point of Bode plot on Y axis of magnitude plot is equal to $20\log_{10}K$ dB.

The starting slope will depend on the type of the system. For a term $(j\omega)^r$ in the denominator, starting slope will be -20dB/dec □r; while in numerator starting slope will be +20dB/decade □r

Before drawing Bode plot, the transfer function should always be brought in the form of

$$G(s)H(s) = \frac{K\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)---}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)----}$$

Instead of using $\log_{10} \omega$ in X-axis, we can use ω if we divide X-axis as given



Now, we will understand the Bode plot and its usage through the following example.

Examples of Bode Plot

Construct the Bode plot of $\frac{10\left(1+\frac{s}{5}\right)}{s^2\left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)}$

Solution:

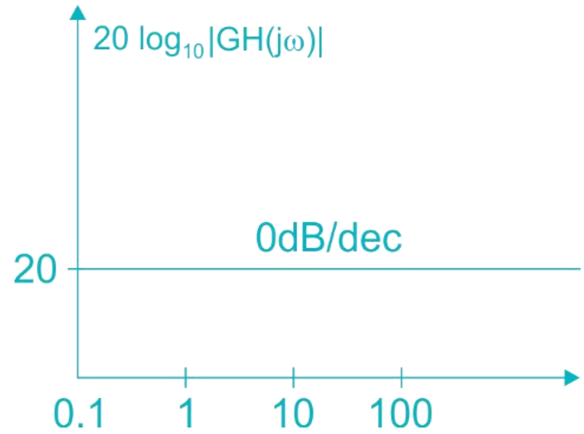
We have $G(s)H(s) = \frac{10\left(1+\frac{s}{5}\right)}{s^2\left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)}$

Before drawing the complete Bode magnitude plot, we will draw the magnitude plot for separate elements which are

$10, \frac{1}{s^2}, \left(1+\frac{s}{5}\right), \left(1+\frac{s}{10}\right)$ and $\left(1+\frac{s}{100}\right)$

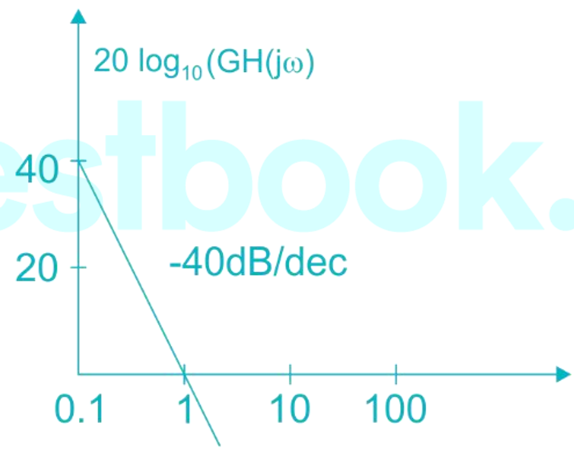
Magnitude plot for 10

$20\log_{10}(10) = 20\text{dB}.$

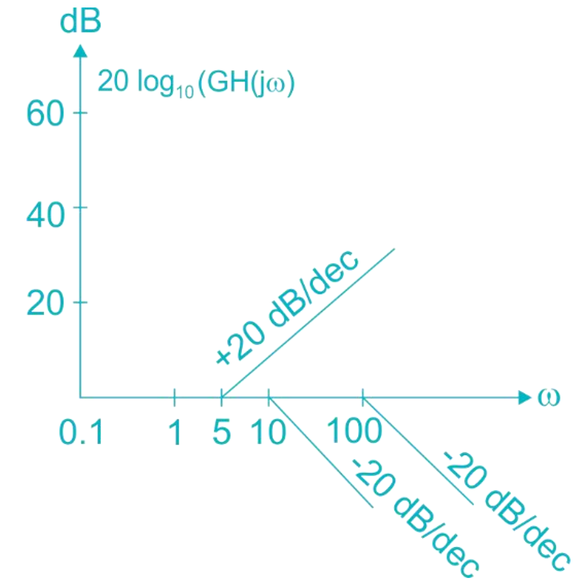


In magnitude plot for $\frac{1}{s^2}$, starting slope will be -40 dB/decade in such a manner that the magnitude will be 0 dB at ω = 1.

So the plot will be as follows

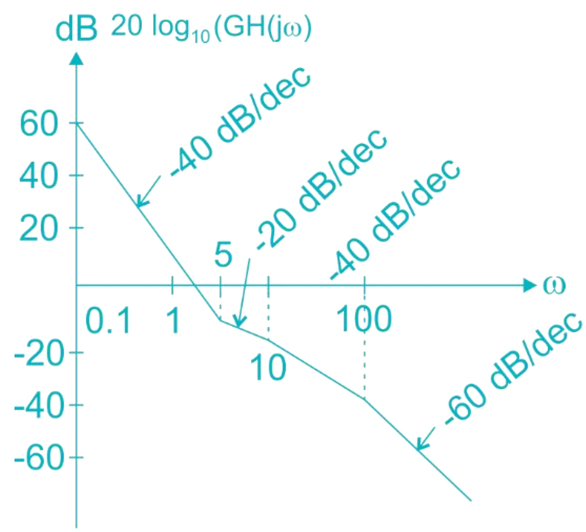


The magnitude plots for $\left(1+\frac{s}{5}\right), \left(1+\frac{s}{10}\right)$ and $\left(1+\frac{s}{100}\right)$ will be shown as (seperately)



Since order of all above elements are 1, hence the slope will be +20 dB/decade or -20 dB/decade.

s=5 is a zero, hence slope is +20 dB/decade for this one
For s=10, 100; slope is -20 dB/decade as these are poles.
Now, combining above plots, we will get the magnitude plot of the given transfer function as



At starting, slope = −40 dB/decade

At ω = 5, slope = −40 + 20 = −20 dB/decade.

At ω = 10, slope = −20 − 20 = −40 dB/decade.

At ω = 100, slope = −40 − 20 = −60 dB/decade.

For the phase plot, the phase at different frequencies will be calculated as

$$\angle GH(j\omega) = \tan^{-1}\left(\frac{\omega}{5}\right) - 2 \times 90° - \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{100}\right)$$

at ω = 0.1 ≈ 0

∠GH(jω) = −180□

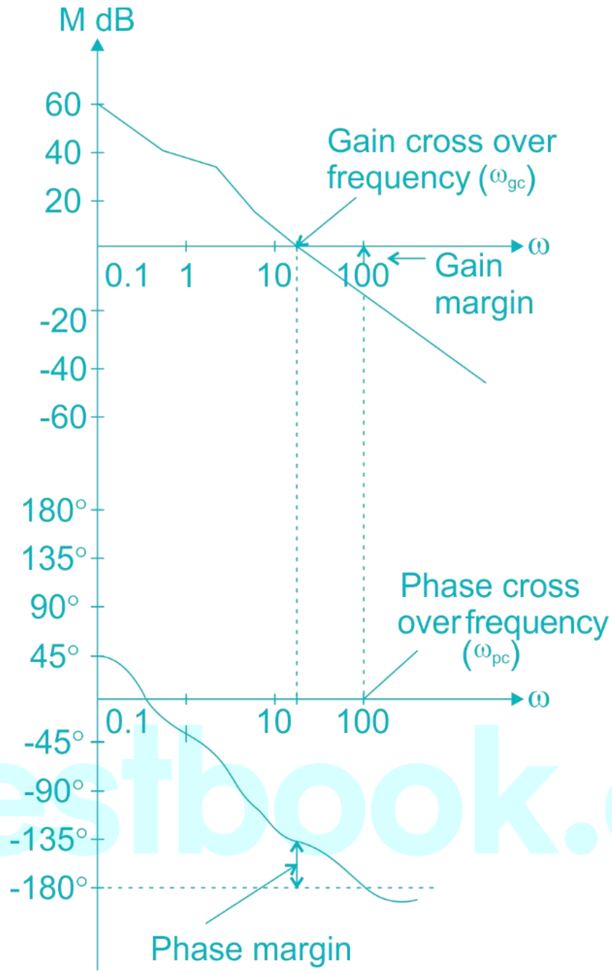
At ω = 5, ∠GH(jω) = −164.43□

At ω = 10, ∠GH(jω) = −167.20□

At ω = 100, ∠GH(jω) = −222.152□

Calculate the phase at some other values in between these frequencies and try to plot the phase plot yourself.

The concept of gain margin and phase margin can also be applied to Bode plot. The magnitude plot and phase plot for a system are shown as below



In the above system, $\omega_{pc} > \omega_{gc}$

Also GM > 1, PM > 0

Hence the above system is stable.

This conclues the frequency response analysis. Further, we will discuss controllers and compensators.

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[**Shortcut to Draw Polar Plot for All Pole Systems**]

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