C.A. Chapter 1.6 Notes
Solving equations/inequalities involving absolute value

$\|u\| = a$	Is equivalent to	$u = a$ or $u = -a$
$\|u\| < a$	Is equivalent to	$-a < u < a$
$\|u\| \leq a$	Is equivalent to	$-a \leq u \leq a$
$\|u\| > a$	Is equivalent to	$u < -a$ or $u > a$
$\|u\| \geq a$	Is equivalent to	$u \leq -a$ or $u \geq a$

When writing in interval notation: $(-\infty, a) \cup (b, \infty)$

1.) Find the real solutions of the equation.

$|3x| = 12$

1st equation: $\frac{3x}{3} = \frac{12}{3}$, $x = 4$

2nd equation: $\frac{3x}{3} = \frac{-12}{3}$, $x = -4$

$4, -4$

* To get rid of the absolute value sign, we will make 2 separate equations.
* 1st equation will look like original problem, just without absolute value bars.
* 2nd equation will look just like 1st equation, except add a negative sign in front of the number after the equal sign.
* Then you will solve both equations for x.
* These will be your 2 answers.

2.) Find the real solutions of the equation.

$|6x + 5| = 7$

$6x + 5 = 7$, $-5 \quad -5$

$\frac{6x}{6} = \frac{2}{6}$

$x = \frac{1}{3}$

$6x + 5 = -7$, $-5 \quad -5$

$\frac{6x}{6} = \frac{-12}{6}$

$x = -2$

$\frac{1}{3}, -2$

* To get rid of the absolute value sign, we will make 2 separate equations.
* 1st equation will look like original problem, just without absolute value bars.
* 2nd equation will look just like 1st equation, except add a negative sign in front of the number after the equal sign.
* Then you will solve both equations for x.
* You will move the number on the left to the right by doing the opposite.
* Then we divide each side by the number in front of x.
* These will be your 2 answers.

3.) Find the real solutions of the equation.

$|1-5t|+5=16$

$|1-5t|+5=16$

$-5 \quad -5$

$|1-5t|=11$

$1-5t=11$ | $1-5t=-11$

$-1 \quad -1$ | $-1 \quad -1$

$-5t=10$ | $-5t=-12$

$\frac{-5t}{-5}=\frac{10}{-5}$ | $\frac{-5t}{-5}=\frac{-12}{-5}$

$t=-2$ | $t=\frac{12}{5}$

$-2, \frac{12}{5}$

* We want the absolute value expression on one side and the numbers on the other side.
* To do this, we will move the number that is outside the absolute value signs to the other side by doing the opposite. Here it was addition, so we will subtract it from both sides.
* Now, to get rid of the absolute value sign, we will make 2 separate equations.
* 1st equation will look like original problem, just without absolute value bars.
* 2nd equation will look just like 1st equation, except add a negative sign in front of the number after the equal sign.
* Then you will solve both equations for x.
* You will move the number on the left to the right by doing the opposite.
* Then we divide each side by the number in front of x.
* These will be your 2 answers.

4. Find the real solutions of the equation.

$2-|6x|=1$

$2-|6x|=1$

$-2 \quad -2$

$-|6x|=-1$

$+ \quad +$

$|6x|=1$

$6x=1$ | $6x=-1$

$\frac{6x}{6}=\frac{1}{6}$ | $\frac{6x}{6}=\frac{-1}{6}$

$x=\frac{1}{6}$ | $x=-\frac{1}{6}$

$\frac{1}{6}, -\frac{1}{6}$

* We want the absolute value expression on one side and the numbers on the other side.
* To do this, we will move the number that is outside the absolute value signs to the other side by doing the opposite. Here it was addition, so we will subtract it from both sides.
* Now we still have a negative sign outside the absolute value sign, so to get rid of it, we will change it to a positive, but then we must change the number on the other side of the equal sign to the opposite sign.
* Now, to get rid of the absolute value sign, we will make 2 separate equations.
* 1st equation will look like original problem, just without absolute value bars.
* 2nd equation will look just like 1st equation, except add a negative sign in front of the number after the equal sign.
* Then you will solve both equations for x.
* You we divide each side by the number in front of x.
* These will be your 2 answers.

5. Solve the following inequality. Graph the solution set.

$$|3t-2| \le 4$$

$$-4 \le 3t-2 \le 4$$

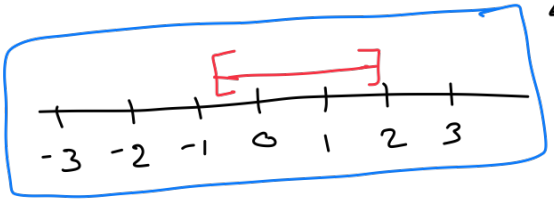
$+2 \quad +2 \quad +2$

$$-2 \le 3t \le 6$$

$\frac{-2}{3} \quad \frac{3t}{3} \quad \frac{6}{3}$

$$\frac{-2}{3} \le t \le 2$$

$$\left[-\frac{2}{3}, 2\right]$$



* If you will look at the chart on the first page, you will see that this will be in the form $-a \le u \le a$
* So, to get rid of the absolute value sign, you will write down the original equation but without the absolute value bars. Then on the write side of the equation, you will write down the same number that is on the right side, but put a negative in front of it. Then place the same absolute value sign in between that number and the original equation.
* Now you want to get the variable all by itself in the center.
* Begin by moving the number in the center to both sides by doing the opposite. Here it was subtracted, so we will add it to all 3 terms.
* Then to get t by itself in the center, we will divide all 3 terms by the number in front of "t".
* Then to write it in interval notation, write down the smaller number and then the bigger number separated by a comma. Then you will put [ ] around the numbers.
* To graph, you will place "[" on the smaller number, and the "]" on the bigger number, and make a line in between.

6.) Solve the following inequality. Graph the solution set.

$$|1-6x|-11 < -2$$

$$|1-6x|-11 < -2$$

$+11 \quad +11$

$$|1-6x| < 9$$

$$-9 < 1-6x < 9$$

$-1 \quad -1 \quad -1$

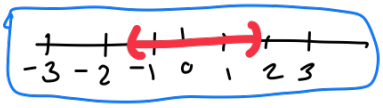
$$-10 < -6x < 8$$

$\frac{-10}{-6} \quad \frac{-6x}{-6} \quad \frac{8}{-6}$

dividing by (-) flip so flip signs

$$\frac{5}{3} > x > -\frac{4}{3}$$

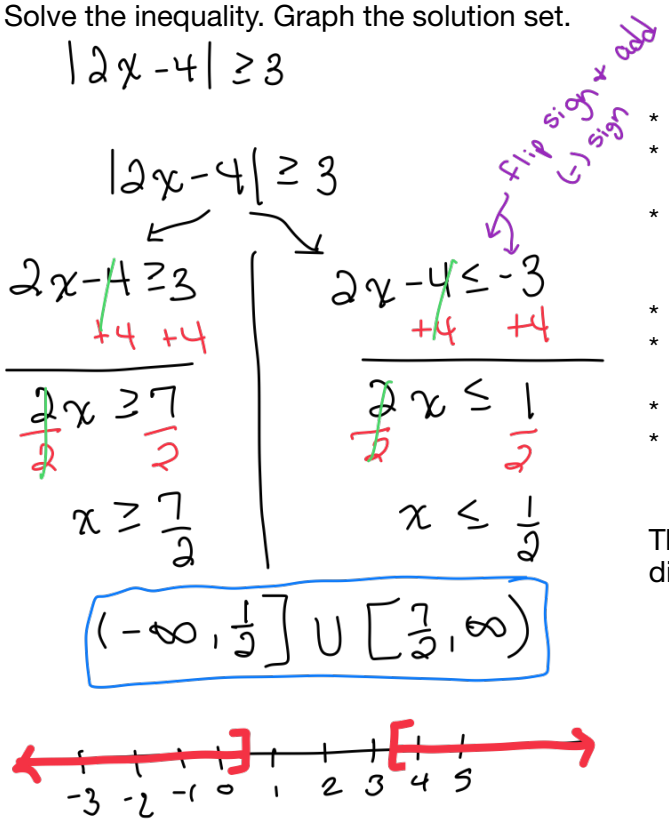
$$\left(-\frac{4}{3}, \frac{5}{3}\right)$$



* First, you must get the absolute value on one side and everything else on the other side. So, we will move the number on the left to the right by doing the opposite.
* Then, If you will look at the chart on the first page, you will see that this will be in the form $-a < u < a$
* So, to get rid of the absolute value sign, you will write down the original equation but without the absolute value bars. Then on the write side of the equation, you will write down the same number that is on the right side, but put a negative in front of it. Then place the same absolute value sign in between that number and the original equation.
* Now you want to get the variable all by itself in the center.
* Begin by moving the number in the center to both sides by doing the opposite. Here it was subtracted, so we will add it to all 3 terms.
* Then to get t by itself in the center, we will divide all 3 terms by the number in front of "t".
* Since we are dividing by a negative, we will flip the sign of both.
* Then to write it in interval notation, write down the smaller number and then the bigger number separated by a comma. Then you will put [ ] around the numbers.
* To graph, you will place "[" on the smaller number, and the "]" on the bigger number, and make a line in between.

7. Solve the inequality. Graph the solution set.

$|2x-4| \geq 3$



* To start we will make 2 equations.
* 1st equation will look exactly like original problem, but without the absolute value bars.
* 2nd equation will look exactly like 1st equation, except flip the sign and change the sign on the number on the left to its opposite.
* Then you will solve both equations for "x".
* Start with moving the numbers on the left to the right by doing the opposite.
* Then divide both sides by the number in front of x.
* Now to write in interval notation, you will write

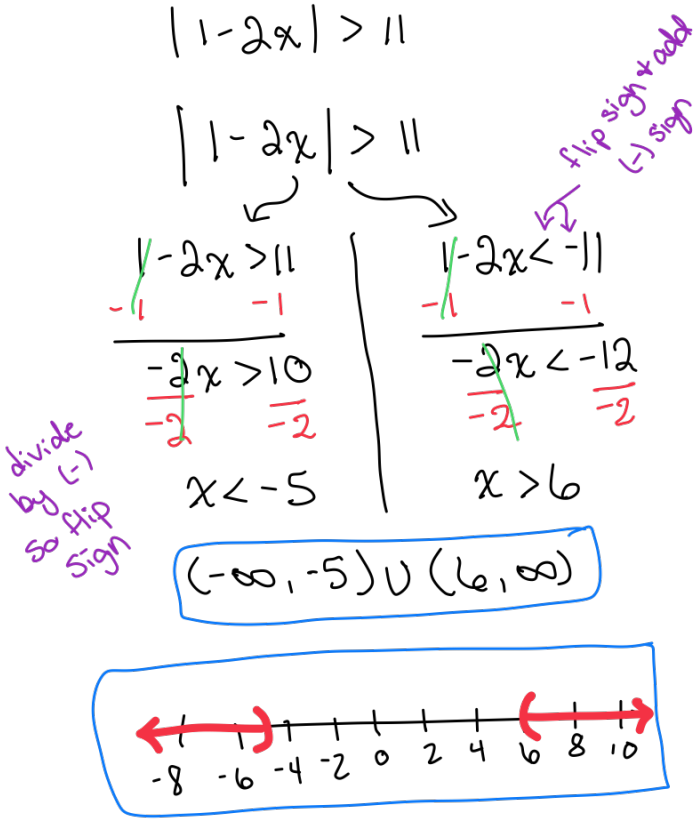
$(-\infty, \text{smallest number}] \cup [\text{biggest \#}, \infty)$

Then when you graph, it will be a split graph going in different directions.

* Tip: You will always use a parenthesis next to an ∞ or $-\infty$ sign.

8. Solve the following inequality. Graph the solution set.

$|1-2x| > 11$



* To start we will make 2 equations.
* 1st equation will look exactly like original problem, but without the absolute value bars.
* 2nd equation will look exactly like 1st equation, except flip the inequality sign and change the sign on the number on the left to its opposite.
* Then you will solve both equations for "x".
* Start with moving the numbers on the left to the right by doing the opposite.
* Then divide both sides by the number in front of x.
* Since we are dividing by a negative number, you will flip the signs.
* Now to write in interval notation, you will write

$(-\infty, \text{smallest \#}) \cup (\text{biggest \#}, \infty)$

Then when you graph, it will be a split graph going in different directions.

9. Solve the following inequality. Graph the solution set.

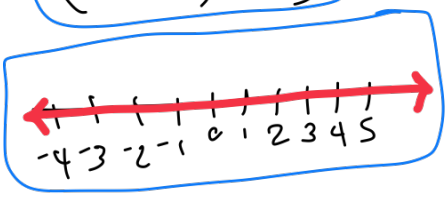
$|2x| \geq -3$

$|2x| \geq -3$

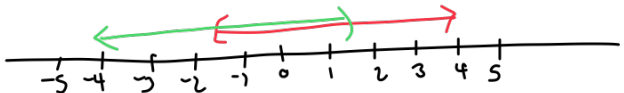
$\frac{2}{2}x \geq \frac{-3}{2}$ | $\frac{2}{2}x \leq \frac{3}{2}$

$x \geq \frac{-3}{2}$ | $x \leq \frac{3}{2}$

$(-\infty, \infty)$



* To begin with, we make 2 equations.
* 1st equation will look just like original equation, but without the absolute value bars.
* 2nd equation will look like 1st equation, but flip the inequality sign and change sign to the opposite. (Here it was a negative so we make it a positive.)
* Then to get "x" alone, divide both sides by the number in front of "x".
* Now, if you were to graph it, it would look like this:



* so, since the arrows would cross each other, then any number could work as the solution.
* So to write that in interval notation: $(-\infty, \infty)$

*Then to graph, you would just have a continuous line.

* Tip: If you have an absolute value being ≥ a negative number it will always be $(-\infty, \infty)$.

10. Solve the inequality. Graph the solution set.

$4 + |x - 3| > \frac{5}{8}$

$4 + |x - 3| > \frac{5}{8}$

$-4 \quad -4$

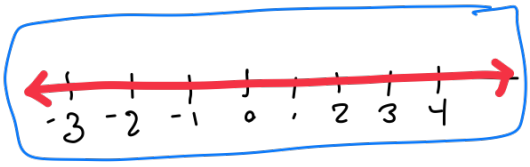
$|x - 3| > -\frac{27}{8}$

$x - 3 > -\frac{27}{8}$ | $x - 3 < \frac{27}{8}$

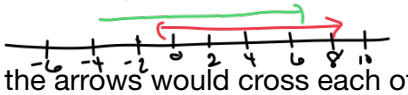
$+3 \quad +3$ | $+3 \quad +3$

$x > -\frac{3}{8}$ | $x < \frac{51}{8}$

$(-\infty, \infty)$



* Remember, we must move the number on the left to right first, so that we only have the absolute value on the left.
* Now looking at this, we have an absolute value being greater than a negative number, so we now that the answer will be
* But to show our work, we will make 2 equations.
* 1st equation will look just like the original equation, but without the absolute value bars.
* 2nd equation will look like 1st equation, but flip the inequality sign and change the sign to the opposite.
* Then to get "x" alone, we will move the number on the left to the right by doing the opposite.
* Now if we were to graph it, it would look like this:



* So, since the arrows would cross each other, then any number could work as the solution.
* So to write as an interval notation: $(-\infty, \infty)$
* Then to graph, it would just be a continuous line.

* Tip: If you have an absolute value being > a negative number it will always be $(-\infty, \infty)$.

11. Find the real solutions of the equation

$$|u-6| = -\frac{1}{8}$$

Tip: An absolute value can never equal a negative number.

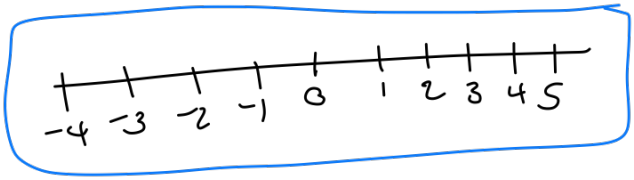
There is no real solution.

12. Solve the absolute value inequality. Graph the solution set on a real number line.

$$|5x+7| < -8$$

Tip: An absolute value can never be less than ($<$) a negative number.

There is no real solution



← since there is no solution, the graph would be blank.