Chapter 8.1 Notes
Systems of linear Equations

* system of equations — a collection of 2 or more equations, each containing one or more variables.

* Consistent — system of equation has at least one solution.
* Inconsistent — system of equation has no solution

* Linear equations may:
  *intersect — has 1 solution, consistent and independent
  *parallel — no solution, inconsistent
  * coincident — lines lie on top of each other; infinitely many solutions, consistent, & dependent.
* solving systems of equation:
  1. Method of substitution
  2. Method of elimination

1. Solve the system of equations. If the system has no solution, say that it is inconsistent.

$$\begin{cases} 3x - 5y = -7 & \leftarrow 1^{st} \text{ equation} \\ 10x + y = 12 & \leftarrow 2^{nd} \text{ equation} \end{cases}$$

$5(10x + y = 12)$

$$\begin{array}{r} 50x + 5y = 60 \\ + \quad 3x - 5y = -7 \quad \leftarrow 1^{st} \text{ equation} \\ \hline 53x = 53 \\ \frac{53x}{53} = \frac{53}{53} \\ x = 1 \end{array}$$

* If we multiply the 2nd equation by 5, we can eliminate the y's.
* Now add the 2 equations together.
  - Then solve for x.

$3x - 5y = -7 \quad \leftarrow 1^{st}$ equation

$3(1) - 5y = -7$

$3 - 5y = -7$

$-3 \qquad -3$

$-5y = -10$

$\frac{-5y}{-5} = \frac{-10}{-5}$

$y = 2$

* Now replace the # for x back into the 1st equation.
  - Solve for y.

$(1, 2)$

$x \quad y$

* This gives you your ordered pair.

2. Solve the system of equations. If the system has no solution, say that it is inconsistent.

$$\begin{cases} 4x + 5y = 24 & \leftarrow 1^{st} \text{ equation} \\ x - y = \frac{3}{2} & \leftarrow 2^{nd} \text{ equation} \end{cases}$$

$5\left(x - y = \frac{3}{2}\right)$

$$5x - 5y = \frac{15}{2}$$

$+ \quad 4x + 5y = 24 \quad \leftarrow 1^{st} \text{ equation}$

$$\frac{9x}{9} = \frac{\frac{63}{2}}{9}$$

$$x = \frac{7}{2}$$

$4x + 5y = 24 \quad \leftarrow 1^{st} \text{ equation}$

$$4\left(\frac{7}{2}\right) + 5y = 24$$

$$14 + 5y = 24$$

$$-14 \qquad -14$$

$$\frac{5y}{5} = \frac{10}{5}$$

$$y = 2$$

$$\left(\frac{7}{2}, 2\right)$$

$x \quad y$

* If we multiply the 2nd equation by 5, we can eliminate the y's.
* Now add the 2 equations together.
  - Then solve for x.
* Now replace the # for x back into the 1st equation.
  - Solve for y.
* This gives you your ordered pair.

3. Solve the system of equations. If the system has no solution, say that it is inconsistent.

$$\begin{cases} 6x - 3y = 6 & \leftarrow 1^{st} \text{ equation} \\ 24x + 3y = 29 & \leftarrow 2^{nd} \text{ equation} \end{cases}$$

$$+ \quad \begin{array}{r} 6x - 3y = 6 \\ 24x + 3y = 29 \end{array}$$

$$\overline{\qquad\qquad}$$

$$\begin{array}{r} 30x = 35 \\ \overline{30} \quad \overline{30} \end{array}$$

$$x = \frac{7}{6}$$

* Since the y's have opposite #'s, we can eliminate them.

* So add the 2 equations together.
- Then solve for x.

$$6x - 3y = 6 \quad \leftarrow 1^{st} \text{ equation}$$

$$6\left(\frac{7}{6}\right) - 3y = 6$$

$$\begin{array}{r} 7 - 3y = 6 \\ -7 \quad\quad -7 \\ \overline{\qquad\qquad} \\ -3y = -1 \\ \overline{-3} \quad \overline{-3} \end{array}$$

$$y = \frac{1}{3}$$

* Now replace the # for x back into the 1st equation.
- Solve for y.

* This gives you your ordered pair.

$$\left(\frac{7}{6}, \frac{1}{3}\right)$$

$\uparrow$ x $\quad$ $\uparrow$ y

4. Solve the system of equations. If the system has no solution, say that it is inconsistent. Graph the lines.

$$\begin{cases} x+y=9 \\ x-y=1 \end{cases}$$

$$\begin{array}{r} x+y=9 \\ +\ x-y=1 \\ \hline 2x=10 \\ \frac{2}{2}\quad\frac{}{2} \\ x=5 \end{array}$$

* Here we can eliminate the "y"s, because they are opposite.
* So, eliminate the "y"s and then combine the other terms.
* Now solve that equation for x, by dividing both sides by the number in front of x.
* Now, pick one of the original equations and replace the x with your number. Then solve for y.
* Then make an ordered pair, placing the number for x first, and then the number for y.

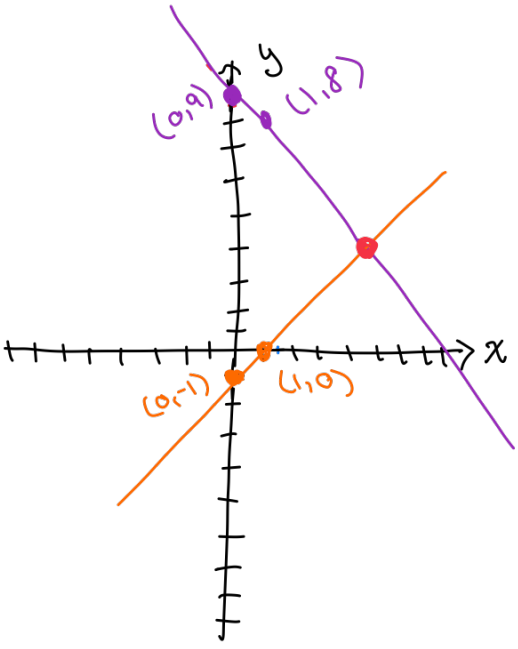
$x+y=9$ ← 1st equation

$$\begin{array}{r} 5+y=9 \\ -5\quad -5 \\ \hline y=4 \end{array}$$

$(5,4)$

x y

* To graph:
* Take the 1st equation and solve it for y. Then replace the x with 0 and solve for y. This gives you one point.
* Then use the same equation and replace x with another number, I would suggest 1, and then solve for y. This is the second point.
* Now take 2nd equation and solve for y. Then replace the x with 0 and solve for y. This gives you one point.
* Then use the same equation and replace x with another number, I would suggest 1. Then solve for y. This is the 2nd point.



$x+y=9$

$-x \quad -x$

$y=-x+9$

Replace x with 0: $y=-0+9$, $y=9$, $(0,9)$

Replace x with 1: $y=-1+9$, $y=8$, $(1,8)$

$x-y=1$

$-x \quad -x$

$-y=-x+1$

$\frac{-y}{-1}=\frac{-x}{-1}+\frac{1}{-1}$

$y=x-1$

Replace x with 0: $y=0-1$, $y=-1$, $(0,-1)$

Replace x with 1: $y=1-1$, $y=0$, $(1,0)$

5. Solve the system of equations. If the system has no solution, say that it is inconsistent. Graph the lines.

$$\begin{cases} 4x - y = 4 \\ 3x + 2y = 14 \end{cases}$$

$2(4x - y = 4)$

$$\begin{array}{r} 8x - 2y = 8 \\ +\ 3x + 2y = 14 \\ \hline 11x = 22 \\ \frac{11x}{11} = \frac{22}{11} \end{array}$$

$x = 2$

$4x - y = 4$ ← 1st equation

$4(2) - y = 4$

$8 - y = 4$

$-8 \quad -8$

$-y = -4$

$\frac{-y}{-1} = \frac{-4}{-1}$

$y = 4$

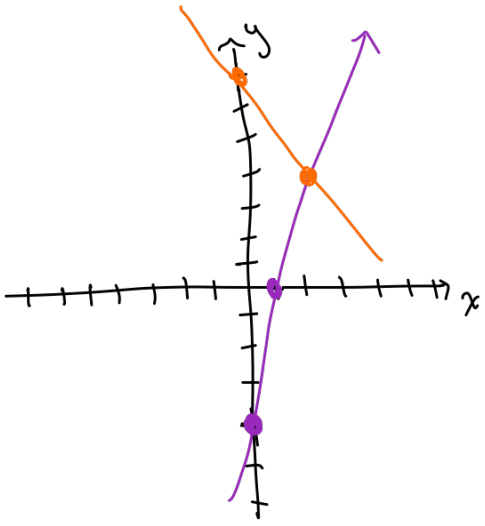
$(2, 4)$

(x, y)

* If we multiply the top equation by 2, then we could eliminate the y's.
* Multiply each term inside the ( ) by 2.
* Now you can eliminate the y's, because they are opposites.
* Then solve for x.

* Now take one of the original equations and replace the x with the number you found.
* Then solve the equation for y.
* Now the solution will be the x number and then the y number.

* To graph:
* Take the 1st equation and solve it for y. Then replace the x with 0 and solve for y. This gives you one point.
* Then use the same equation and replace x with another number, I would suggest 1, and then solve for y. This is the second point.
* Now take 2nd equation and solve for y. Then replace the x with 0 and solve for y. This gives you one point.
* Then use the same equation and replace x with another number. Here it was a fraction, so I would multiple by the same number as the denominator. Then solve for y. This is the 2nd point.



Line 1

$4x - y = 4$

$-4x \quad -4x$

$-y = -4x + 4$

$\frac{-y}{-1} = \frac{-4x}{-1} + \frac{4}{-1}$

$y = 4x - 4$

Replace x with 0: $y = 4(0) - 4$

$(0, -4)$ $y = -4$

Replace x with 1: $y = 4(1) - 4$

$(1, 0)$ $y = 4 - 4 = 0$

Line 2

$3x + 2y = 14$

$-3x \quad -3x$

$\frac{2y}{2} = \frac{-3x}{2} + \frac{14}{2}$

$y = -\frac{3}{2}x + 7$

Replace x with 0: $y = -\frac{3}{2}(0) + 7 = 7$

$(0, 7)$

Replace x with 2: $y = -\frac{3}{2}(2) + 7$

$(2, 4)$ $y = 4$

6. Solve the system of equations. If the system has no solution, say that it is inconsistent. Graph the lines.

$$\begin{cases} 6x = 48 \\ x + 4y = 0 \end{cases}$$ ← 2nd equation

$\frac{6x}{6} = \frac{48}{6}$

$x = 8$

$x + 4y = 0$ ← 2nd equation

$8 + 4y = 0$

$-8 \quad -8$

$\frac{4y}{4} = \frac{-8}{4}$

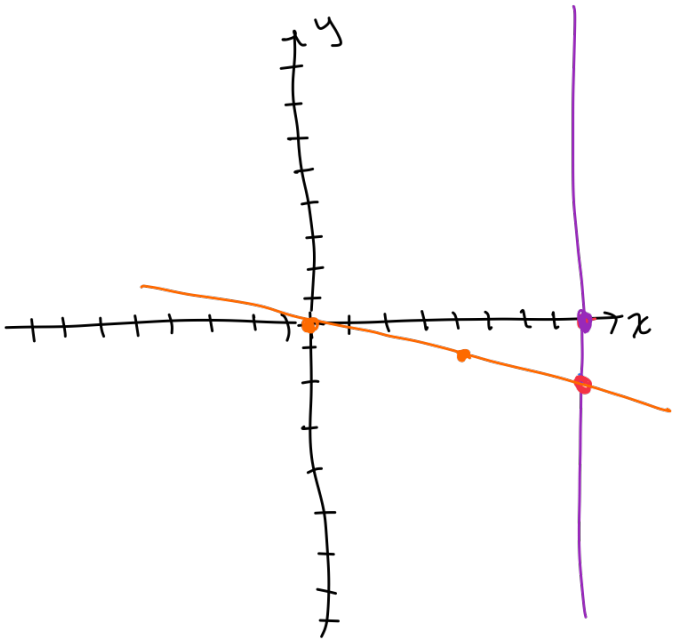
$y = -2$

$(8, -2)$

↑ x ↑ y

* The 1st equation only has one variable. So, we will solve that equation for x.
* Now take one of the original equations and replace the x with the number you found.
* Then solve the equation for y.
* Now the solution will be the x number and then the y number.

* To graph:
* Take the 1st equation and solve it for y. Then replace the x with 0 and solve for y. This gives you one point.
* Then use the same equation and replace x with another number, I would suggest 1, and then solve for y. This is the second point.
* Now take 2nd equation and solve for y. Then replace the x with 0 and solve for y. This gives you one point.
* Then use the same equation and replace x with another number. Here it was a fraction, so I would multiple by the same number as the denominator. Then solve for y. This is the 2nd point.



$\frac{6x}{6} = \frac{48}{6}$

$x = 8$

* This is a vertical line

$x + 4y = 0$

$-x \quad -x$

$\frac{4y}{4} = \frac{-x}{4}$

$y = -\frac{1}{4}x$

Replace x with 0 → (0,0)

$y = -\frac{1}{4}(0)$

$y = 0$

Replace x with 4 → (4, -1)

$y = -\frac{1}{4}(4)$

$y = -1$

7. Solve the system of equations. If the system has no solution, say that it is inconsistent. Graph the lines.

$$\begin{cases} 15x - 10y = 10 & \leftarrow 1^{st} \text{ equation} \\ 8x + 8y = 2 & \leftarrow 2^{nd} \text{ equation} \end{cases}$$

$4(15x - 10y = 10)$

$60x - 40y = 40$

$5(8x + 8y = 2)$

$40x + 40y = 10$

* If we multiply the top equation by 4, and the bottom equation by 5, then we could eliminate the y's.
* Now you can eliminate the y's, because they are opposites.
* Then solve for x.

$$\begin{array}{r} 60x - 40y = 40 \\ +\ 40x + 40y = 10 \\ \hline 100x = 50 \end{array}$$

$\frac{100x}{100} = \frac{50}{100}$

$x = \frac{1}{2}$

* Now take one of the original equations and replace the x with the number you found.
* Then solve the equation for y.
* Now the solution will be the x number and then the y number.

$8x + 8y = 2$ ← 2nd equation

$8\left(\frac{1}{2}\right) + 8y = 2$

$4 + 8y = 2$

$-4 \qquad -4$

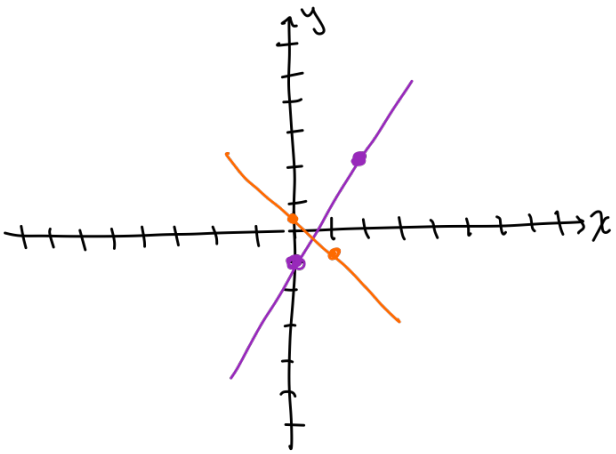
$8y = -2$

$\frac{8y}{8} = \frac{-2}{8}$

$y = -\frac{1}{4}$

$\left(\frac{1}{2}, -\frac{1}{4}\right)$

* To graph:
* Take the 1st equation and solve it for y. Then replace the x with 0 and solve for y. This gives you one point.
* Then use the same equation and replace x with another number, I would suggest 1, and then solve for y. This is the second point.
* Now take 2nd equation and solve for y. Then replace the x with 0 and solve for y. This gives you one point.
* Then use the same equation and replace x with another number. Here it was a fraction, so I would multiple by the same number as the denominator. Then solve for y. This is the 2nd point.



$15x - 10y = 10$

$-15x \qquad -15x$

$-10y = -15x + 10$

$\frac{-10y}{-10} = \frac{-15x}{-10} + \frac{10}{-10}$

$y = \frac{3}{2}x - 1$

Replace x with 0: $(0, -1)$

$y = \frac{3}{2}(0) - 1$

Replace x with 2: $(2, 2)$

$y = \frac{3}{2}(2) - 1$

$y = 2$

$8x + 8y = 2$

$-8x \qquad -8x$

$8y = -8x + 2$

$\frac{8y}{8} = \frac{-8x}{8} + \frac{2}{8}$

$y = -1x + \frac{1}{4}$

Replace x with 0: $(0, \frac{1}{4})$

$y = -1(0) + \frac{1}{4}$

$y = \frac{1}{4}$

Replace x with 1: $(1, -\frac{3}{4})$

$y = -1(1) + \frac{1}{4}$

$y =$

8. Solve the system of equations. If the system has no solution, say that it is inconsistent. Graph the lines.

$\begin{cases} 3x - y = 0 \\ 9x + 3y = 12 \end{cases}$ ← 1st equation

$3(3x - y = 0)$

$+ \quad 9x - 3y = 0$

$9x + 3y = 12$

$18x = 12$

$\frac{18x}{18} = \frac{12}{18}$

$x = \frac{2}{3}$

* If we multiply the top equation by 3, then we could eliminate the y's.
* Multiply each term inside the ( ) by 2.
* Now you can eliminate the y's, because they are opposites.
* Then solve for x.

$3x - y = 0$ ← 1st equation

$3\left(\frac{2}{3}\right) - y = 0$

$2 - y = 0$

$-2 \qquad -2$

$-y = -2$

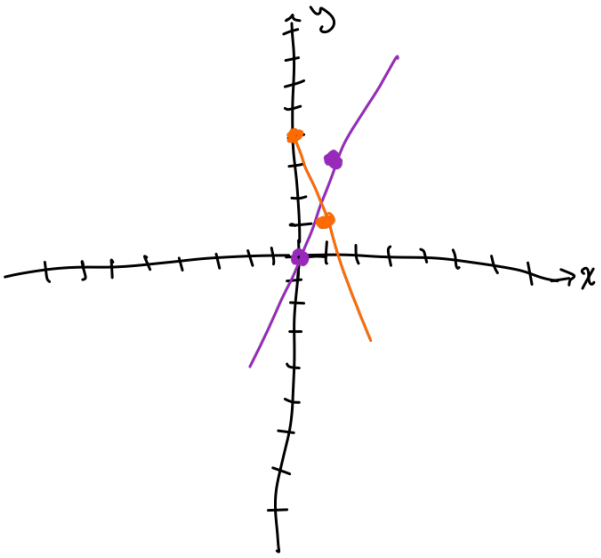
$\frac{-y}{-1} = \frac{-2}{-1}$

$y = 2$

$\left(\frac{2}{3}, 2\right)$

* Now take one of the original equations and replace the x with the number you found.
* Then solve the equation for y.
* Now the solution will be the x number and then the y number.

* To graph:
* Take the 1st equation and solve it for y. Then replace the x with 0 and solve for y. This gives you one point.
* Then use the same equation and replace x with another number, I would suggest 1, and then solve for y. This is the second point.
* Now take 2nd equation and solve for y. Then replace the x with 0 and solve for y. This gives you one point.
* Then use the same equation and replace x with another number. Here it was a fraction, so I would multiple by the same number as the denominator. Then solve for y. This is the 2nd point.



$3x - y = 0$

$-3x \qquad -3x$

$-y = -3x$

$\frac{-y}{-1} = \frac{-3x}{-1}$

$y = 3x$

Replace x with 0: $y = 3(0)$

$(0,0)$ $y = 0$

Replace x with 1: $y = 3(1)$

$(1,3)$ $y = 3$

$9x + 3y = 12$

$-9x \qquad -9x$

$3y = -9x + 12$

$\frac{3y}{3} = \frac{-9x}{3} + \frac{12}{3}$

$y = -3x + 4$

Replace x with 0: $y = -3(0) + 4$

$(0,4)$ $y = 4$

Replace x with 1: $y = -3(1) + 4$

$(1,1)$ $y = 1$