

Chapter 6.4 Notes  
Logarithmic Functions

\* Logarithms is just another way of thinking about exponents.

\*  $2^x = 16 \rightarrow x = 4$

\* This asks "2 raised to what power = 16"

\* Logarithms say the same thing just written a little different.

\*  $\log_2 16 = x$

↑  
what power do we raise 2 to to get 16

ex.)  $\log_2 64 \rightarrow \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_6 = 64$  so  $2^6 = 64$   
or  
 $\log_2 64 = 6$

$\log_b(a) = c \iff b^c = a$

- $b$  = base
- $c$  = exponent
- $a$  = argument

1. Evaluate the expression without using a calculator.

$\log_8 1$

• This asks: 8 to what power = 1

$8^x = 1$

$8^0 = 1$

So ... 0

$\log_b(a) = c \rightarrow b^c = a$   
↑                      ↑  
b to the c power = a

2. Find the exact value of the logarithm without using a calculator.

$$\log_8 64$$

· This asks: 8 to what power = 64

$$8^x = 64$$

$$\begin{array}{ccc} 8^x & = & 8^2 \\ \downarrow & & \downarrow \\ x & = & 2 \end{array}$$

$$\begin{array}{c} 8 \cdot 8 = 64 \\ \text{' 2} \\ 8^2 \end{array}$$

\* Want to get bases the same on both sides of =, so that exponents will = each other.

3. Find the value of the logarithmic expression.

$$\log_2 \left( \frac{1}{16} \right)$$

· This asks: 2 to what power =  $\frac{1}{16}$

$$2^x = \frac{1}{16}$$

Bases are the same so exponents = each other.

$$\begin{array}{ccc} 2^x & = & 2^{-4} \\ \downarrow & & \downarrow \\ x & = & -4 \end{array}$$

$$\begin{array}{c} 2 \cdot 2 \cdot 2 \cdot 2 = 16 \\ \text{' 2 3 4} \end{array}$$

$$2^4 = 16$$

$$2^{-4} = \frac{1}{16}$$

\* If going from a whole number to a fraction, the exponent will be negative.

4. Find the exact value of the logarithm without using a calculator.

$$\log_6 \sqrt{6}$$

This asks 6 raised to what power =  $\sqrt{6}$ .

$$6^x = \sqrt{6}$$

$$6^x = 6^{\frac{1}{2}}$$

$$x = \frac{1}{2}$$

$$\sqrt[2]{6} = 6^{\frac{1}{2}}$$

5. Change the exponential statement to an equivalent statement involving a logarithm.

$$16 = 4^2$$
$$\log_4(16) = 2$$

$$\log_4(16) = 2$$

$$a = b^c \rightarrow \log_b(a) = c$$

\* Just replace the letters with your numbers.

6. Change the exponential statement to an equivalent statement involving a logarithm.

$$1.3 = a^3$$

Diagram showing the mapping from the exponential form to the logarithmic form:

- The base  $a$  in the exponential form maps to the base  $b$  in the logarithmic form.
- The exponent  $3$  in the exponential form maps to the result  $c$  in the logarithmic form.
- The result  $1.3$  in the exponential form maps to the argument  $a$  in the logarithmic form.

$$\log_b(a) = c$$

$$\log_a(1.3) = 3$$

$$a = b^c \rightarrow \log_b(a) = c$$

\* Just replace the letters with your numbers.

7. Change the logarithmic statement to an equivalent statement involving a exponent.

$$\log_4 16 = 2$$

Diagram showing the mapping from the logarithmic form to the exponential form:

- The base  $4$  in the logarithmic form maps to the base  $b$  in the exponential form.
- The argument  $16$  in the logarithmic form maps to the result  $a$  in the exponential form.
- The result  $2$  in the logarithmic form maps to the exponent  $c$  in the exponential form.

$$b^c = a$$

$$4^2 = 16$$

$$\log_b(a) = c \rightarrow b^c = a$$

\* Just replace the letters with your numbers.

8. Change the logarithmic statement to an equivalent statement involving a exponent.

$$\log_3 81 = x$$

Diagram showing the mapping from the logarithmic form to the exponential form:

- The base  $3$  in the logarithmic form maps to the base  $b$  in the exponential form.
- The argument  $81$  in the logarithmic form maps to the result  $a$  in the exponential form.
- The result  $x$  in the logarithmic form maps to the exponent  $c$  in the exponential form.

$$b^c = a$$

$$3^x = 81$$

$$\log_b(a) = c \rightarrow b^c = a$$

\* Just replace the letters with your numbers.

9. Solve the following equation. Write the answer in terms of the natural logarithm.

$$e^{6x} = 5$$

$$\log_b(a) = c$$

Diagram showing the relationship between the equation  $e^{6x} = 5$  and the general logarithmic form  $\log_b(a) = c$ . Arrows indicate:  $e$  maps to  $b$ ,  $6x$  maps to  $c$ , and  $5$  maps to  $a$ .

$$b^c = a \rightarrow \log_b(a) = c$$

$$\log_e(5) = 6x$$

$$\frac{\ln 5}{6} = \frac{6x}{6}$$

\* Replace  $\log_e$  with  $\ln$

← solve for  $x$ .

$$\boxed{\frac{\ln 5}{6} = x}$$

10. Solve the following equation. Write the answer in terms of the natural logarithm.

$$e^{5x+9} = 6$$

$$\log_b(a) = c$$

Diagram showing the relationship between the equation  $e^{5x+9} = 6$  and the general logarithmic form  $\log_b(a) = c$ . Arrows indicate:  $e$  maps to  $b$ ,  $5x+9$  maps to  $c$ , and  $6$  maps to  $a$ .

$$b^c = a \rightarrow \log_b(a) = c$$

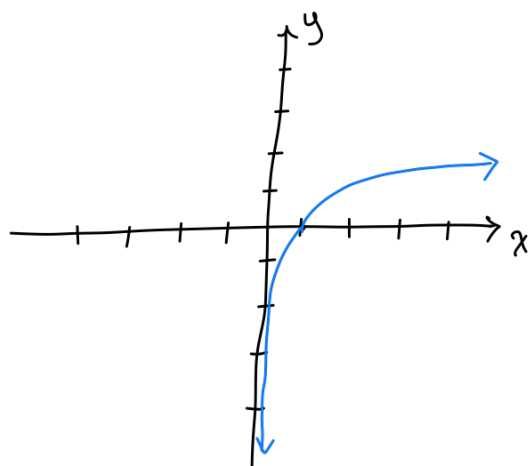
$$\log_e(6) = 5x+9$$

\* Replace  $\log_e$  with  $\ln$

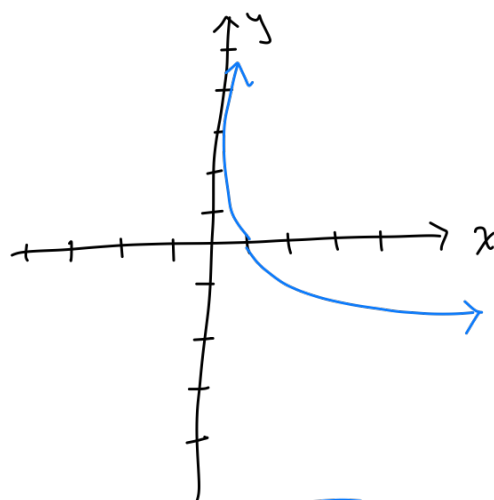
$$\begin{array}{r} \ln 6 = 5x+9 \\ -9 \qquad -9 \\ \hline \ln 6 - 9 = 5x \\ \underline{\quad 5 \quad} \quad \underline{\quad 5 \quad} \end{array}$$

$$\boxed{\frac{\ln 6 - 9}{5} = x}$$

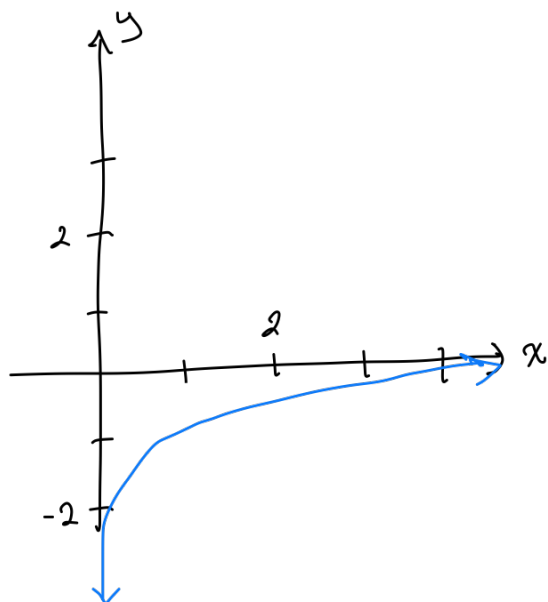
11. The graph of a logarithmic function is given. Select the function for each graph from the given options.



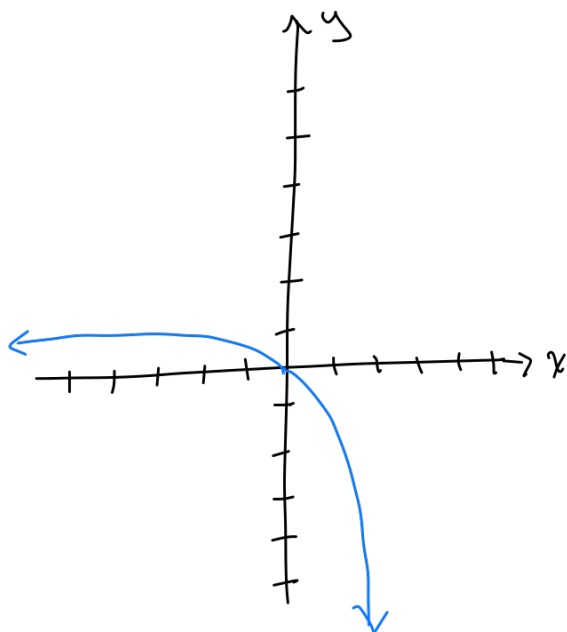
$$\log_8 x$$



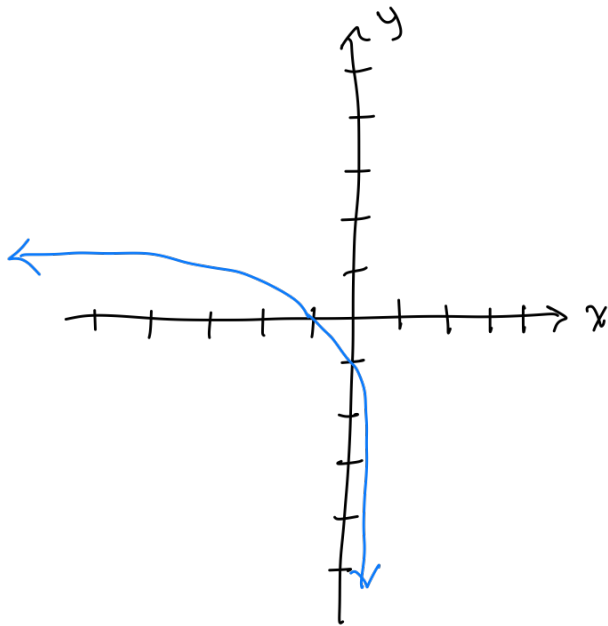
$$-\log_8 x$$



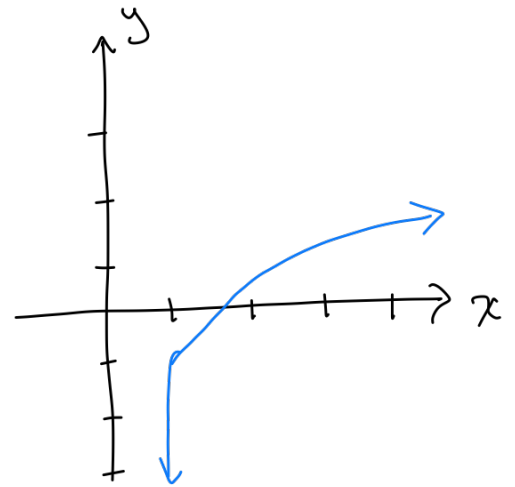
$$\log_8 x - 1$$



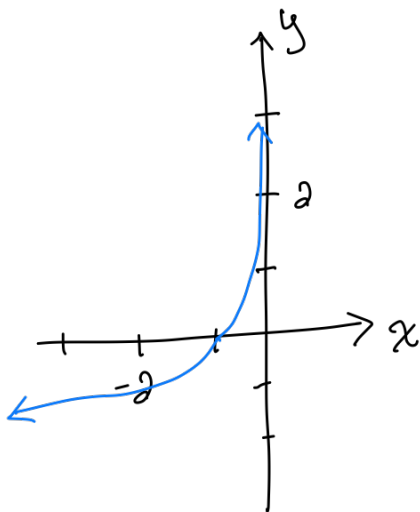
$$\log_8 (1-x)$$



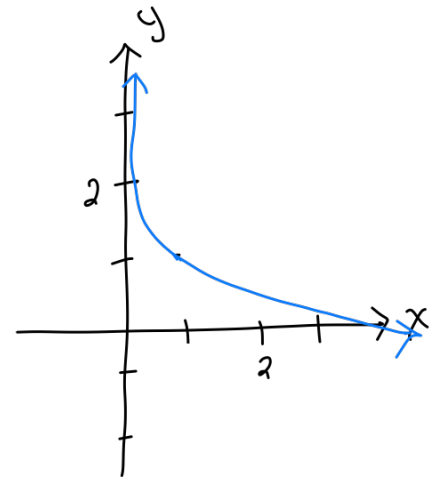
$$\log_8(-x)$$



$$\log_8(x-1)$$



$$-\log_8(-x)$$



$$1 - \log_8 x$$