

Chapter 6.1 Notes
Composite Functions

1. Given $f(x) = 3x$ and $g(x) = 3x^2 + 5$, find the following expressions.

$$f(x) = 3x \quad g(x) = 3x^2 + 5$$

a.) $(f \circ g)(4) =$

$$\begin{aligned} g(4) &= 3x^2 + 5 \\ &= 3(4)^2 + 5 \quad \leftarrow \text{Type into calculator} \\ &= 53 \end{aligned}$$

$$\begin{aligned} f(53) &= 3x \\ &= 3(53) \quad \leftarrow \text{Type into calculator} \\ &= 159 \end{aligned}$$

c.) $f \circ f(1) =$

$$\begin{aligned} f(1) &= 3x \\ &= 3(1) \\ &= 3 \end{aligned}$$

$$\begin{aligned} f(3) &= 3x \\ &= 3(3) \\ &= 9 \end{aligned}$$

b.) $(g \circ f)(2) =$

$$\begin{aligned} f(2) &= 3x \\ &= 3(2) \quad \leftarrow \text{Type into calculator} \\ &= 6 \end{aligned}$$

$$\begin{aligned} g(6) &= 3x^2 + 5 \\ &= 3(6)^2 + 5 \quad \leftarrow \text{Type into calculator} \\ &= 113 \end{aligned}$$

d.) $g \circ g(0) =$

$$\begin{aligned} g(0) &= 3x^2 + 5 \\ &= 3(0)^2 + 5 \\ &= 5 \end{aligned}$$

$$\begin{aligned} g(5) &= 3x^2 + 5 \\ &= 3(5)^2 + 5 \\ &= 80 \end{aligned}$$

2. Given $f(x) = 5x^2 - 4$ and $g(x) = 5 - \frac{1}{2}x^2$, find the following expressions.

$$f(x) = 5x^2 - 4 \quad g(x) = 5 - \frac{1}{2}x^2$$

a.) $(f \circ g)(4) =$

$$\begin{aligned} g(4) &= 5 - \frac{1}{2}x^2 \\ &= 5 - \frac{1}{2}(4)^2 \end{aligned}$$

← Type into calculator

$$\begin{aligned} &= -3 \\ f(-3) &= 5x^2 - 4 \\ &= 5(-3)^2 - 4 \end{aligned}$$

← Type into calculator

$$\boxed{41}$$

c.) $(f \circ f)(1) =$

$$\begin{aligned} f(1) &= 5x^2 - 4 \\ &= 5(1)^2 - 4 \end{aligned}$$

$$\begin{aligned} &= 1 \\ f(1) &= 5x^2 - 4 \\ &= 5(1)^2 - 4 \end{aligned}$$

$$\boxed{1}$$

b.) $(g \circ f)(2) =$

$$\begin{aligned} f(2) &= 5x^2 - 4 \\ &= 5(2)^2 - 4 \end{aligned}$$

← Type into calculator

$$\begin{aligned} &= 16 \\ g(16) &= 5 - \frac{1}{2}x^2 \end{aligned}$$

$$5 - \frac{1}{2}(16)^2$$

← Type into calculator

$$\boxed{-123}$$

d.) $(g \circ g)(0) =$

$$\begin{aligned} g(0) &= 5 - \frac{1}{2}x^2 \\ &= 5 - \frac{1}{2}(0)^2 \end{aligned}$$

$$\begin{aligned} &= 5 \\ g(5) &= 5 - \frac{1}{2}x^2 \end{aligned}$$

$$5 - \frac{1}{2}(5)^2$$

$$\boxed{-\frac{15}{2}}$$

3. Given $f(x) = 6\sqrt{x}$ and $g(x) = 2x$, find the following expressions.

$$f(x) = 6\sqrt{x} \quad g(x) = 2x$$

a.) $(f \circ g)(4) =$

$$g(4) = 2x$$

$$= 2(4)$$

$$= 8$$

$$f(8) = 6\sqrt{x}$$

$$6\sqrt{8}$$

$$\boxed{12\sqrt{2}}$$

← Type into calculator

← Type into calculator

c.) $(f \circ f)(1) =$

$$f(1) = 6\sqrt{x}$$

$$6\sqrt{1}$$

$$f(6\sqrt{1}) = 6\sqrt{x}$$

$$6\sqrt{6\sqrt{1}}$$

$$\boxed{6\sqrt{6}}$$

b.) $(g \circ f)(2) =$

$$f(2) = 6\sqrt{x}$$

$$6\sqrt{2}$$

$$g(6\sqrt{2}) = 2x$$

$$2(6\sqrt{2})$$

$$\boxed{12\sqrt{2}}$$

← Type into calculator

← Type into calculator

d.) $(g \circ g)(0) =$

$$g(0) = 2x$$

$$2(0)$$

$$0$$

$$g(0) = 2(0)$$

$$\boxed{0}$$

4. Given $f(x) = |x|$ and $g(x) = \frac{2}{x^2+1}$, find the following expressions.

$$f(x) = |x| \quad g(x) = \frac{2}{x^2+1}$$

a.) $(f \circ g)(4) =$

$$g(4) = \frac{2}{x^2+1}$$

$$= \frac{2}{(4)^2+1}$$

$$= \frac{2}{17}$$

$$f\left(\frac{2}{17}\right) = |x|$$

$$= \left|\frac{2}{17}\right|$$

$$\boxed{\frac{2}{17}}$$

b.) $(g \circ f)(2) =$

$$f(2) = |x|$$

$$= |2|$$

$$= 2$$

$$g(2) = \frac{2}{x^2+1}$$

$$= \frac{2}{2^2+1}$$

$$\boxed{\frac{2}{5}}$$

c.) $(f \circ f)(1) =$

$$f(1) = |x|$$

$$= |1|$$

$$= 1$$

$$f(1) = |x|$$

$$= |1|$$

$$\boxed{1}$$

d.) $(g \circ g)(0) =$

$$g(0) = \frac{2}{x^2+1}$$

$$= \frac{2}{0^2+1}$$

$$= 2$$

$$g(2) = \frac{2}{x^2+1}$$

$$= \frac{2}{2^2+1}$$

$$\boxed{\frac{2}{5}}$$

5. If $f(x) = x + 4$ and $g(x) = \frac{3}{x}$, then which of the following does $(f \circ g)(x)$ equal?

$$f(x) = \sqrt{x+4} \quad g(x) = \frac{3}{x}$$

$$(f \circ g)(x) =$$

$$g(x) = \frac{3}{x}$$

$$= \frac{3}{x}$$

$$f\left(\frac{3}{x}\right) = \sqrt{x+4}$$

$$\sqrt{\frac{3}{x} + 4}$$

6. Find functions f and g so that $f \circ g = H$.

$$H(x) = \sqrt{x^2 + 17}$$

$$f \circ g = H$$

$$\rightarrow x^2 + 17$$

$$\rightarrow \sqrt{x}$$

$$= \sqrt{x^2 + 17}$$

$$\text{So: } \begin{cases} f(x) = \sqrt{x} \\ g(x) = x^2 + 17 \end{cases}$$

* To go from H to $f(x)$ and $g(x)$ you must think backwards from what you were doing in the previous questions.

* The expression under the $\sqrt{\quad}$ will be the $g(x)$.

* The $\sqrt{\quad}$ will be the $f(x)$.

* The $f(x)$ will be what's on the outside, ex.

$$\sqrt{x}, (x)^2, |x|$$

* The $g(x)$ will be what's inside the $\sqrt{\quad}$, $(\quad)^2$, or $||$.

7. Find functions f and g so that $f \circ g = H$.

$$H = (9x+1)^6$$

$$f \circ g = H$$

$$\begin{array}{l} \hookrightarrow 9x+1 \\ \searrow \downarrow \\ \rightarrow (x)^6 \end{array}$$

$$= (9x+1)^6$$

so: $f(x) = (x)^6$
 $g(x) = 9x+1$

- * To go from H to $f(x)$ and $g(x)$ you must think backwards from what you were doing in the previous questions.
- * The expression inside the $()$ will be the $g(x)$.
- * The $()^6$ will be the $f(x)$.

8. Find functions f and g so that $f \circ g = H$.

$$H(x) = |9x+2|$$

$$f \circ g = H$$

$$\begin{array}{l} \hookrightarrow 9x+2 \\ \searrow \downarrow \\ \rightarrow |x| \end{array}$$

$$= |9x+2|$$

so: $f(x) = |x|$
 $g(x) = 9x+2$

- * To go from H to $f(x)$ and $g(x)$ you must think backwards from what you were doing in the previous questions.
- * The expression inside the $| |$ will be the $g(x)$.
- * The $| |$ will be the $f(x)$.

