Chapter 5.3 Notes
Finding vertical, horizontal, and oblique asymptotes

* Vertical asymptotes — Take the bottom of the fraction (denominator) and make it = 0. Then solve for x.

ex.) $\frac{3x}{x+7} \longrightarrow$

$$\begin{aligned} x+7&=0 \\ -7\ \ &\ \ -7 \\ x&=-7 \end{aligned}$$

* horizontal asymptotes— if the degree of the top (numerator) is less than the bottom (denominator), then y = 0 is the horizontal asymptote.
  * If the degree of the numerator and denominator are equal, then the horizontal asymptote is the ratio of the leading coefficients.

ex.) $\frac{x+2}{x^2+5} = \frac{x}{x^2} =$ top is less than bottom so $y=0$

ex.) $\frac{3x}{x+7} = \frac{3x}{x} =$ same degree so $\frac{3}{1} = 3$

* Oblique asymptotes — if the degree of the numerator is 1 more than the degree of the denominator, then you will use long or synthetic division to find the asymptote.

ex.) $\frac{x^2+2x+1}{x-1} =$ top bigger than bottom so divide

$$\begin{array}{r} x+3 \\ x-1 \overline{\big)\, x^2+2x+1} \\ -(x^2-1x)\downarrow \\ \hline 3x+1 \\ 3x-3 \\ \hline \end{array}$$

* **You can not have a horizontal and oblique asymptote at the same time.**

* Domain — set the denominator ≠ "0", and solve for x

ex.) $\frac{10x}{x+11}$

$$\begin{aligned} x+11 &\neq 0 \\ -11\ \ &\ \ -11 \\ x &\neq -11 \end{aligned}$$

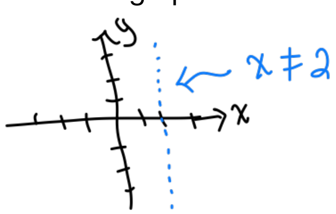
ex.) $\frac{-8x^2}{(x-7)(x+7)}$

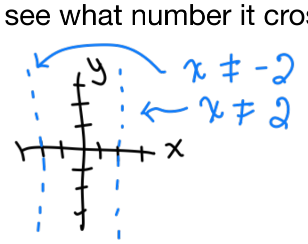
$$\begin{aligned} x-7 &\neq 0 \\ +7\ \ &\ \ +7 \\ x &\neq 7 \end{aligned} \qquad \begin{aligned} x+7 &\neq 0 \\ -7\ \ &\ \ -7 \\ x &\neq -7 \end{aligned}$$

ex.) $\frac{3(x^2-x-30)}{4(x^2-36)}$

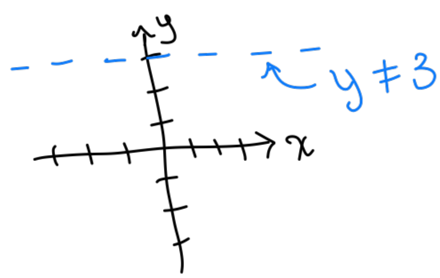
$$\begin{aligned} x^2-36 &\neq 0 \\ +36\ \ &\ \ +36 \\ \sqrt{x^2} &\neq \sqrt{36} \\ x &\neq 6, -6 \end{aligned}$$

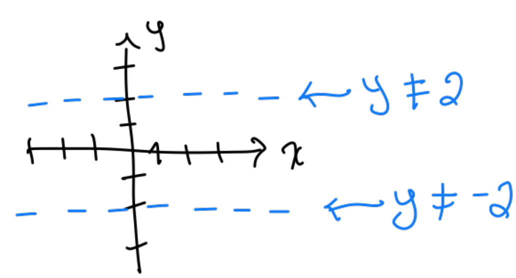
* Finding domain on the graph — look for the vertical dotted line and see what number it crosses on the x-axis.





* Find range on a graph — look for the horizontal dotted line and see what number on the y axis it crosses.





1. Find the vertical, horizontal, and oblique asymptotes, if any, for the following rational function.

$$R(x) = \frac{17x}{x+5}$$

a. Find the vertical asymptotes.

Set bottom of fraction = 0.

$$\frac{17x}{x+5} \rightarrow \begin{array}{r} x+5=0 \\ -5 \quad -5 \\ \hline x=-5 \end{array}$$

* Write down the bottom of the fraction and set it = 0.
* Solve for x.

* The function has one vertical asymptote, $\boxed{x=-5}$

b. Find the horizontal asymptote.

Write down term on top and bottom with highest exponent

$$\frac{17x}{x+5} = \frac{17\cancel{x}}{\cancel{x}} = \frac{17}{1}$$

$$y = 17$$

* write down the leading term for the top and bottom of the fraction (this is the x with the highest degree.)
* Since both the top and bottom are the same degree (x to the same power) then you write down the number in front of each x.

* The function has one horizontal asymptote, $\boxed{y=17}$

c. Find the oblique asymptote.

* The function has no oblique asymptote.

* Since there is a horizontal asymptote, there can't be a oblique asymptote.

2. Find the vertical, horizontal, and oblique asymptotes, if any, for the following rational function.

$$T(x) = \frac{3x^2}{x^4-1}$$

a. Find the vertical asymptotes.

$$\frac{3x^2}{x^4-1} \rightarrow x^4 - 1 = 0$$

$$\begin{aligned} x^4 - 1 &= 0 \\ +1 \quad & \quad +1 \\ \hline x^4 &= 1 \\ \sqrt[4]{x^4} &= \sqrt[4]{1} \\ x &= 1, -1 \end{aligned}$$

* Write down the bottom of the fraction and set it = 0.
* Solve for x.

* The function has two vertical asymptotes. The leftmost asymptote is $\boxed{x=-1}$, and the rightmost asymptote is $\boxed{x=1}$.

* This is from smallest to biggest.

b. Find the horizontal asymptote.

$$\frac{3x^2}{x^4-1} = \frac{3x^2}{x^4}$$

← top degree less than bottom.

$$y = 0$$

* write down the leading term for the top and bottom of the fraction (this is the x with the highest degree.)
* Since the top degree is less than the bottom degree, the horizontal asymptote is y=0.

* The function has one horizontal asymptote at $\boxed{y=0}$

c. Find the oblique asymptote.

* The function has no oblique asymptote.

* There's a horizontal asymptote, so no oblique

3. Find the vertical, horizontal, and oblique asymptotes, if any, for the following rational function.

$$Q(x) = \frac{2x^2 - 7x - 15}{5x^2 - 24x - 5}$$

a. Find the vertical asymptotes.

top of fraction $2x^2 - 7x - 15$ ← quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 2 \quad b = -7 \quad c = -15$

$$\frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-15)}}{2(2)}$$

Top: $x = 5, -\frac{3}{2}$

① Take the top of the fraction + use the quadratic formula to find the x's.

Bottom of fraction $5x^2 - 24x - 5$

$a = 5 \quad b = -24 \quad c = -5$

$$\frac{-(-24) \pm \sqrt{(-24)^2 - 4(5)(-5)}}{2(5)}$$

Bottom: $x = 5, -\frac{1}{5}$

② Take the bottom of the fraction + use the quadratic formula to find the x's.

③ Write down the Top x's on the top of the fraction. Write down the Bottom x's on the bottom of the fraction.

$$\frac{\text{Top: } \cancel{5}, -\frac{3}{2}}{\text{Bottom: } \cancel{5}, -\frac{1}{5}}$$

④ Mark out numbers that are the same.

⑤ The vertical asympote will be any number left on the bottom.

* The function has one vertical asymptote. $\boxed{x = -\frac{1}{5}}$

b. Find the horizontal asymptote.

$$\frac{2x^2 - 7x - 15}{5x^2 - 24x - 5} = \frac{2\cancel{x^2}}{5\cancel{x^2}} = \frac{2}{5}$$

$$y = \frac{2}{5}$$

- write down the leading term for the top and bottom of the fraction (this is the x with the highest degree.)
- Since both the top and bottom are the same degree (x to the same power) then you write down the number in front of each x.

* The function has one horizontal asymptote $\boxed{y = \frac{2}{5}}$

c. Find the oblique asymptote.

* The function has no oblique asymptote.

4. Find the vertical, horizontal, and oblique asymptotes, if any, for the following rational function.

$$H(x) = \frac{x^3-125}{x^2-6x+5}$$

a. Find the vertical asymptotes.

formula

$$a^3-b^3 = (a-b)$$

Top Equation: $x^3 - 125$

$$\underset{a}{(x)^3} - \underset{b}{(5)^3} = (x-5)$$

$$x - 5 = 0$$
$$+5 \quad +5$$

Top: $x = 5$

Bottom Equation: $\underset{a}{x^2} - \underset{b}{6x} + \underset{c}{5}$

Quadratic formula

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$$

Bottom: $x = 5, 1$

Top x's: $\cancel{5}$

Bottom x's: $\cancel{5}, 1$

① Take the top of the fraction + use the formula to factor it out.

② Then take the expression in the 1st set of ( ) and set it $=0$ & Solve for x

③ Write down the bottom of the equation + use the quadratic formula to find the x's.

④ Write down the Top x on the top of the fraction, then write down the bottom x's on the bottom of the fraction.

⑤ Mark out what they have in common. The x's will be what's left on the bottom.

* The function has one vertical asymptote $\boxed{x = 1}$

b. Find the horizontal asymptote.

$$\frac{x^3-125}{x^2-6x+5} = \frac{x^3}{x^2}$$

① Look only at the leading terms of the top + bottom.

② Since the top is bigger than bottom, No horizontal asymptote.

* The function has no horizontal asymptote.

c. Find the oblique asymptote.

$$\frac{x^3-125}{x^2-6x+5}$$

$$\begin{array}{r} x+6 \\ x^2-6x+5 \overline{\big) x^{\cancel{3}}+0x^2+0x-125} \\ -(\cancel{x}^3-6x^2+5x) \downarrow \\ \hline 6x^2-5x-125 \end{array}$$

$\frac{x^3}{x^2} = x$ ← Step #1 take 1st term from equation under √ + put it over 1st term outside √. Then divide. This goes on top.

Step 2: Multiply each term outside √ by answer you just put on top. Write answer underneath √ equation.

Step 3: Subtract the 2 equations under the √ sign.

$\frac{6x^{\cancel{2}}}{\cancel{x^2}} = 6$ Step 4: take 1st term in the equation you just found under √ + put it over 1st term outside √. Then divide. This goes on top

* The function has one oblique asymptote. $\boxed{y = x+6}$

* We will use long division.
* Take the top part of the fraction and write it down under the division sign. Be sure to place 0's in as needed to make the equation go down evenly.
* Then write the bottom part of the fraction on the outside of the division sign.
* Now divide the 1st term inside the division by the 1st term outside the division sign. Write the answer on top.
* Now multiply each term outside the division sign by the number you wrote on top.
* Write it down and then then subtract.
* Now divide again.

5. Find the vertical, horizontal, and oblique asymptotes, if any, for the following rational function.

$$\frac{15x^2+32x-7}{5x-1}$$

a. Find the vertical asymptotes.

Quadratic formula

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Top Equation

$$\underset{a}{15x^2}+\underset{b}{32x}\underset{c}{-7}$$

$$\frac{-(32)\pm\sqrt{(32)^2-4(15)(-7)}}{2a}$$

Top: $x=\frac{1}{5}, \frac{-7}{3}$

① Write down the top part of the fraction. Use the quadratic formula to find the x's

Bottom Equation

$$5x-\cancel{1}=0$$
$$\cancel{+1} \quad +1$$
$$\frac{\cancel{5}x}{\cancel{5}}=\frac{1}{5}$$

Bottom: $x=\frac{1}{5}$

② Write down the bottom part of the fraction, and make it $=0$. Then solve for x.

$$\frac{\text{Top} \quad \cancel{\frac{1}{5}}, \frac{-7}{3}}{\text{Bottom} \quad \cancel{\frac{1}{5}}}$$

③ write down the top x's on the top of the fraction + the bottom x's on the bottom of the fraction.

④ mark out what they have in common.

* Since there is nothing left on the bottom, there's no vertical asymptote.

* The function has no vertical asymptote.

b. Find the horizontal asymptote.

$$\frac{15x^2+32x-7}{5x-1} = \frac{15x^2}{5x}$$

① Look only at the leading terms of the top + bottom.

② Since the top is bigger than bottom, No horizontal asymptote.

$\boxed{\text{* The function has no horizontal asymptote.}}$

c. Find the oblique asymptote.

$$\frac{15x^2+32x-7}{5x-1}$$

$$\begin{array}{r} 3x+7 \\ 5x-1 \,\big)\, \overline{15x^2+32x-7} \\ -(15x^2-3x) \downarrow \\ \hline 35x-7 \end{array}$$

$$\frac{15x^2}{5x} = 3x$$

step 1:
Take 1st term under √ + divide by 1st term outside √

step 2:
- multiply the expression outside the √, by the term you just put on top.
- write the answer under the expression under the √

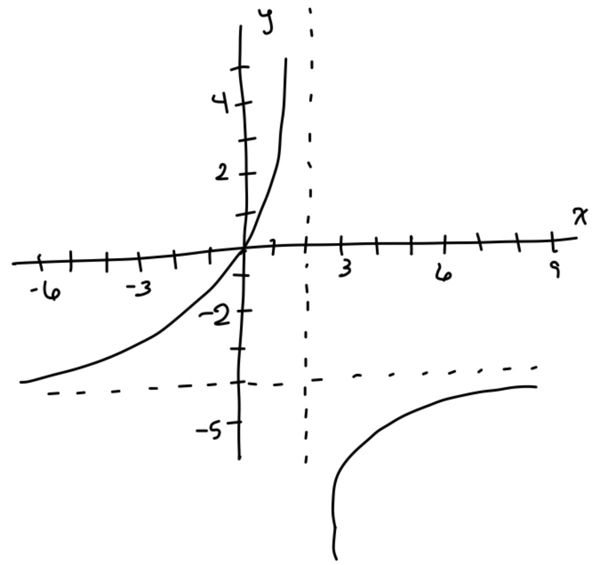
step 3:
- subtract the 2 expressions.

$$\frac{35x}{5x} = 7$$

step 4:
- Divide 1st term under √ by 1st term outside √

* The function has one oblique asymptote. $\boxed{y=3x+7}$

6. Use the graph to find the following.



a. What is the domain?

To find domain on graph, look for vertical (going up + down) dotted line. Then see what number on the x-axis it crosses.

$\boxed{x \neq 2}$

b. What is the range?

To find range on graph, look for horizontal (going side to side) dotted line. Then see what number it crosses.

$\boxed{y \neq -4}$

c. Find the x intercepts.

Look at the graph and see where it crosses the x-axis.

$\boxed{0}$

d. Find the y-intercepts.

Look at the graph and see where it crosses the y-axis.

$\boxed{0}$

e. Find the horizontal asymptote.

Look for horizontal (side to side) dotted line & see what number it crosses.

$\boxed{-4}$

f. Find the vertical asymptote.

Look for vertical (up + down) dotted line & see what number it crosses.

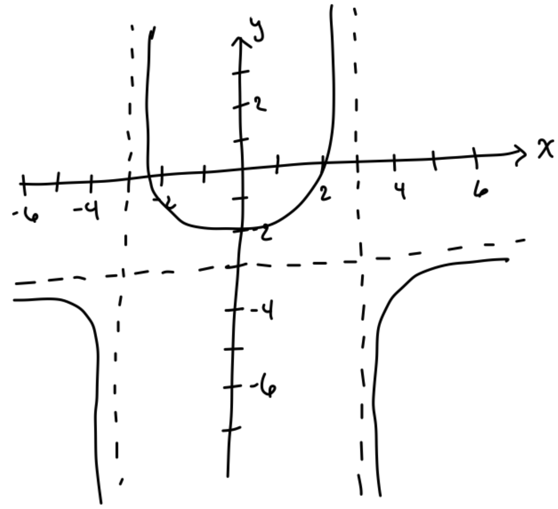
$\boxed{2}$

g. Find the oblique asymptote.

Look for a diagonal dotted line.

$\boxed{\text{No oblique}}$

7. Use the graph to find the following.



a. What is the domain?

To find domain on graph, look for vertical (going up + down) dotted line. Then see what number on the x-axis it crosses.

$\boxed{x \neq -3,\ x \neq 3}$

b. What is the range?

To find range on graph, look for horizontal (going side to side) dotted line. Then see what number it crosses.

$\boxed{y < -3,\ y \geq -2}$

c. Find the x intercepts.

Look at the graph and see where it crosses the x-axis.

$\boxed{-2.4,\ 2.4}$

d. Find the y-intercepts.

Look at the graph and see where it crosses the y-axis.

$\boxed{-2}$

e. Find the horizontal asymptote.

Look for horizontal (side to side) dotted line & see what number it crosses.

$\boxed{-3}$

f. Find the vertical asymptote.

Look for vertical (up + down) dotted line & see what number it crosses.

$\boxed{-3, 3}$

g. Find the oblique asymptote.

Look for a diagonal dotted line.

$\boxed{\text{No oblique}}$