Time Signals & Signal Transformation - GATE Study Material in PDF

In these **free** GATE 2018 **Notes**, we move to the subject of **Signals and Systems**. Specifically, we will try and understand the basic definitions of different types of signals as well as certain properties of signals. We will also learn about **Signal Transformation** such as **Time Shifting, Time Scaling, Time Inversion/Time Reversal**. These GATE Material on **Introduction to Time Signals & Signal Transformation** is important for GATE EC and GATE EE. You can have all this material **downloaded in PDF** so that your exam preparation is made easy and you can ace your paper.

Recommended Reading –

Types of Matrices

Properties of Matrices

Rank of a Matrix & Its Properties

Solution of a System of Linear Equations

Eigen Values & Eigen Vectors

Linear Algebra Revision Test 1

Laplace Transforms

Limits, Continuity & Differentiability

Mean Value Theorems

[Differentiation](#)

[Partial Differentiation](#)

[Maxima and Minima](#)

[Methods of Integration & Standard Integrals](#)

[Vector Calculus](#)

[Vector Integration](#)

What is a Signal

Anything which contains some information is known as a signal. A signal may be a function of one or more independent variables like time, pressure, distance, position, etc. For electrical purpose, signal can be current or voltage which is function of time as the independent variable.

Signals can be classified into two broad categories. These are

1. Continuous Time Signals

2. Discrete Time Signals

Continuous Time Signals

A continuous signal may be defined as a continuous function of independent variable. In case of continuous time signal, the independent variable is time. Signals are continuous function of time. They can also be termed as Analog Signals.

Discrete Time Signals

For discrete time signals, the independent variable is discrete. So, they are defined only at certain time instants. These signals have both discrete amplitude and discrete time. They are also known as Digital Signals.

Signal Energy and Power

Another important parameter for a signal is the signal energy and power.

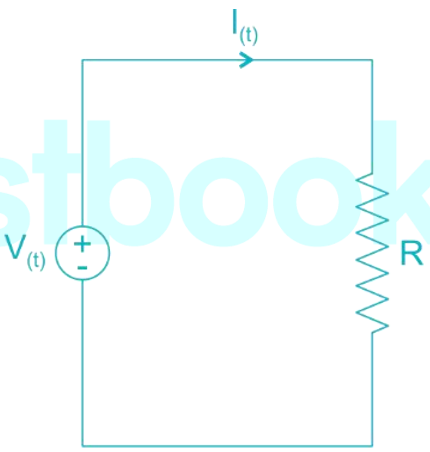
Energy of a signal $\triangleq \int_{-\infty}^{\infty} |x(t)|^2 dt$ for continuous time signals

$\triangleq \sum_{n=-\infty}^{\infty} |x[n]|^2$ for discrete time signals

Power of a signal $\triangleq \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$ for continuous time signals

$\triangleq \lim_{N\to\infty} \frac{1}{2N+1} \sum_{\eta=-N}^{N} |x[n]|^2$ for discrete time signals

If we take an example of an electrical circuit given as follows



Instantaneous power is $P(t) = V(t)I(t) = \frac{1}{R}V^2(t) = I^2(t).R$

Energy dissipated in this circuit, $E = \int_{-\infty}^{\infty} P(t)dt$

$= \frac{1}{R}\int_{-\infty}^{\infty} V^2(t)dt$

The total energy dissipated in the time interval $t_1 \le t \le t_2$ is

$\int_{t_1}^{t_2} P(t)dt = \frac{1}{R}\int_{t_1}^{t_2} V^2(t)dt$

Average power other this time interval, $P_{av} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \frac{1}{R} V^2(t)dt$

Signal Transformations through Variations of the Independent Variable

A signal can undergo several transformations some of which are:

1. Time Shifting
2. Time Scaling
3. Time Inversion / Time Reversal

Time Shifting

Time shifting is a very basic operation that you never stop to come across if you are handling a signals and systems problem. We seek to settle all doubts regarding it for one last time.

Consider that we are given a signal x(t) then how do you implement time shifting and scaling to obtain the signal $x(-\alpha t - \beta)$, $x(-\alpha t + \beta)$, $x(\alpha t - \beta)$ or $x(\alpha t + \beta)$, where α and β are both positive quantities. The first thing we would want to clear forever is that a negative time shift implies a right shift and a positive time shift implies a left shift.

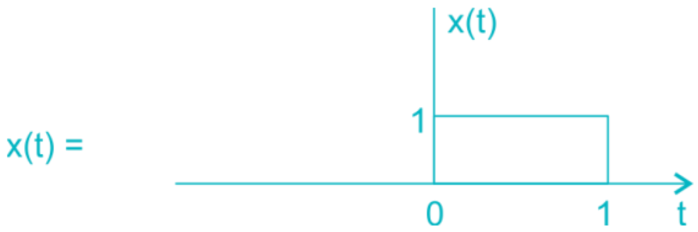
Remember it by the thinking of creating an arrow out of negative sign, - to → which implies right shift for negative time shift. The other (+ time shift) would obviously mean a left shift.

Returning to our original agenda, the next thing to know is that time shifting and scaling can be implemented, starting from both the left and right side. Since, we are discussing time shifting, we set $\alpha = 1$ which is responsible for scaling.

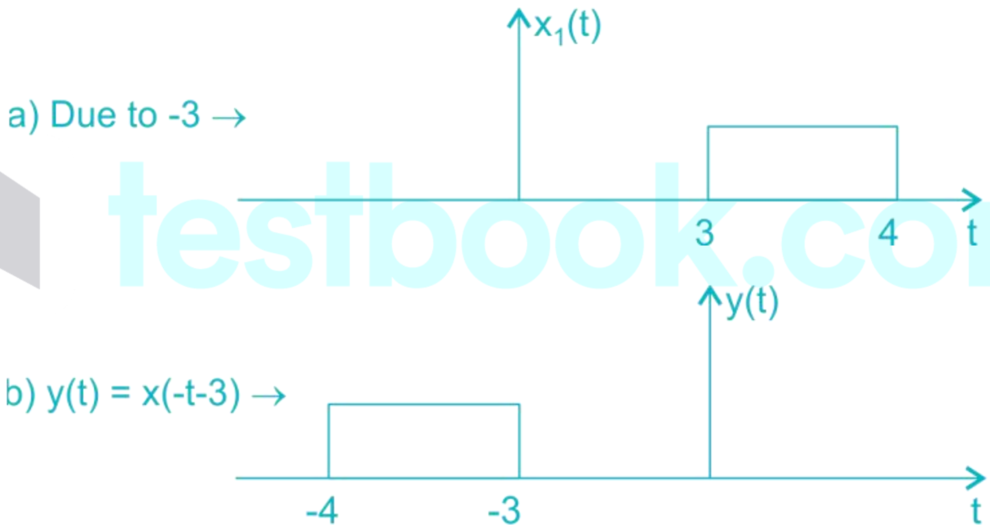
Time Shift - Working from the Right

This is general method which always works.

Let, x(t) = u(t) – u(t – 1)

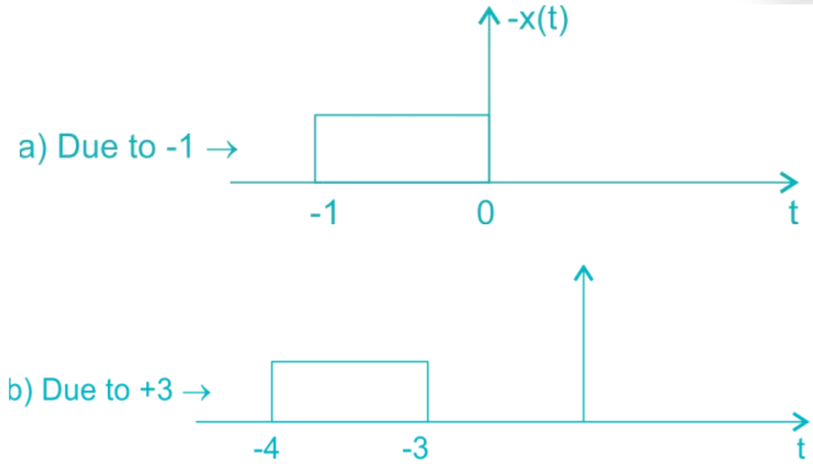


Then, to implement x(–t –3), working from the right, we first implement right shift by 3 (due to -3) and then do time reversal (due to -1 coefficient of t) .



Time Shift - Working from the Left

When we work from left side, first we take common everything that is coefficient of t. So, x(± αt ± β) becomes x[± α(t ± β/α)]. For x(-t-3), we get x[– (t+3)]. So, we first implement time inversion [due to -1 getting multiplied with (t + 3)] and then do left shift by 3 (due to +3).



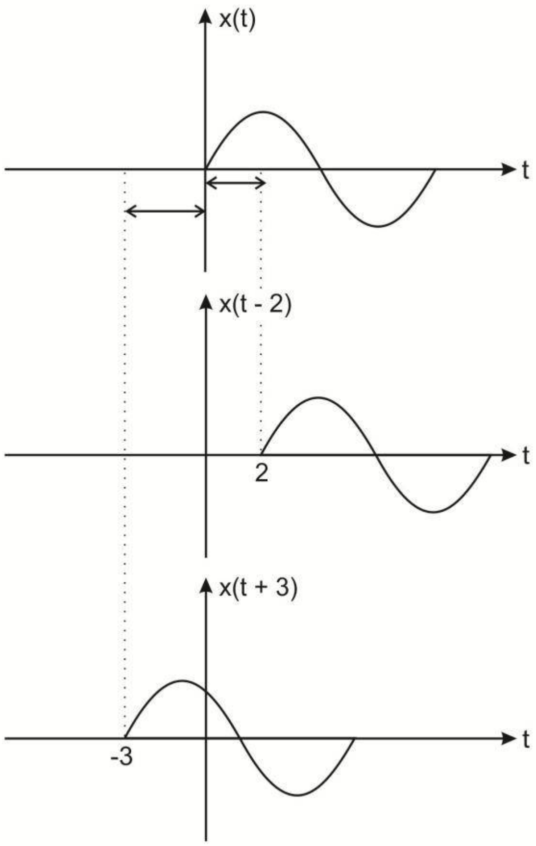
This method works always for continuous time signals but not always for discrete time signals. For discrete time signals, when taking common creates a fraction inside the bracket, the method fails. The last thing to verify that the final results from both approaches is same.

Origin Shifting

Suppose we want to shift the origin from (0, 0) and (0, a) (let a be positive), then it is a right shift for axis, or left shift for the signal relative to the axis. And hence x(t) change to x(t + a) contrary to what we intuitively expect it to be x(t – a) due to right shift for axis. Similarly, if we want to shift origin to (0, –a) then this is left shift for axis but a right shift for signal relative to axis. So, x(t) change to x(t – a). This lack of understanding causes us to believe that shifting signals and shifting axes are two different concepts and then remember two different sets of rule for them. Now, we see that they are just one thing and makes easies to remember.

Example 1:

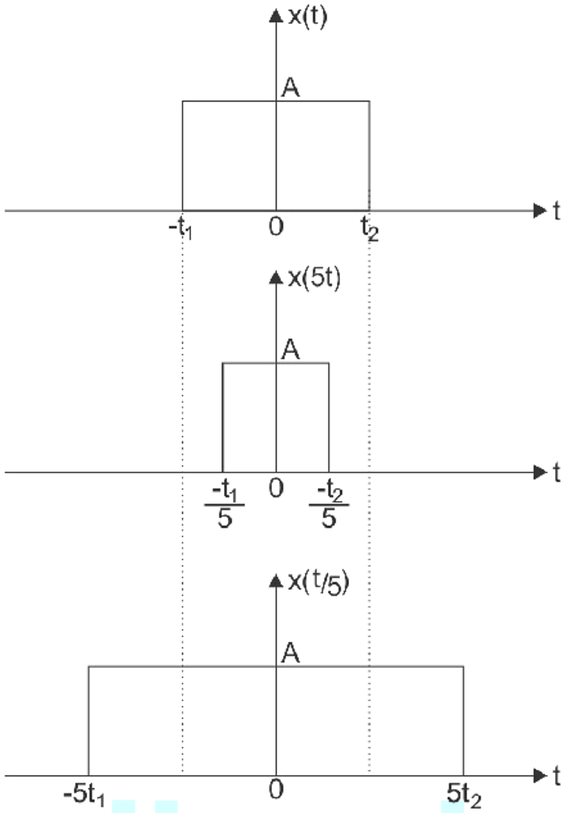
A continuous time signal x(t) is given in the figure. Plot the functions x(t – 2) and x(t + 3).



Time Scaling

Expansion or compression of a signal with respect to time is known as time scaling. Let x(t) be a continuous time signal, and then x(5t) will be the compressed version of x(t) by a factor of 5 in time. And x(t/5) will be the expanded version of x(t) by a factor of 5 in time.

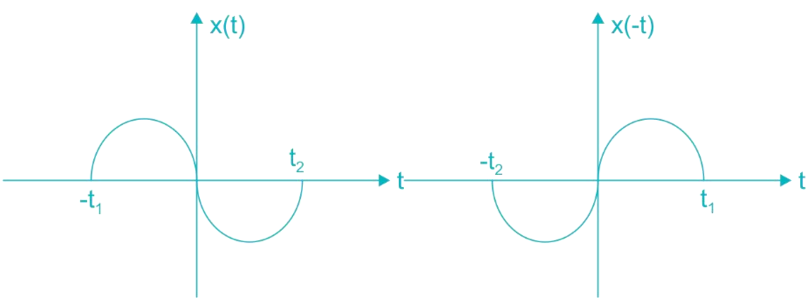
In general, if we consider x(at) then for a > 1, the signal will be compressed by the factor 'a' and for a < 1, the signal will be expanded by the factor (1/a).



Time Inversion / Time Reversal

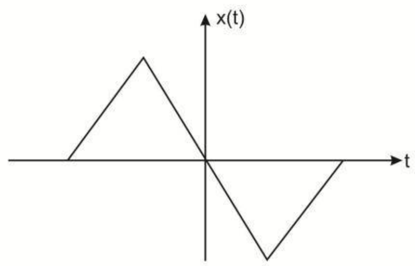
Time inverted signal is denoted by x(–t) which is achieved by replacing the independent variable time t by its negative counterpart i.e. –t. So, time inverted signal of x(t) is x(–t). This can also be considered as special case of time scaling with a = –1.

Time inverted signal can also be achieved by taking the reflection / mirror image about the vertical axis. It can be explained as follows:



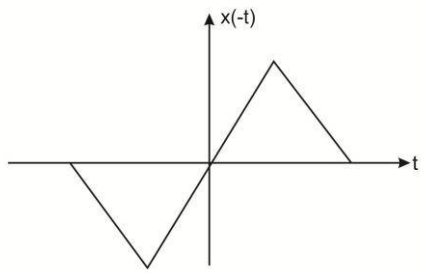
Example 2:

Plot x(−t) for the given x(t).



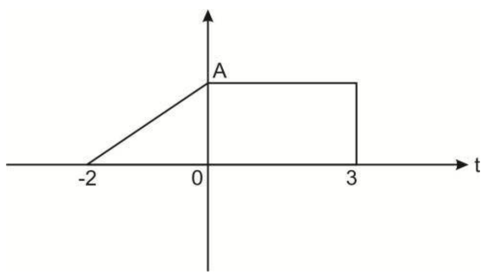
Solution:

Take mirror image about vertical axis.



Example 3:

For the continuous time signal x(t) given in the figure, plot x(t+2), x(-t), x(t+2), x(t-2), x(-t-2), x(2t), $x\left(\frac{t}{2}\right)$, $x(3t+2)$, $x\left(\frac{t}{3}-2\right)$.



Solution:

x(t - 2) can be obtained by shifting x(t) to the right by 2 units.

x(t + 2) can be obtained by shifting x(t) to the left by 2 units.

x(-t) → taking mirror image about the vertical axis.

x(-t+2) can be obtained be shifting x(t) to the left by 2 units and then taking the mirror image about the vertical axis.

x(-t+2) → shifting left + time inversion/time reversal i.e. First plot x (t+2) then do the time inversion.

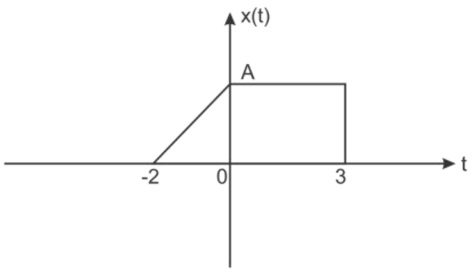
Similarly, x(-t-2) → Shifting right + time inversion/time reversal, i.e. first plot x (t-2) then time inversion.

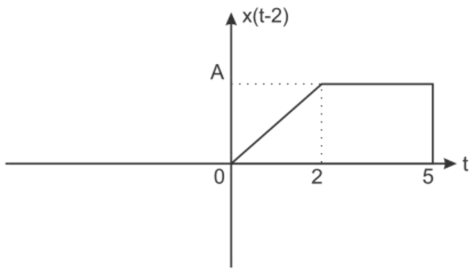
x(2t) can be obtained by compression of signal x(t) by the factor 2.

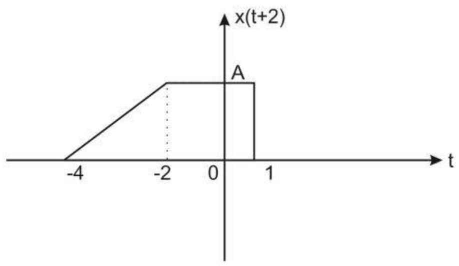
x (t/2) can be obtained by expansion of signal x(t) by the factor 2.

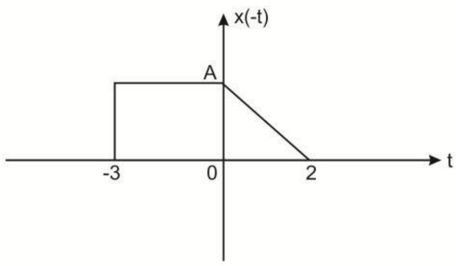
x (3t+2) can be obtained by shifting + time scaling. First plot x(t+2) then compress this signal by the factor 3.

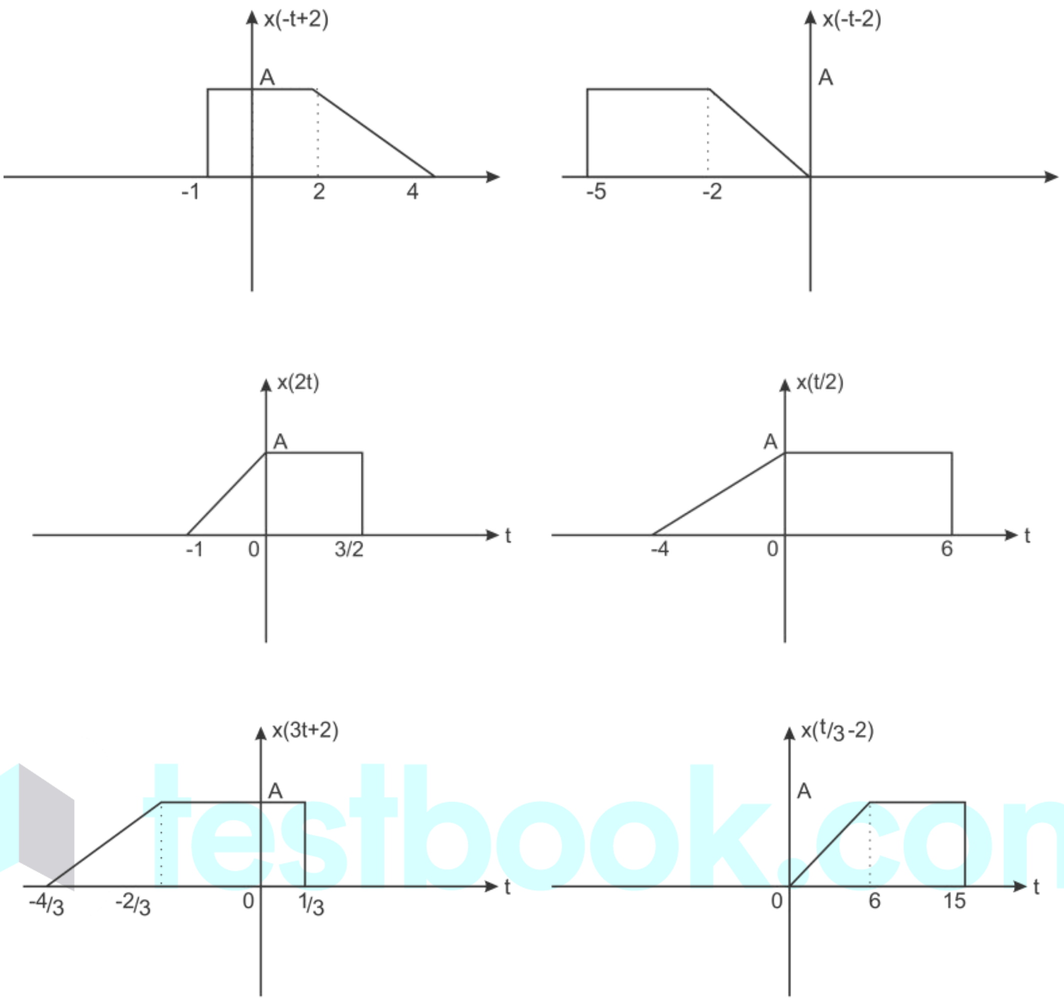
x(t/3-2) can be obtained by shifting + time scaling. First plot x(t-2) then expand this signal by the factor 3.











Note:

When a combination of time shifting and time scaling has to be performed on a signal x(t) then to realize an expression of the form x(at-b), the natural order is time shifting followed by time scaling. However, time shifting can also be performed after time scaling with a precaution of dividing the visible shift amount by the scaling factor.

Now, we are familiar with the Signals and Signal Transformation. In next article, we will deal with different kinds of standard signals used.

Did you enjoy this article on Time Signals & Signal Transformation? Let us know in the comments. You may also enjoy reading –

Important Standard DC Signals

Signal Classification

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