Chapter 1.3
Complex Numbers

* Imaginary unit = i $\quad i^2 = -1$
* Complex numbers in form = a + bi , where a= real part, b = imaginary part, and i= imaginary unit.

1. Write the expression in the standard form a + bi.

i^{17}

$17 \div 4 = 4\overline{)17}$ R. 1

-16

1

$= i$

Chart

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

* Take exponent and divide by 4.
* Look at remainder.
* Now look at chart. The remainder will be the same number as the exponent of i.

2. Perform the indicated operation, and write the expression in the standard form a + bi.

i^{-48}

$i^{-48} = \frac{1}{i^{48}}$

$48 \div 4 = 4\overline{)48}$ R. 0

-48

0

$= \frac{1}{1} = 1$

If the remainder is 0, then the answer is 1.

* First, since the exponent is a negative, we move it to the bottom of the fraction to make it a positive.
* Next, divide the exponent by 4.
* Look at remainder. If it is 0, then the answer is 1.

3. Write the expression in the standard form a + bi. $(-2+6i)-(6-4i)$

$(-2+6i)-(6-4i)$

$-2+6i-6+4i$

Combine like terms

$-8+6i+4i$

$-8+10i$

* To begin, we must distribute the negative sign that is outside the 2nd set of parentheses to each term inside the parentheses. When we do this, it will just change the sign of each term inside the parentheses.
* Then we will combine like terms.
* We begin by combining the numbers that are by themselves.
* Then we will combine the terms with the i's.
* Make sure in your answer that you always put the number first, and then the term with the i.

4. Write the expression in the standard form a + bi. $(1-9i)-(3+3i)$

$(1-9i)-(3+3i)$

Combine like terms

$1-9i-3-3i$

$-2-9i-3i$

$-2-12i$

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5. Write the expression in the standard form a + bi. $(4-6i)(8+i)$

$(4-6i)(8+i)$

$32+4i-48i-6i^2$

$32-44i-6i^2$

$32-44i-6(-1)$

$32-44i+6$

$38-44i$

Replace i^2 with (-1)

* To begin with, we will use the distributive property to get rid of the parentheses.
* To do this, we will multiply the 1st term in the 1sr set of parentheses by each term in the 2nd set of parentheses.
* Then we will multiply the 2nd term in the 1st set of parentheses by each term in the 2nd set of parentheses. (Remember
* ... $= i^2$
* Now we will combine like terms beginning with the i's.
* Now we will replace the i^2 With a -1.
* Then we will combine the like terms, which will be the numbers.
* Make sure in your answer that you always put the number first, and then the term with the i.

6. Write the expression in the standard form a + bi. $(6+i)^2$

$(6+i)(6+i)$

$36+6i+6i+i^2$

$36+12i+i^2$

$36+12i+(-1)$

$35+12i$

* Anytime you see something in parentheses squared, that just means to write it down twice where they are multiplied by each other.

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* Then we will multiply the 2nd term in the 1st set of parentheses by each term in the 2nd set of parentheses. (Remember
* $i \cdot i = i^2$
* Now we will combine like terms beginning with the i's.
* Now we will replace the i^2 With a -1.
* Then we will combine the like terms, which will be the numbers.
* Make sure in your answer that you always put the number first, and then the term with the i.

7. Write the expression in the standard form a + bi. $\frac{4}{7-6i}$

$\frac{4}{7-6i} \cdot \frac{7+6i}{7+6i}$ change the sign

$\frac{4(7+6i)}{(7-6i)(7+6i)}$

$\frac{28+24i}{49+42i-42i-36i^2}$

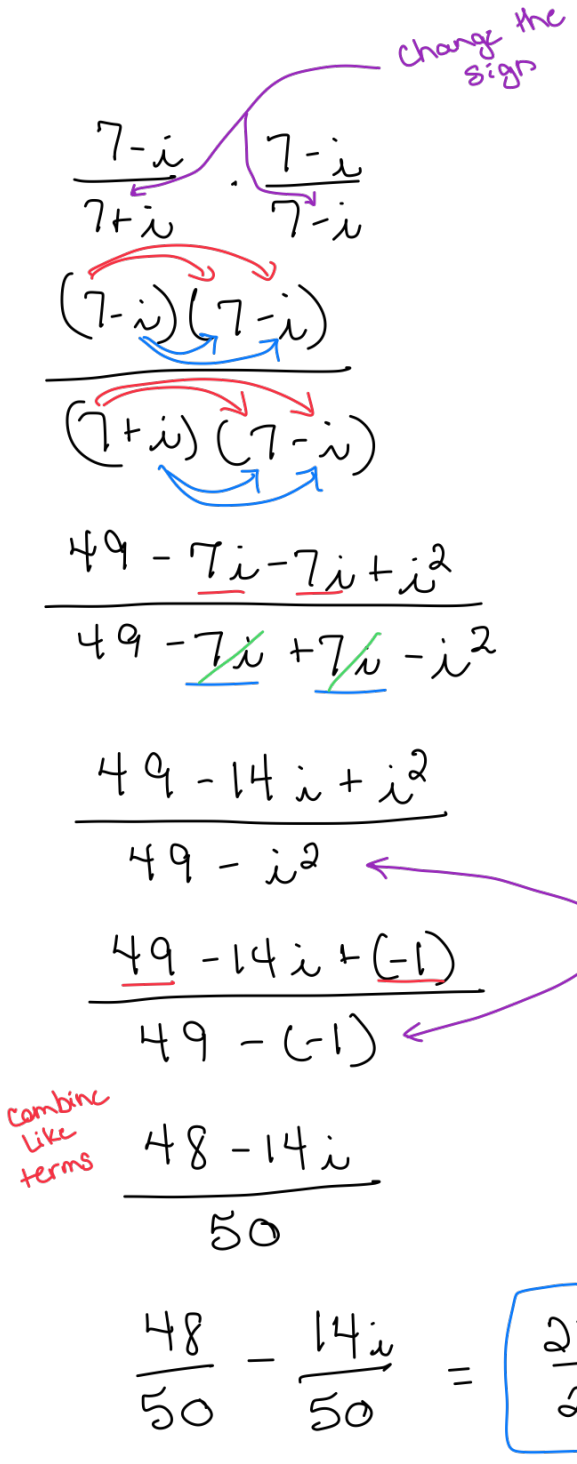
* Since we have a fraction with i on the bottom, we want to get the i off the bottom.
* To do this, we multiply both the top and bottom by the conjugate. This just means that you will change the sign to the opposite of the term that has i in it.
*Then rewrite the problem as multiplication problem.
* Then we will use the distributive property to get rid of the parentheses.
* Remember: $i \cdot i = i^2$
* Then we will combine like terms. The i's on the bottom should cancel each other out.
* Then you will replace the i^2 With a (-1).
* Then combine together the numbers on the bottom.
* Now to get it in the appropriate form, you will write down each term on the top as a separate fraction. Make sure to include the i with the 2nd fraction.

$\frac{28+24i}{49-36i^2} = \frac{28+24i}{49-36(-1)}$ ← Replace i^2 with (-1)

combine like terms

$\frac{28+24i}{49+36} = \frac{28+24i}{85} = \frac{28}{85} + \frac{24}{85}i$

8. Write the expression in the standard form a + bi. $\frac{7-i}{7+i}$



* Since we have a fraction with i on the bottom, we want to get the i off the bottom.
* To do this, we multiply both the top and bottom by the conjugate. This just means that you will change the sign to the opposite of the term that has i in it.

*Then rewrite the problem as multiplication problem.

* Then we will use the distributive property to get rid of the parentheses.
* Remember: $i \cdot i = i^2$
* Then we will combine like terms on the top and then the bottom. The i's on the bottom should cancel each other out.
* Then you will replace the i^2 With a (-1).
* Then combine together the numbers on the top and the bottom.
* Now to get it in the appropriate form, you will write down each term on the top as a separate fraction. Make sure to include the i with the 2nd fraction.
* You will need to simplify the fractions for the final answer. Just use a calculator for this. (Type in each fraction and hit enter.)

9. Solve the equation in the complex number system.

$x^2 - 16x + 68 = 0$

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$a = 1 \quad b = -16 \quad c = 68$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ quadratic formula

$$\frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(68)}}{2(1)}$$

$$\frac{16 \pm \sqrt{-16}}{2}$$

* Type whats under the $\sqrt{}$ sign in the calculator

divide $\frac{16}{2} \pm \frac{\sqrt{-16}}{2}$

* a (-) sign under a $\sqrt{}$ will turn into an i outside the $\sqrt{}$ sign

$$8 \pm \frac{i\sqrt{16}}{2}$$

$8 + 2i, \; 8 - 2i$

10. Solve the equation in the complex number system.

$2x^2 + 1 = 2x$

$2x^2 + 1 = 2x$

$-2x \quad -2x$

$2x^2 - 2x + 1 = 0$

$a = 2 \quad b = -2 \quad c = 1$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ { quadratic formula

$\frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(1)}}{2(2)}$

$\frac{2 \pm \sqrt{-4}}{4}$

* Type whats under the $\sqrt{}$ sign in the calculator

Simplify $\frac{2}{4} \pm \frac{\sqrt{-4}}{4}$

* as (-) sign under as $\sqrt{}$ will turn into an i outside the $\sqrt{}$ sign

$\frac{1}{2} \pm \frac{i\sqrt{4}}{4}$

$\frac{1}{2} \pm \frac{2i}{4}$ ← Simplify

$\frac{1}{2} \pm \frac{1}{2}i$

$\frac{1}{2} + \frac{1}{2}i \quad , \quad \frac{1}{2} - \frac{1}{2}i$