

Chapter 5.5 Notes  
Finding Real Zeros of a Polynomial Function

1. Solve the following inequality.

$$(x-1)(x-2)(x-3) \geq 0$$

$$\begin{array}{r} x-1 \geq 0 \\ +1 \quad +1 \\ \hline x \geq 1 \end{array}$$

$$\begin{array}{r} x-2 \geq 0 \\ +2 \quad +2 \\ \hline x \geq 2 \end{array}$$

$$\begin{array}{r} x-3 \geq 0 \\ +3 \quad +3 \\ \hline x \geq 3 \end{array}$$

1. Write down what's in each set of ( ) and set it " $\geq 0$ ".
2. Now solve for x.

3. Now write down those numbers.

$-\infty$     $1$     $2$     $3$     $\infty$

4. Write down the intervals between each of those numbers.

$[-\infty, 1]$

$[1, 2]$

$[2, 3]$

$[3, \infty)$

5. Write down a number that could go in that interval.

$0$

$2.5$

$4$

$$(0-1)(0-2)(0-3)$$

$$= -6$$

$$(2.5-1)(2.5-2)(2.5-3)$$

$$= -0.375$$

$$(4-1)(4-2)(4-3)$$

$$= 6$$

6. Now replace each one of those numbers back in the original equation replace each x with that number.

7. Write down the answer and circle it if it is  $\geq 0$ , as was stated in original problem.

$$(1.5-1)(1.5-2)(1.5-3)$$

$$= 0.375$$

\*Circle answers that are " $\geq 0$ ".

$$[1, 2] \cup [3, \infty)$$

8. Now write down the interval that goes with the numbers you circled.

2. Solve the following inequality.  $(x+9)^2(x-7) < 0$

$$(x+9)^2(x-7) < 0$$

$$\begin{array}{r} x+9 < 0 \\ \underline{-9 \quad -9} \\ x < -9 \end{array}$$

$$\begin{array}{r} x-7 < 0 \\ \underline{+7 \quad +7} \\ x < 7 \end{array}$$

1. Write down what's in each set of ( ) and set it "> 0"

\* Then write down each expression and then write down the inequality sign and 0.

\* now solve each one for x.

$$\begin{array}{ccccccc} -\infty & & -9 & & 7 & & \infty \\ & \downarrow & & \downarrow & & \downarrow & \\ (-\infty, -9) & & (-9, 7) & & (7, \infty) & & \\ & \downarrow & \downarrow & & \downarrow & & \\ -10 & & 0 & & 8 & & \end{array}$$

\* Now write down those numbers.

\* Write down the intervals between each of those numbers.

\* Write down a number that could go in that interval

$$\begin{array}{ccc} (-10+9)^2(-10-7) & (0+9)^2(0-7) & (8+9)^2(8-7) \\ \text{= } -17 & \text{= } -567 & = 289 \end{array}$$

\* Now replace each one of those numbers back in the original equation replace each x with that number.

\* Write down the answer and then since the original equation says < 0, circle the number if it is a negative number.

$$(-\infty, -9) \cup (-9, 7)$$

Now write down the interval that goes with the numbers you circled. That is your answer. Be sure and place a U between set of ( ).

3. Solve the following inequality.  $x^3 - 4x^2 < 0$

$$x^3 - 4x^2 < 0$$

$$x^2(x - 4) < 0$$

\* First, factor out what they have in common. Write it down and then make a set of ( ) and write down in it what's left after factoring.

$$x^2 < 0$$

$$\sqrt{x^2} < \sqrt{0}$$

$$x < 0$$

$$x - 4 < 0$$

$$\frac{+4 \quad +4}{x < 4}$$

\* Then write down each expression and then write down the inequality sign and 0.

\* now solve each one for x.

$-\infty$

0

4

$\infty$

\* Now write down those numbers.

$$(-\infty, 0)$$

$$(0, 4)$$

$$(4, \infty)$$

\* Write down the intervals between each of those numbers.

$\downarrow$   
-1

$\downarrow$   
1

$\downarrow$   
5

\* Write down a number that could go in that interval

$$(-1)^3 - 4(-1)^2 = -5$$

$$(1)^2 - 4(1)^2 = -3$$

$$(5)^3 - 4(5)^2 = 25$$

\* Now replace each one of those numbers back in the original equation replace each x with that number.

\* Write down the answer and then since the original equation says  $< 0$ , circle the number if it is a negative number.

$$(-\infty, 0) \cup (0, 4)$$

Now write down the interval that goes with the numbers you circled. That is your answer. Be sure and place a U between set of ( ).

4. Solve the following inequality.

$$x^3 - 2x^2 - 63x < 0$$

$$x^3 - 2x^2 - 63x < 0$$

$$x(x^2 - 2x - 63) < 0$$

$$\underline{x < 0}$$

$$x^2 - 2x - 63 < 0$$

$\uparrow$  a       $\uparrow$  b       $\uparrow$  c

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

← Quadratic function

$$\frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-63)}}{2(1)}$$

$$\underline{9} < 0, \quad -7 < 0$$

$$-\infty \quad -7 \quad 0 \quad 9 \quad \infty$$

$$(-\infty, -7)$$

$$(-7, 0)$$

$$(0, 9)$$

$$(9, \infty)$$

$$\downarrow$$

$$-8$$

$$\downarrow$$

$$(-8)^3 - 2(-8)^2 - 63(-8)$$

$$\downarrow$$

$$\underline{-136}$$

$$\downarrow$$

$$-1$$

$$\downarrow$$

$$(-1)^3 - 2(-1)^2 - 63(-1)$$

$$\downarrow$$

$$60$$

$$\downarrow$$

$$1$$

$$\downarrow$$

$$(1)^3 - 2(1)^2 - 63(1)$$

$$\downarrow$$

$$\underline{-64}$$

$$\downarrow$$

$$10$$

$$\downarrow$$

$$(10)^3 - 2(10)^2 - 63(10)$$

$$\downarrow$$

$$170$$

\* Now write down those numbers in order from least to greatest.

\* Write down the intervals between each of those numbers.

\* Write down a number that could go in that interval

\* Now replace each one of those numbers back in the original equation replace each x with that number.

\* Write down the answer and then since the original equation says  $< 0$ , circle the number if it is a negative number.

$$\underline{(-\infty, -7) \cup (0, 9)}$$

Now write down the interval that goes with the numbers you circled. That is your answer. Be sure and place a U between set of ( ).

5. Solve the following inequality.

$$x^4 > 169x^2$$

$$x^4 > 169x^2$$

$$\underline{-169x^2 \quad -169x^2}$$

$$x^4 - 169x^2 > 0$$

← factor

$$x^2(x^2 - 169) > 0$$

$$x^2 > 0$$

$$\sqrt{x^2} > \sqrt{0}$$

$$x > 0$$

$$x^2 - 169 > 0$$

$$\underline{+169 \quad +169}$$

$$x^2 > 169$$

$$\sqrt{x^2} > \sqrt{169}$$

$$x > 13, -13$$

\* First move the term on the right to the left by adding or subtracting.

\* Then factor out what they have in common. Write it down and then make a set of ( ) and write down in it what's left after factoring.

\* Then write down each expression and then write down the inequality sign and 0.

\* now solve each one for x.

$$-\infty \quad -13 \quad 0 \quad 13 \quad \infty$$

\* Now write down those numbers in order from least to greatest.

$$(-\infty, -13) \quad (-13, 0) \quad (0, 13) \quad (13, \infty)$$

\* Write down the intervals between each of those numbers.

$$-14 \quad -1 \quad 1 \quad 14$$

$$(-14)^4 - 169(-14)^2 \quad (-1)^4 - 169(-1)^2 \quad (1)^4 - 169(1)^2 \quad (14)^4 - 169(14)^2$$

\* Write down a number that could go in that interval

$$5292 \quad -168 \quad -168 \quad 5292$$

\* Write down the answer and then since the original equation says  $> 0$ , circle the number if it is a positive number.

$$(-\infty, -13) \cup (13, \infty)$$

Now write down the interval that goes with the numbers you circled. That is your answer. Be sure and place a U between set of ( ).

6. Solve the following inequality.

$$x^4 > 1$$

$$x^4 > 1$$

$$\sqrt[4]{x^4} > \sqrt[4]{1}$$

$$x > 1, -1$$

\* Take the 4th root of each side.

\* In the calculator, type in 4, then push the green 2nd button, then the  $\wedge$  button, and then the number (here it is 1).

\* Remember to write down the + and - of the number.

$-\infty$	$-1$	$ $	$\infty$
↓	↓		↓
$(-\infty, -1)$	$(-1, 1)$		$(1, \infty)$
↓	↓		↓
$-2$	$0$		$2$
↓	↓		↓
$(-2)^2 > 1$	$(0)^2 > 1$		$(2)^2 > 1$
↓	↓		↓
$4 > 1$	$0 > 1$		$4 > 1$

\* Now write down those numbers in order from least to greatest.

\* Write down the intervals between each of those numbers.

\* Write down a number that could go in that interval

\* Write down the answer and circle the one(s) that are true.

$$(-\infty, -1) \cup (1, \infty)$$

Now write down the interval that goes with the numbers you circled. That is your answer. Be sure and place a U between set of ( ).

7. Solve the following inequality.  $3x^3 > 18x^2$

$$\begin{array}{l}
 3x^3 > 18x^2 \\
 \underline{-18x^2 \quad -18x^2} \\
 3x^3 - 18x^2 > 0 \\
 x^2(3x - 18) > 0 \\
 \swarrow \quad \searrow \\
 x^2 > 0 \qquad 3x - 18 > 0 \\
 \sqrt{x^2} > \sqrt{0} \qquad \underline{\quad +18 \quad +18 \quad} \\
 x > 0 \qquad \qquad \frac{3x}{3} > \frac{18}{3} \\
 \qquad \qquad \qquad x > 6
 \end{array}$$

\* First move the term on the right to the left by adding or subtracting.

\* Then factor out what they have in common. Write it down and then make a set of ( ) and write down in it what's left after factoring.

\* Then write down each expression and then write down the inequality sign and 0.

\* now solve each one for x.

$$\begin{array}{ccccc}
 -\infty & & 0 & & 6 & & \infty \\
 \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 (-\infty, 0) & & (0, 6) & & (6, \infty) & & \\
 \downarrow & & \downarrow & & \downarrow & & \\
 -1 & & 1 & & 7 & & \\
 \downarrow & & \downarrow & & \downarrow & & \\
 3(-1)^3 > 18(-1)^2 & & 3(1)^3 > 18(1)^2 & & 3(7)^3 > 18(7)^2 & & \\
 \downarrow & & \downarrow & & \downarrow & & \\
 -3 > 18 & & 3 > 18 & & 1029 > 882 & & 
 \end{array}$$

\* Now write down those numbers in order from least to greatest.

\* Write down the intervals between each of those numbers.

\* Write down a number that could go in that interval

\* Write down the answer and circle the one(s) that are true.

$$(6, \infty)$$

Now write down the interval that goes with the numbers you circled. That is your answer. Be sure and place a U between set of ( ).

8. Solve the following inequality.

$$\frac{x+2}{x-4} < 0$$

$$\frac{x+2}{x-4} < 0$$

$$\begin{array}{r} x+2 < 0 \\ -2 \quad -2 \\ \hline x < -2 \end{array}$$

$$\begin{array}{r} x-4 < 0 \\ +4 \quad +4 \\ \hline x < 4 \end{array}$$

\* You will make 2 equations. One with the top part of the fraction, and one with the bottom part of the fraction.

\* Then write down each expression and then write down the inequality sign and 0.

\* now solve each one for x.

$-\infty$ $\downarrow$ $(-\infty, -2)$ $\downarrow$ $-3$ $\downarrow$ $\frac{-3+2}{-3-4} < 0$ $\frac{1}{7} < 0$	$-2$ $\downarrow$ $(-2, 4)$ $\downarrow$ $0$ $\downarrow$ $\frac{3+2}{3-4} < 0$ $-5 < 0$	$4$ $\downarrow$ $(4, \infty)$ $\downarrow$ $5$ $\downarrow$ $\frac{5+2}{5-4} < 0$ $7 < 0$
--------------------------------------------------------------------------------------------------------------------------------------	---------------------------------------------------------------------------------------------------------------	-----------------------------------------------------------------------------------------------------------------

\* Now write down those numbers in order from least to greatest.

\* Write down the intervals between each of those numbers.

\* Write down a number that could go in that interval

\* Write down the answer and circle the one(s) that are true.

$$(-2, 4)$$

Now write down the interval that goes with the numbers you circled. That is your answer. Be sure and place a U between set of ( ).



9. Solve the following inequality.

$$\frac{(x-9)(x+5)}{x} \leq 0$$

$$\begin{array}{r} x-9 \leq 0 \\ +9 \quad +9 \\ \hline x \leq 9 \\ = \end{array}$$

$$\begin{array}{r} x+5 \leq 0 \\ -5 \quad -5 \\ \hline x \leq -5 \\ = \end{array}$$

$$x \leq 0$$

\* Make equations from both the top and bottom parts of fraction.

\* Write down each expression and then write down the inequality sign and 0.

\* now solve each one for x.

$$-\infty \quad -5 \quad 0 \quad 9 \quad \infty$$

\* Now write down those numbers in order from least to greatest.

$$(-\infty, -5) \quad (-5, 0) \quad (0, 9) \quad (9, \infty)$$

\* Write down the intervals between each of those numbers.

$$\begin{array}{c} \downarrow \\ -6 \\ \downarrow \\ \frac{(-6-9)(-6+5)}{-6} \end{array}$$

\* Write down a number that could go in that interval

$$\frac{(-6-9)(-6+5)}{-6}$$

$$\frac{(1-9)(1+5)}{1}$$

$$\frac{(10-9)(10+5)}{10}$$

$$\frac{(-1-9)(-1+5)}{-1}$$

$$\frac{-5}{2} \leq 0$$

$$40 \leq 0$$

$$-48 \leq 0$$

$$\frac{3}{2} \leq 0$$

\* Write down the answer and circle the one(s) that are true.

$$(-\infty, -5) \cup (0, 9)$$

Now write down the interval that goes with the numbers you circled. That is your answer. Be sure and place a U between set of ( ).

10. Solve the following inequality.

$$\frac{(x-5)^2}{x^2-1} \geq 0$$

$$\begin{array}{r} x-5 \geq 0 \\ +5 \quad +5 \\ \hline x \geq 5 \end{array}$$

$$\begin{array}{r} x^2-1 \geq 0 \\ +1 \quad +1 \\ \hline x^2 \geq 1 \\ \sqrt{x^2} \geq \sqrt{1} \\ x \geq 1, -1 \end{array}$$

\* Make equations from both the top and bottom parts of fraction.

\* Write down each expression and then write down the inequality sign and 0.

\* now solve each one for x.

$$\begin{array}{ccccccc} -\infty & & -1 & & 1 & & 5 & & \infty \\ & \downarrow & & \downarrow & & \downarrow & & \downarrow & \end{array}$$

$$(-\infty, -1) \quad (-1, 1) \quad (1, 5) \quad (5, \infty)$$

\* Now write down those numbers in order from least to greatest.

\* Write down the intervals between each of those numbers.

\* Write down a number that could go in that interval

$$\begin{array}{c} \downarrow \\ -2 \\ \downarrow \\ \frac{(-2-5)^2}{(-2)^2-1} \end{array}$$

$$\begin{array}{c} \downarrow \\ 0 \\ \downarrow \\ \frac{(0-5)^2}{(0)^2-1} \end{array}$$

$$\begin{array}{c} \downarrow \\ 2 \\ \downarrow \\ \frac{(2-5)^2}{2^2-1} \end{array}$$

$$\begin{array}{c} \downarrow \\ 6 \\ \downarrow \\ \frac{(6-5)^2}{6^2-1} \end{array}$$

$$\frac{49}{3} \geq 0$$

$$-25 \geq 0$$

$$3 \geq 0$$

$$\frac{1}{35} \geq 0$$

\* Since these two are side by side (continuous) we combine them.

\* Write down the answer and circle the one(s) that are true.

$$(-\infty, -1) \cup (1, \infty)$$

Now write down the interval that goes with the numbers you circled. That is your answer. Be sure and place a U between set of ( ).