Chapter 5.2 Notes
Properties of Rational Functions

1. Analyze the polynomial function f(x) = x^2(x-8).

a. Determine the end behavior of the graph of the function.

$y = x^2(x-8)$

$x^3 - 8x^2$

$\uparrow$

$y = \boxed{x^3}$

· The end behavior is the x to the largest degree.

b. Find the x and y-intercepts of the graph of the function.

$y = x^2(x-8)$

$0 = x(x-8)$

$x^2 = 0$

$x - 8 = 0$
$+8 \quad +8$

$x = 8$

x-intercepts = $\boxed{0, 8}$

· for x-intercepts: make y = 0 and solve for x.

$y = x^2(x-8)$

$y = 0^2(0-8)$

$y = 0$

y-intercepts = $\boxed{0}$

· for y-intercepts: Replace x with 0 and solve for y.

c. Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.

* The zeros are the same as the x-intercepts.

$x = 0 \quad x = 8$ The zero's of f are $\boxed{0, 8}$

· The multiplicity is equal to the exponent that goes with the zero.

$x^2 = 0$ is 2 $\quad x = 8$ is 1

· If the multiplicity is even, then the graph touches the x-axis at that point + then turns.

· If the multiplicity is odd, then the graph crosses the x-axis at that point.

* The lesser zero of the function is of the multiplicity $\boxed{2}$, so the graph of f $\boxed{\text{touches}}$ the x-axsis at $x = \boxed{0}$.

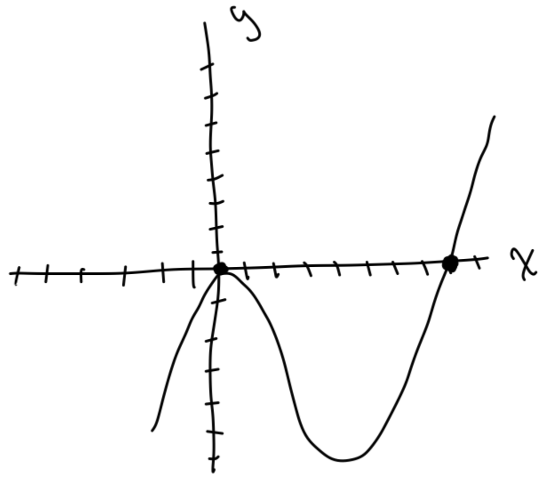
* The greater zero of the function is of multiplicity $\boxed{1}$, so the graph of f $\boxed{\text{crosses}}$ the x-axis at $x = \boxed{8}$.

d. Determine the maximum number of turning points on the graph of the function.

- To determine the maximum number of turning points, find the highest degree of the function & subtract 1.

- $x^3 \rightarrow 3-1 = \boxed{2}$

e. Use the above information to draw a complete graph of the function. Choose the correct graph.



- Looking for graph where touches x-axis on 0 & crosses at 8.

- also since the end behavior was a (+) x^3, the right side of the graph will point upwards.

2. Analyze the polynomial function $f(x) = (x + 6)(x - 4)^2$.

a. Determine the end behavior of the graph of the function.

option #1

$y = (x+6)(x-4)^2$

$(x-4)(x-4)$

$x^2 - 4x - 4x + 16$

$(x+6)(x^2 - 8x + 16)$

$x^3 - 8x^2 + 16x + 16x^2 - 48x + 96$

$x^3 + 8x^2 - 32x + 96$

$y = \boxed{x^3}$

-or-

Option #2

$(x+6)(x-4)^2$

$(x+6)(x-4)(x-4)$

$x \cdot x \cdot x$

x^3

· you basicly are counting your x's

b. Find the x and y-intercepts of the graph of the function.

$y = (x+6)(x-4)^2$

$0 = (x+6)(x-4)^2$

. for x-intercepts: make $y=0$ and solve for x.

$x + 6 = 0$
$-6 \quad -6$
$x = -6$

$x - 4 = 0$
$+4 \quad +4$
$x = 4$

x-intercepts = $\boxed{-6, 4}$

$y = (x+6)(x-4)^2$

$y = (0+6)(0-4)^2$

$y = 96$

· for y-intercepts: Replace x with 0 and solve for y.

Type into calculator

y-intercepts = $\boxed{96}$

c. Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.

* The zero are the same as the x-intercepts.

$x = -6 \quad x = 4$ The zero's of f are $\boxed{-6, 4}$

· The multiplicity is equal to the exponent that goes with the zero.

$(x+6)(x-4)^2$

-6 came from here + the exponent is 1 so...
$x = -6$ has multiplicity of 1

4 came from here & the exponent is 2 so...
$x = 4$ has multiplicity of 2

· If the multiplicity is even, then the graph touches the x-axis at that point + then turns.

· If the multiplicity is odd, then the graph crosses the x-axis at that point.

The smaller x

* The lesser zero of the function is of the multiplicity $\boxed{1}$, so the graph of f $\boxed{\text{crosses}}$ the x-axsis at $x = \boxed{-6}$.

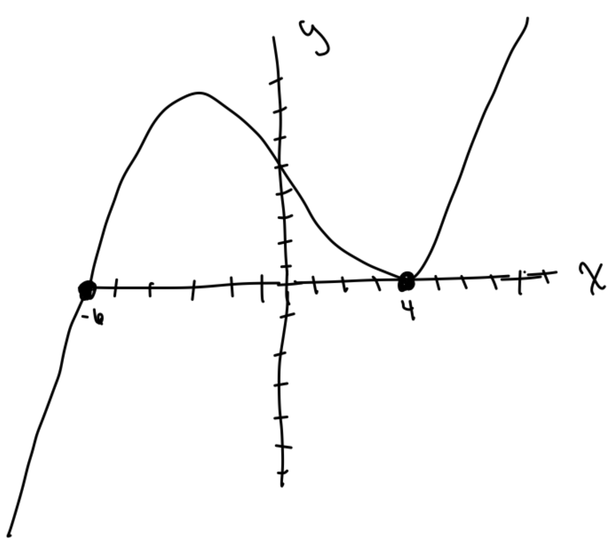
The bigger x

* The greater zero of the function is of multiplicity $\boxed{2}$, so the graph of f $\boxed{\text{touches}}$ the x-axis at $x = \boxed{4}$.

d. Determine the maximum number of turning points on the graph of the function.

- To determine the maximum number of turning points, find the highest degree of the function & subtract 1.
- $x^3 \rightarrow 3-1 = \boxed{2}$

e. Use the above information to draw a complete graph of the function. Choose the correct graph.



- Looking for graph where crosses x-axis on -6 & touches on 4
- also since the end behavior was a (+) x^3, the right side of the graph will point upwards.

3. Graph the polynomial function f(x) = x(3-x)(7-x).

a. Determine the end behavior of the graph of the function.

option #1

$y = x(3-x)(7-x)$

$(3x - x^2)(7-x)$

$21x - 3x^2 - 7x^2 + x^3$

$21x - 10x^2 + x^3$

$x^3 - 10x^2 + 21x$

-or-

option #2

$x(3-x)(7-x)$

$\downarrow \quad \downarrow \quad \downarrow$

$(x) \cdot (-x) \cdot (-x)$

x^3

Basically multiply x's together

$y = \boxed{x^3}$

b. Find the x and y-intercepts of the graph of the function.

$y = x(3-x)(7-x)$

$0 = x(3-x)(7-x)$

· for x-intercepts: make y=0 and solve for x.

$x = 0$

$3 - x = 0$
$+x \quad +x$
$3 = x$

$7 - x = 0$
$+x \quad +x$
$7 = x$

x-intercepts = $\boxed{0, 3, 7}$

· for y-intercepts: Replace x with 0 and solve for y.

$y = x(3-x)(7-x)$

$y = 0(3-0)(7-0)$

$y = 0$

y-intercepts = $\boxed{0}$

c. Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.

* The zero are the same as the x-intercepts.

$x = 0, \ x = 3, \ x = 7$ The zero's of f are $\boxed{0, 3, 7}$

· The multiplicity is equal to the exponent that goes with the zero.

$x = 0$ is 1 $\quad x = 3$ is 1 $\quad x = 7$ is 1

· If the multiplicity is even, then the graph touches the x-axis at that point + then turns.

· If the multiplicity is odd, then the graph crosses the x-axis at that point.

* The smallest zero is a zero of multiplicity $\boxed{1}$, so the graph of f $\boxed{\text{crosses}}$ the x-axis at $x = \boxed{0}$.

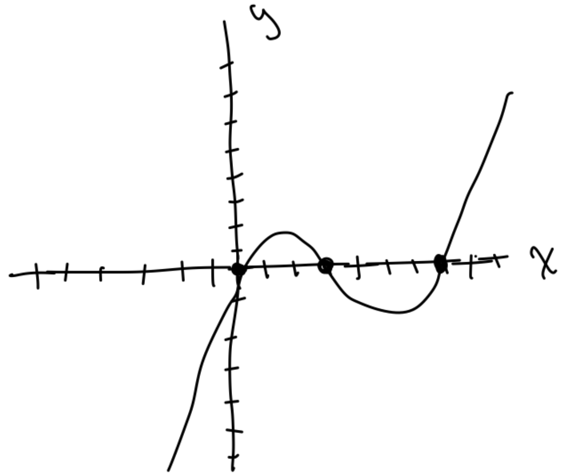
* The middle zero is a zero of multiplicity $\boxed{1}$, so the graph of f $\boxed{\text{crosses}}$ the x-axis at $x = \boxed{3}$

* The largest zero is a zero of multiplicity $\boxed{1}$, so the graph of f $\boxed{\text{crosses}}$ the x-axis at $x = \boxed{7}$

d. Determine the maximum number of turning points on the graph of the function.

- To determine the maximum number of turning points, find the highest degree of the function & subtract 1.
- $x^3 \rightarrow 3 - 1 = \boxed{2}$

e. Use the above information to draw a complete graph of the function. Choose the correct graph.



- Looking for graph where crosses x-axis on 0, 3, + 7.
- also since the end behavior was a (+) x^3, the right side of the graph will point upwards.

4. Graph the polynomial function $f(x) = (x + 3)^2 (x - 4)^2$

a. Determine the end behavior of the graph of the function.

option #1

$y = (x+3)^2 (x-4)^2$

$y = (x+3)(x+3)(x-4)(x-4)$

$y = (x^2+3x+3x+9)(x^2-4x-4x+16)$

$y = (x^2+6x+9)(x^2-8x+16)$

$y = x^4 - 8x^3 + 16x^2 + 6x^3 - 48x^2 + 96x + 9x^2 - 72x + 144$

$y = x^4 - 2x^3 - 23x^2 + 24x + 144$ $\qquad y = \boxed{x^4}$

-OR-

option #2

$(x+3)^2 (x-4)^2$

$(x+3)(x+3)(x-4)(x-4)$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$(x) \cdot (x) \cdot (x) \cdot (x)$

x^4

b. Find the x and y-intercepts of the graph of the function.

$y = (x+3)^2 (x-4)^2$

$0 = (x+3)^2 (x-4)^2$

· for x-intercepts: make $y=0$ and solve for x.

$x+3 = 0$
$-3 \quad -3$
$x = -3$

$x-4=0$
$+4 \quad +4$
$x = 4$

x-intercepts = $\boxed{-3, 4}$

$y = (x+3)^2 (x-4)^2$

$y = (0+3)^2 (0-4)^2$

$y = 144$

· for y-intercepts: Replace x with 0 and solve for y.

y-intercepts = $\boxed{144}$

c. Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.

* The zero are the same as the x-intercepts.

$x = -3 \qquad x = 4$ $\qquad$ The zero's of f are $\boxed{-3, 4}$

· The multiplicity is equal to the exponent that goes with the zero.

$x = -3$ is 2 $\qquad x = 4$ is 2

· If the multiplicity is even, then the graph touches the x-axis at that point + then turns.

· If the multiplicity is odd, then the graph crosses the x-axis at that point.

* The lesser zero of the function is of the multiplicity $\boxed{2}$, so the graph of f $\boxed{\text{touches}}$ the x-axsis at $x = \boxed{-3}$.

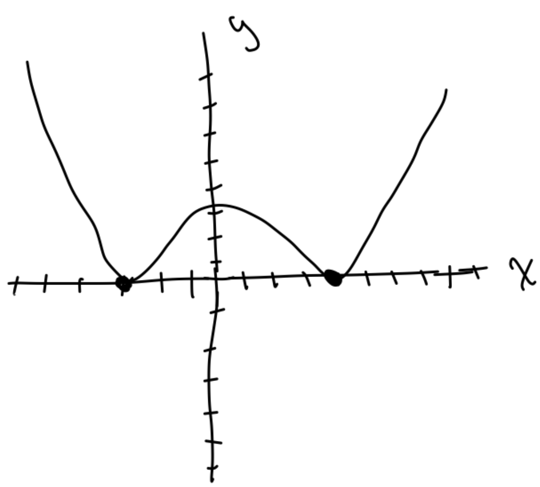
* The greater zero of the function is of multiplicity $\boxed{2}$, so the graph of f $\boxed{\text{touches}}$ the x-axis at $x = \boxed{4}$.

d. Determine the maximum number of turning points on the graph of the function.

- To determine the maximum number of turning points, find the highest degree of the function & subtract 1.

- $x^4 \rightarrow 4 - 1 = \boxed{3}$

e. Use the above information to draw a complete graph of the function. Choose the correct graph.



- Looking for graph where touches x-axis on -3 & 4.

- also since the end behavior was a $(+)\ x^4$, so both sides of the graph will point upwards.