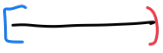
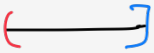
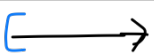
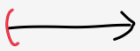
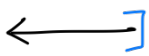
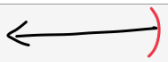
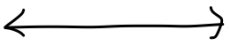
Chapter 1.5 Notes
Solving Inequalities

* Open interval = (a,b). This just mean that the endpoint goes to the number, but does not include the number.
* Closed interval = [a,b] This just means that the endpoint goes to that number and includes that number.
* Half open or half closed interval = (a,b] or [a,b). a is the left endpoint, b is the right endpoint.

	Interval	Inequality or set notation	Graph
1.	(a,b)	$a < x < b$	
2.	$[a,b]$	$a \leq x \leq b$	
3.	$[a,b)$	$a \leq x < b$	
4.	$(a,b]$	$a < x \leq b$	
5.	$[a,\infty)$	$x \geq a$	
6.	(a,∞)	$x > a$	
7.	$(-\infty,a]$	$x \leq a$	
8.	$(-\infty,a)$	$x < a$	
9.	$(-\infty,\infty)$	all real numbers	

*Multiplying or dividing by a negative number reverses the inequality.

ex.) $-2x \leq -14$

$\frac{}{-2} \quad \frac{}{-2}$ ← notice that since we are dividing by a negative number, you must flip the sign.

$x \geq 7$

1. Find the solution of the inequality and express the solution set in set notation and interval notation. Graph the solution set on the teal number line.

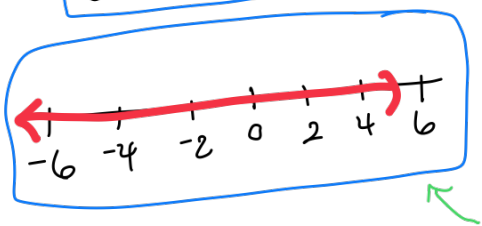
$x + 1 < 6$

$x + 1 < 6$

$-1 \quad -1$

$x < 5$ ← Inequality/ Set notation

$(-\infty, 5)$ ← Interval notation



graph

* when dealing with an inequality, we treat it kind of like an equal sign, in that we do the opposite to move something from one side to the other.
* Here we begin with moving the number on the left to the right by doing the opposite. Since here it is addition, we will subtract it from both sides.
* This will give us the set notation form.
* Then using the chart, find the inequality/set notation that looks similar to the form in your answer.
* Here it looks like, x < a, which is in row 8.
* Looking in the same row, look under the column titled 'interval'. Write down the interval notion just like it is ,except replace the "a" with the number in your inequality.
* Then to graph, looking in the same row, you will draw a graph just like that, with the) being on the number in your inequality.

2. Solve the following inequality. Graph the solution set.

$5x - 2 > 33$

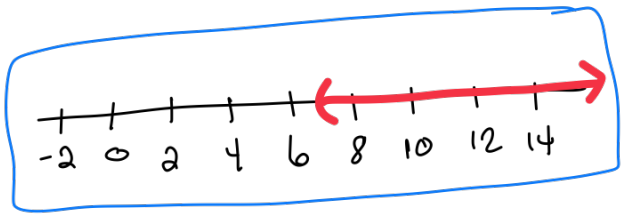
$5x - 2 > 33$

$+2 \quad +2$

$\frac{5x}{5} > \frac{35}{5}$

$x > 7$

$(7, \infty)$



* Here we will begin with moving the number on the left side to the right by doing the opposite. Here it is subtraction, so we will add it to both sides.
* Then to get the x by itself, we will divide both sides by the number in front of x.
* This will be the set notation form.
* Then using the chart, find the set notation that looks similar to the form in your answer.
* Here it looks like x > a, which is in row 6.
* Looking in the same row, look under the column titled "interval". Write down the interval notation just like it is except replace the 'a' with the number in your inequality. This will be the interval form.
* To graph, looking in the same row, you will draw a graph just like that, with the "(" being on the number in your inequality.

3. Solve the inequality. Express your answer using set notation or interval notation. Graph the solution set.

$3 - 8x \leq -5$

$3 - 8x \leq -5$

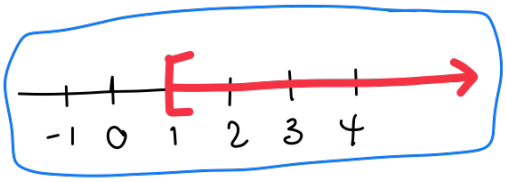
$-3 \quad -3$

$-8x \leq -8$

$\frac{-8x}{-8} \leq \frac{-8}{-8}$ ← dividing by negative, flip the sign

$x \geq 1$

$[1, \infty)$



* Here we will begin with moving the number on the left side to the right by doing the opposite. Here it is addition, so we will subtract it from both sides.
* Then to get the x by itself, we will divide both sides by the number in front of x.
* Since we are dividing by a negative number, we must flip the sign.
* Now this will be the set notation form.
* Then using the chart, find the set notation that looks similar to the form in your answer.
* Here it looks like x ≥ a, which is in row 5.
* Looking in the same row, look under the column titled "interval". Write down the interval notation just like it is except replace the 'a' with the number in your inequality. This will be the interval form.
* To graph, looking in the same row, you will draw a graph just like that, with the "[" being on the number in your inequality.

4. Solve the inequality. Express the answer using set notation or interval notation. Graph the solution set.

$-3(2x - 3) > -3$

$-3(2x-3) > -3$

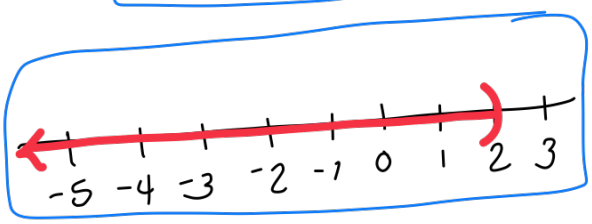
$-6x + 9 > -3$

$-9 \quad -9$

$\frac{-6x}{-6} > \frac{-12}{-6}$ ← dividing by negative, flip the sign

$x < 2$

$(-\infty, 2)$



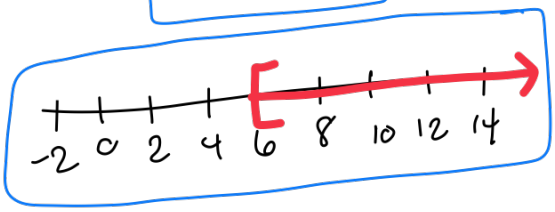
* First we must do the distributive property to get rid of the parenthesis. We do this by multiplying the term outside the parenthesis by both terms inside the parentheses.
* Now we move the number on the left side to the right by doing the opposite. Here it is addition, so we will subtract it from both sides.
* Then to get the x by itself, we will divide both sides by the number in front of x.
* Since we are dividing by a negative number, we must flip the sign.
* Now this will be the set notation form.
* Then using the chart, find the set notation that looks similar to the form in your answer.
* Here it looks like x < a, which is in row 8.
* Looking in the same row, look under the column titled "interval". Write down the interval notation just like it is except replace the 'a' with the number in your inequality. This will be the interval form.
* To graph, looking in the same row, you will draw a graph just like that, with the ")" being on the number in your inequality.

5. Solve the inequality. Express your answer using set notation or interval notation. Graph the solution set.

$$7x - 8 \geq 4 + 5x$$

$$7x - 8 \geq 4 + 5x$$
$$-5x \qquad -5x$$
$$2x - 8 \geq 4$$
$$+8 \qquad +8$$
$$\frac{2x}{2} \geq \frac{12}{2}$$
$$x \geq 6$$

$$[6, \infty)$$



* Here we begin by moving the x's on the right side to the left by doing the opposite. Since it is added here, we will subtract it from both sides.
* Then we will move the number on the left side to the right by doing the opposite. Here it is subtraction, so we will add it to both sides.
* Then to get the x by itself, we will divide both sides by the number in front of x.
* Now this will be the set notation form.
* Then using the chart, find the set notation that looks similar to the form in your answer.
* Here it looks like x ≥ a, which is in row 5.
* Looking in the same row, look under the column titled "interval". Write down the interval notation just like it is except replace the 'a' with the number in your inequality. This will be the interval form.
* To graph, looking in the same row, you will draw a graph just like that, with the "[" being on the number in your inequality.

6. Solve the inequality. Express your answer using set notation or interval notation. Graph the solution set.

$$\frac{1}{4}(x-6) < x+12$$

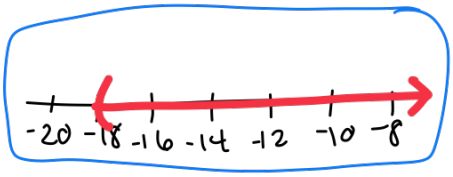
$$\frac{1}{4}(x-6) < x+12$$

$$\frac{1}{4}x - \frac{6}{4} < x + 12$$
$$-1x \qquad -x$$
$$-\frac{3}{4}x - \frac{6}{4} < 12$$
$$+\frac{6}{4} \qquad +\frac{6}{4}$$
$$-\frac{3}{4}x < \frac{27}{2}$$
$$\frac{}{-\frac{3}{4}} \qquad \frac{}{-\frac{3}{4}}$$

dividing by negative, flip the sign

$$x > -18$$

$$(-18, \infty)$$



* First we must do the distributive property to get rid of the parenthesis. We do this by multiplying the term outside the parenthesis by both terms inside the parentheses.
* Now we move the x's on the right to the left by doing the opposite. Here they are added, so we will subtract it from both sides.
* Now we move the number on the left side to the right by doing the opposite. Here it is subtraction, so we will add it to both sides.
* Then to get the x by itself, we will divide both sides by the number in front of x. (Just type as fraction into your calculator)
* Since we are dividing by a negative number, we must flip the sign.
* Now this will be the set notation form.
* Then using the chart, find the set notation that looks similar to the form in your answer.
* Here it looks like x > a, which is in row 6.
* Looking in the same row, look under the column titled "interval". Write down the interval notation just like it is except replace the 'a' with the number in your inequality. This will be the interval form.
* To graph, looking in the same row, you will draw a graph just like that, with the "(" being on the number in your inequality.

7. Solve the inequality. Express your answer using set notation and interval notation. Graph the solution set.

$0 \le 3x - 15 \le 9$

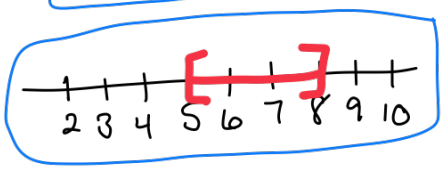
$0 \le 3x - 15 \le 9$

$+15 \quad +15 \quad +15$

$\frac{15}{3} \le \frac{3x}{3} \le \frac{24}{3}$

$5 \le x \le 8$

$[5, 8]$



* Here we have 2 inequality signs in the same problem. Just think about it as working 2 problems at the same time.
* We look at the expression in the middle 1st. We want "x" to be by itself in the middle.
* So we will move the number in the middle to both the right side and the left side by doing the opposite. Here the number is subtracted, so we will add it to all 3 columns. (the left, middle, & right sides.)
* Now, to get "x" by itself, we must divide by the number in front of "x". Be sure and divide all 3 numbers by the number in front of "x".
* This will be the inequality/set notation form.
* For the interval notation, look at the chart and find the set notation that looks similar to yours and write down the interval notation, replacing "a" with the smaller number, and "b" with the bigger number.
* Then look at the graph in the same row, and draw it with the "[" on the smaller number, and "]" on the bigger number with a line in between.

8. Solve the inequality. Express your answer using interval notation. Graph the solution set.

$-2 < \frac{2x-3}{5} < 0$

$5 \cdot -2 < 5 \cdot \frac{2x-3}{5} < 0 \cdot 5$

$-10 < 2x - 3 < 0$

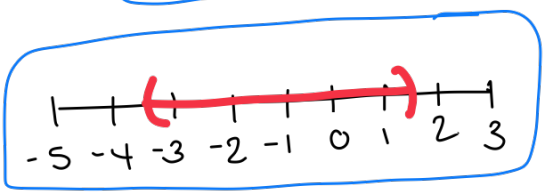
$+3 \quad +3 \quad +3$

$-7 < 2x < 3$

$\frac{-7}{2} < \frac{2x}{2} < \frac{3}{2}$

$\frac{-7}{2} < x < \frac{3}{2}$

$\left(-\frac{7}{2}, \frac{3}{2}\right)$



* Here we have 2 inequality signs in the same problem. Just think about it as working 2 problems at the same time.
* We look at the expression in the middle 1st. We want "x" to be by itself in the middle.
* First we want to get rid of the fraction in the middle. To do this we will multiply the middle expression by the denominator. This will cancel out the denominator. But we must also multiply both sides, (left and right) by that same number.
* Now we will move the number in the middle to both the right side and the left side by doing the opposite. Here the number is subtracted, so we will add it to all 3 columns. (the left, middle, & right sides.)
* Now, to get "x" by itself, we must divide by the number in front of "x". Be sure and divide all 3 numbers by the number in front of "x".
* This will be the inequality/set notation form.
* For the interval notation, look at the chart and find the set notation that looks similar to yours and write down the interval notation, replacing "a" with the smaller number, and "b" with the bigger number.
* Then look at the graph in the same row, and draw it with the "(" on the smaller number, and ")" on the bigger number with a line in between.