Types of Time Systems - GATE Study Material in PDF

In the previous articles, we discussed all about Time Signals. In these free GATE 2018 Notes, we will discuss some **Types of Time Systems** as well as some basics**.** These study material are useful for **GATE EC** and **GATE EE** as well as other exams like BARC, BSNL, DRDO, ISRO, IES etc. These notes may also be **downloaded in PDF** so that your exam preparation is made easy and you ace your exam.

You should probably go through the basics covered in previous articles, before starting off with this module.

Recommended Reading –

Laplace Transforms

Limits, Continuity & Differentiability

Mean Value Theorems

Differentiation

Partial Differentiation

Maxima and Minima

Methods of Integration & Standard Integrals

Vector Calculus

Vector Integration

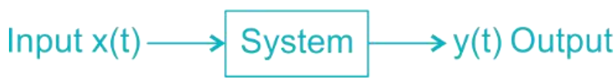
Time Signals & Signal Transformation

Standard Time Signals

Signal Classification

What are Systems?

It this article, we will discuss the basics of systems and their types. System is defined as a physical entity containing a set of elements or functional blocks which are connected together to perform some operation on input signals and produce an output in response to that input signal. A general system can be represented as below.



Mathematically they can be written as

y(t) = f(x(t))

Systems can also be divided into continuous time and discrete time systems. Apart from this there are various categories of classification of systems based on various properties.

Continuous time systems are defined as those systems in which input and output are continuous time signals or we can say that all the associated signals are continuous–time. Continuous–time systems are represented using differential equation. Systems whose input and output both are discrete time signals are known as discrete–time systems or all the associated signals are discrete. Discrete–time systems are represented by difference equation.

Nomenclature of Time System Entities

The nomenclature of various entities in the system to be used are being given as follows.

For Continuous Time systems, the naming is done as -

$x(t) \rightarrow$ input

$y(t) \rightarrow$ output

$H(t) \rightarrow$ system function

$x(t), y(t) \rightarrow$ present values

$x(t - t_o), y(t - t_o)$; $t_o > 0 \rightarrow$ past / previous values

$x(t - t_o), y(t - t_o)$; $t_o < 0 \rightarrow$ future values

For Discrete Time systems replace 't' with 'n' and 't_o' with 'n_o' in the equations above.

Classification of Time Systems

Systems can be classified on the basis of the system properties as follows-

1. Static & Dynamic System; or System With or Without Memory

2. Causal & Non-Causal System; Non-Anticipatory & Anticipatory System

3. Time Variant & Time Invariant System

4. Linear & Non-Linear System

5. Invertible System & Non-Invertible System

6. Stable & Unstable System

Static and Dynamic Systems

1. Static systems are also known as memory less system because static system does not contain any storage element.

2. A system is said to be static or memory less if output at any instant depends only on input at that instant otherwise it is known as dynamic system.

3. Since static system does not contain any storage element it means the differential equation does not contain any derivative or the integral term. For example, $y(t) = Ax(t)$ is a static / memory less system.

4. Dynamic systems are called as system with memory because these systems contain some energy–storage elements to store the values other than present value.

5. Basically these stored energies are the initial conditions. Differential equations of dynamic systems contain derivative and integral terms.

Some real time examples of dynamic systems have been given as

i) Voltage across a capacitor

$v(t) = \frac{1}{c}\int_{-\infty}^{\infty} i(\tau)d\tau$

$i(t) \rightarrow$ input current

$v_c(t) \rightarrow$ output voltage

$c \rightarrow$ capacitance of capacitor

ii) Current through an inductor

$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau$

$i_L(t) \rightarrow$ input current

$v(t) \rightarrow$ output voltage

$L \rightarrow$ inductance of the inductor

iii) Accumulator is an example of dynamic system.

$y[n] = \sum_{k=-\infty}^{n} x[k]$

We will further understand the mathematics of these systems by the following example.

Example 1:

Categorise the following systems as static or dynamic

(i) $y[n] = x^2[n]$

(ii) $y[n] = x[n^2]$

Solution:

(i) $y[n] = x^2[n]$

$y[1] = x^2[1]$

$y[0] = x^2[0]$

$y[-1] = x^2[-1]$

The system output depends only on present values. Hence it is static system.

(ii) $y[n] = x[n^2]$

y[-1] = x[1]

y[2] = x[4]

The systems output depends on input values other than present ones. Hence it is a dynamic system.

Causal and Non-Causal Systems

1. A system is said to be causal system if the output does not begin before the input signal is applied or output does not depend on future values. All real-time systems are causal systems. A delay element is an example of causal system.

2. A system is said to be non-casual if the output begins before input signal is applied or we can say that output depends on future value of input signal.

3. Non-causal systems are also known as anticipatory.

4. Causal systems are physically realizable but non-causal systems are not physically realizable.

Example 2:

Categorise the following systems according to causality

(i) y[n] = x[n] - x[n-1]

(ii) y[n] = x[n^2]

Solution:

(i) y[n] = x[n] - x[n-1]

In this output depends only on past and present value of input. Hence the system is causal.

(ii) y[n] = x[n^2]

y[-1] = x[1]

Since the output depends on future values of input, hence the systems is non causal.

Time Invariant and Time Variant Systems

A system is said to be time invariant if relationship between input and output does not vary with time. i.e time shift in input results same time shift in output (or delay in input = delay in output). If relationship between input and output varies with time, then the system is known as time variant system.

Let, $y(t) = f(x(t))$

Then for time invariant systems

$y(t - t_o) = f(x(t - t_o))$

Steps to check time invariance-

Step 1: Find $y(t - t_o)$ which is the delayed output by t_o units.

Step 2: Find $y(t, t_o)$ which is the output for the delayed input by t_o units.

Step 3: If $y(t - t_o) = y(t, t_o)$ then system is time invariant otherwise system is time variant.

For example, $y(t) = tx(t)$, $y(t - t_o) = (t - t_o)\, x(t - t_o)$

$y(t, t_o) = tx(t - t_o) \neq y(t, t_o) \Rightarrow$ Time Variant system

In general, if 't' is multiplied by -1 or some constant or t, t^2, t^3..... then the system is time variant.

For example: $y(t) = x(-t)$, $y(t) = x(2t)$, $y(t) = x(t^2)$

These are time variant systems.

If amplitude is changing with t, t^2, ... then the system is time variant.

For example, $y(t) = t\, x(t)$

We will try to understand the phenomenon of time variance and invariance by the example given below –

Example 3:

Test the time invariance of the following signals.

1) $y(t) = x(t^2)$ **2)** $y(t) = x^2(t)$

3) $y(t) = x(t) + x(-t - 2)$

Solution:

1) $y(t) = x(t^2)$

$y(t - t_0) = x((t - t_0)^2) \neq y(t, t_0)$

Therefore, this system is not time invariant.

2) $y(t) = x^2(t)$

$y(t - t_0) = x^2(t - t_0)$

$y(t, t_0) = x^2(t - t_0) = y(t - t_0)$

Therefore, this system is a time invariant systems.

3) $y(t) = x(t) + x(-t - 2)$

$y(t - t_0) = x(t - t_0) + x(-(t - t_0) - 2)$

$y(t - t_0) = x(t - t_0) + x(-t + t_0 - 2)$

$y(t, t_0) = x(t - t_0) + x(-t - 2 - t_0) \neq y(t, -t_0)$

Therefore, this system is time variant.

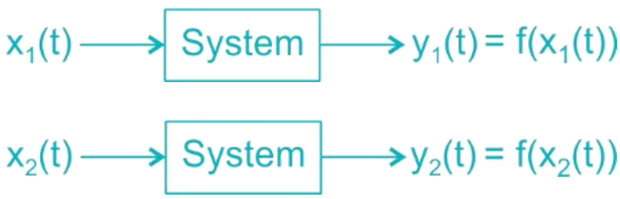
Linear and Non-Linear Systems

A system is said to be linear if it obeys the principle of superposition and homogeneity, i.e. response of the sum of weighted inputs is same as the sum of weighted responses. Any system with finite initial conditions is non-linear.

Addition of the similar quantities is homogeneity. Addition of the dissimilar quantities is superposition. A linear system can be explained as follows. Suppose there are two systems given as –

$y_1 = f(x_1(t))$

$y_2 = f(x_2(t))$



Then for a linear system, we will have

$$ax_1(t)+bx_2(t) \longrightarrow \boxed{\text{System}} \longrightarrow ay_1(t)+by_2(t)$$

A Linear System

Then the system is known as linear system Any system with finite initial conditions is non–linear. Inductor and capacitor are Linear when there is no stored energy initially otherwise these are non – linear.

Example 4:

Check the Linearity of the following systems.

1) $y(t) = x(t^2)$ **2)** $y(t)= x(t)- x(t-2)$ **3)** $y(t) = \log x(t)$ **4)** $y[n] = x^2[n]$

Solution:

1) $y(t) = x(t^2)$

$y_1(t) = x_1(t^2)$, $y_2(t) = x_2(t^2)$

$y_3(t) = f(ax_1(t) + bx_2(t)) = ax_1(t^2) + bx_2(t^2) = ay_1(t) + by_2(t)$

Since, $f(ax_1(t) + bx_2(t)) = ay_1(t) + by_2(t)$

Therefore, this system is a linear system.

2) $y(t) = x(t) - x(t - 2)$

$y_1(t) = x_1(t) = f(x_1(t)) = x_1(t) - x_1(t - 2)$

$y_2(t) = f(x_2(t)) = x_2(t) - x_2(t - 2)$

$y_3(t) = f(ax_1(t) + bx_2(t)) = ax_1(t) + bx_2(t) + ax_1(t - 2) + bx_2(t-2))$

$= ax_1(t) - ax_1(t - 2) + bx_2(t) - bx_2(t - 2)$

$= a[x_1(t) - x_1(t - 2)] + b [x_2(t) - x_2(t - 2)] = ay_1(t) + by_2(t)$

Therefore, this is a linear system.

3) $y(t) = \log x(t)$

$y_1(t) = \log(x_1(t), y_2(t)) = \log (x_1(t))$

$y_3(t) = ay_1(t) + by_2(t)$

$y_4(t) = f(ax_1(t) + bx_2(t)) = \log(ax_1(t) + bx_2(t)) \neq ay_1(t) + by_2(t)$

Therefore, this system is non-linear.

4) $y[n] = x^2[n]$

$y_1[n] = x_1^2[n],\ \ y_2[n] = x_2^2[n]$

$y_3[n] = f(ax_1[n] + bx_2[n]) = (ax_1[n] + bx_2[n])^2$

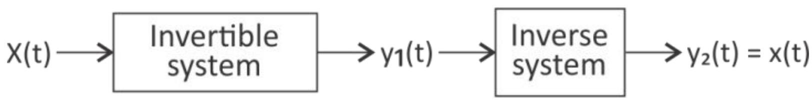
$= a^2x_1^2[n] + b^2x_2^2[n] + 2abx_1[n]x_2[n]$

$\neq ax_1^2[n] + bx_2^2[n]$

$\neq ay_1[n] + by_2[n]$

Hence the given system is non-linear.

Invertible and Non-Invertible Systems



If there is one – to – one correspondence between the input and output of a system, then the system is known as invertible system. If the system is invertible then its inverse system exists. And the cascading of these two systems results in identity system.

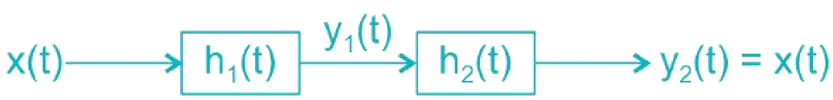
$h_1(t) * h_2(t) = \delta(t)$, if $h_2(t)$ is the inverse system of $h_1(t)$.

$y_2(t) = h_1(t) * h_2(t) * x(t)$

$= \delta(t) * x(t)$

$y_2(t) = x(t)$

A general invertible system can be shown as



An example of the invertible system is shown as

$$x(t) \rightarrow \boxed{x(t-2)} \rightarrow y_1(t) \rightarrow \boxed{y_1(t+2)} \rightarrow y_2(t) = x(t)$$

$y(t) = x^2(t)$ is not invertible as there is no one to one correspondence between x(t)and y(t). As at x(t) = 1, y(t) = 1, at x(t) = -1, y(t) = 1 i.e. many to one correspondence between input and output.

Stable and Unstable Systems

$$X(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$$

1. A system is said to be BIBO (Bounded Input Bounded Output) Stable if it produces bounded output for any bounded input.

2. A system is stable if its natural response tends to zero as $|t| \rightarrow \infty$

3. Complete solution of any differential equation is – Auxiliary solution + Particular solution.

Auxiliary Solution:

1. Also known as Natural response / zero input response. Natural response is the system's response to initial conditions with all externals forces (i.e. inputs) set to zero.

2. To find auxiliary solution, first we write characteristic equation. The roots of the characteristic equation are known as characteristic roots or Eigen values or natural frequencies.

3. If natural response decreases with increasing time then system is stable otherwise system is unstable.

Particular Solution:

Also known as forced response or zero state response. This is system's response to input will all initial conditions set to zero.

Order of a System:

Order of the system is equal to number of storage elements presents in the system.

For continuous time system, highest derivative of output signal present in the input – output differential equation corresponds to order of the system.

Consider x(t) is bounded, i.e $|x(t)| \le B_x$

If y(t) is bounded for this input, then system is stable $|y(t)| \leq B_y$

Example 5:

Test the stability of the following systems.

1) $y(t) = x^2(t)$ **2)** $y(t) = \cos(x(t))$

Solution:

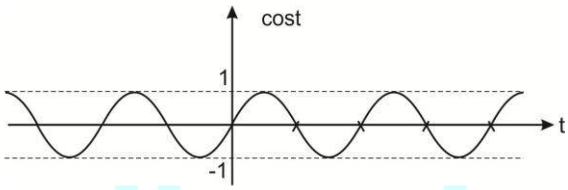
1) $y(t) = x^2(t)$

Since x(t) is bounded i.e. $|x(t)| \leq B_x$

$|x^2(t)| \leq B_x^2 \Rightarrow |y[n]| \leq B_x^2$

i.e. output is bounded. Therefore, this is a stable system.

2) $y(t) = \cos(x(t))$



Cosine function is always bounded between -1 and 1 therefore this system is an absolutely stable system. Even if input is unbounded then also output is bounded.

Stable and Unstable systems are also discussed in detail in control systems article with examples. Among these systems, Linear Time Invariant (LTI) system is very important for exam point of view. We will discuss about it in the next few articles.

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