Chapter 8.6 Notes
Systems of Nonlinear Equations

1. Solve the system. Use any method you wish.

$$\begin{cases} y = x + 3 \\ 14x^2 + y^2 = 9 \end{cases}$$

$$14x^2 + y^2 = 9$$

$$14x^2 + (x+3)^2 = 9$$

$$14x^2 + (x+3)(x+3) = 9$$

$$14x^2 + x^2 + 3x + 3x + 9 = 9$$

$$15x^2 + 6x + 9 = 9$$

$$-9 \quad -9$$

$$15x^2 + 6x + 0 = 0$$

$$a = 15 \quad b = 6 \quad c = 0$$

Quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(6) \pm \sqrt{(6)^2 - 4(15)(0)}}{2(15)}$$

$$x = 0, -\frac{2}{5}$$

$$y = x + 3 \qquad y = x + 3$$

$$y = 0 + 3 \qquad y = -\frac{2}{5} + 3$$

$$y = 3 \qquad y = \frac{13}{5}$$

$$(0, 3), \left(-\frac{2}{5}, \frac{13}{5}\right)$$

* Since 1 equation already equals y, we will replace that equation in for the y in the 2nd equation.
* Now we solve for x.
  * since we have something in ( ) squared, we will write it down twice and then use the foil method to get rid of the ( ).
  * Then we combine like terms.
  * we want everything on the left side and zero on the right.
* Now we use the quadratic formula to solve for x.
* Now take one of your original equations and write it down twice.
  * Replace the x in one equation with one of the x's you found, and in the other equation, replace x with the 2nd number you found.
* Now you will make an ordered pair for each one, writing down the x value first and then a comma and then the y value.

2. Solve the system. Use any method you wish.

$$\begin{cases} x+y+2=0 \\ x^2+y^2+4y-3x=-4 \end{cases}$$

* Take the 1st equation and solve for y, by moving everything to the right side but y.
* Now we will write down the 2nd equation and then replace all the y's with the expression you just found.
* Now we solve for x.
    * since we have something in ( ) squared, we will write it down twice and then use the foil method to get rid of the ( ).
    * Then we combine like terms.
    * we want everything on the left side and zero on the right.
* Now we use the quadratic formula to solve for x.

Solve for y

$$\cancel{x}+y+2=0$$
$$-\cancel{x} \qquad\qquad -x$$
$$y+\cancel{2}=-x$$
$$-\cancel{2} \qquad -2$$
$$y=-x-2$$

$$x^2+y^2+4y-3x=-4$$
$$x^2+(-x-2)^2+4(-x-2)-3x=-4$$
$$x^2+(-x-2)(-x-2)+4(x-2)-3x=-4$$
$$x^2+x^2+2x+2x+4+4x-8-3x=-4$$

$$2x^2+5x-4=-\cancel{4}$$
$$+4 \qquad +\cancel{4}$$
$$2x^2+5x+0=0$$
$$a \qquad b \qquad c$$

Quadratic formula

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\frac{-(5)\pm\sqrt{(5)^2-4(2)(0)}}{2(2)}$$

$$x=0,\ -\frac{5}{2}$$

* Now take one of your original equations and write it down twice (go with the simplest one)
    * Replace the x in one equation with one of the x's you found, and in the other equation, replace x with the 2nd number you found.
* Now you will make an ordered pair for each one, writing down the x value first and then a comma and then the y value.

$$x+y+2=0$$
$$0+y+2=0$$
$$y+2=0$$
$$-2 \qquad -2$$
$$y=-2$$

$$x+y+2=0$$
$$-\tfrac{5}{2}+y+2=0$$
$$y-\tfrac{1}{2}=0$$
$$+\tfrac{1}{2} \qquad +\tfrac{1}{2}$$
$$y=\tfrac{1}{2}$$

$$\boxed{\begin{matrix}(0,-2)\\ \left(-\frac{5}{2},\frac{1}{2}\right)\end{matrix}}$$

3. Solve the system. Use any method you wish.

$$\begin{cases} 9x^2 - xy + y^2 = 35 \\ 3x + y = 7 \end{cases}$$

* Take the 2nd equation and solve for y, by moving everything to the right side but y.
* Now we will write down the 1st equation and then replace all the y's with the expression you just found.
* Now we solve for x.
  * since we have something in ( ) squared, we will write it down twice and then use the foil method to get rid of the ( ).
  * Then we combine like terms.
  * we want everything on the left side and zero on the right.
* Now we use the quadratic formula to solve for x.

$$3\cancel{x} + y = 7$$
$$-3\cancel{x} \qquad -3x$$
$$y = \underbrace{-3x+7}$$

$$9x^2 - xy + y^2 = 35$$

$$9x^2 - x(-3x+7) + (-3x+7)^2 = 35$$

$$9x^2 - x(-3x+7) + (-3x+7)(-3x+7) = 35$$

$$9x^2 + 3x^2 - 7x + 9x^2 - 21x - 21x + 49 = 35$$

$$21x^2 - 49x + 49 = 3\cancel{5}$$
$$-35 \quad -\cancel{35}$$

$$21x^2 - 49x + 14 = 0$$

$a = 21 \qquad b = -49 \qquad c = 14$

Quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-49) \pm \sqrt{(-49)^2 - 4(21)(14)}}{2(21)}$$

$$x = 2, \frac{1}{3}$$

* Now take one of your original equations and write it down twice (go with the simplest one)
  * Replace the x in one equation with one of the x's you found, and in the other equation, replace x with the 2nd number you found.

* Now you will make an ordered pair for each one, writing down the x value first and then a comma and then the y value.

$$3x + y = 7$$
$$3(2) + y = 7$$
$$6 + y = 7$$
$$-6 \qquad -6$$
$$y = 1$$

$$3x + y = 7$$
$$3\left(\tfrac{1}{3}\right) + y = 7$$
$$1 + y = 7$$
$$-1 \qquad -1$$
$$y = 6$$

$$\boxed{(2,1), \left(\tfrac{1}{3}, 6\right)}$$

4. Solve the system. Use any method you wish.

$$x^2 + 3xy = 24$$
$$4x^2 - xy = 5$$

$$3(4x^2 - xy = 5)$$

$$\begin{aligned} 12x^2 - \cancel{3xy} &= 15 \\ + \quad x^2 + \cancel{3}xy &= 24 \\ \hline 13x^2 &= 39 \end{aligned}$$

$$\frac{\cancel{13}x^2}{13} = \frac{39}{13}$$

$$x^2 = \frac{39}{13}$$

$$\sqrt{x^2} = \sqrt{\frac{39}{13}}$$

$$x = \sqrt{3}, -\sqrt{3}$$

* If we multiply the 2nd equation by 3, we could eliminate the "xy".
  * So, multiply the the 2nd equation by 3.
  * Then bring down the 1st equation and add them together.
  * This will leave you with x squared equaling a number.
  * Then take the square root of both sides.
  * Remember your answer will be both positive and negative.

* Now take one of your original equations and write it down twice (go with the simplest one)
  * Replace the x in one equation with one of the x's you found, and in the other equation, replace x with the 2nd number you found.
* Now you will make an ordered pair for each one, writing down the x value first and then a comma and then the y value.

$$x^2 + 3xy = 24$$
$$(\sqrt{3})^2 + 3(\sqrt{3})y = 24$$
$$\begin{aligned} \cancel{3} + 3\sqrt{3}y &= 24 \\ \cancel{-3} \qquad\quad &\;\; -3 \\ \hline 3\sqrt{3}y &= 21 \end{aligned}$$
$$\frac{3\cancel{\sqrt{3}}y}{\cancel{3}\sqrt{3}} = \frac{21}{3\sqrt{3}}$$
$$y = \frac{7\sqrt{3}}{3}$$

$$x^2 + 3xy = 24$$
$$(-\sqrt{3})^2 + 3(-\sqrt{3})y = 24$$
$$\begin{aligned} \cancel{3} - 3\sqrt{3}y &= 24 \\ \cancel{-3} \qquad\quad &\;\; -3 \\ \hline -3\sqrt{3}y &= 21 \end{aligned}$$
$$\frac{-3\cancel{\sqrt{3}}y}{-\cancel{3}\sqrt{3}} = \frac{21}{-3\sqrt{3}}$$
$$y = \frac{-7\sqrt{3}}{3}$$

$$\boxed{\left(\sqrt{3}, \frac{7\sqrt{3}}{3}\right), \quad \left(-\sqrt{3}, \frac{-7\sqrt{3}}{3}\right)}$$

5. Solve the system. Use any method you wish.

$$\begin{cases} x^2 - 2y^2 - 1 = 0 \\ 5x^2 + y^2 = 49 \end{cases}$$

$$2(5x^2 + y^2 = 49)$$

$$\begin{array}{r} 10x^2 + \cancel{2y^2} = 98 \\ + \quad x^2 - \cancel{2y^2} - 1 = 0 \\ \hline 11x^2 - \cancel{1} = 98 \\ \cancel{+1} \quad +1 \\ \hline \end{array}$$

$$\frac{\cancel{11}x^2}{\cancel{11}} = \frac{99}{11}$$

$$x^2 = 9$$

$$\sqrt{x^2} = \sqrt{9}$$

$$x = 3, -3$$

$$\begin{array}{r} 5x^2 + y^2 = 49 \\ 5(3)^2 + y^2 = 49 \\ \cancel{45} + y^2 = 49 \\ \cancel{-45} \quad -45 \\ \hline y^2 = 4 \\ \sqrt{y^2} = \sqrt{4} \\ y = 2, -2 \end{array}$$

$$\begin{array}{r} 5x^2 + y^2 = 49 \\ 5(-3)^2 + y^2 = 49 \\ \cancel{45} + y^2 = 49 \\ \cancel{-45} \quad -45 \\ \hline y^2 = 4 \\ \sqrt{y^2} = \sqrt{4} \\ y = 2, -2 \end{array}$$

$(3, 2), (3, -2), (-3, 2), (-3, -2)$

* If we multiply the 2nd equation by 2, we could eliminate the "y^2".
    * So, multiply the the 2nd equation by 2.
    * Then bring down the 1st equation and add them together.
    * This will leave you with x squared equaling a number.
    * Then take the square root of both sides.
    * Remember your answer will be both positive and negative.

* Now take one of your original equations and write it down twice (go with the simplest one)
    * Replace the x in one equation with one of the x's you found, and in the other equation, replace x with the 2nd number you found.
* Now you will make an ordered pair for each one, writing down the x value first and then a comma and then the y value.
* Since there are 2 y values for each equation, you will have 2 ordered pairs for each equation.

6. Solve the system. Use any method you wish.

$$9x^2 - 5y^2 + 5 = 0$$
$$4x^2 + 4y^2 = 74$$

$$4(9x^2 - 5y^2 + 5 = 0) \qquad 5(4x^2 + 4y^2 = 74)$$

$$20x^2 + 20y^2 = 370$$

$$\begin{array}{r} 36x^2 - \cancel{20y^2} + 20 = 0 \\ + \quad 20x^2 + \cancel{20y^2} \quad = 370 \\ \hline 56x^2 \quad + \cancel{20} = 370 \\ -\cancel{20} \quad -20 \\ \hline \end{array}$$

$$\frac{\cancel{56}x^2}{\cancel{56}} = \frac{350}{56}$$

$$x^2 = \frac{25}{4}$$

$$\sqrt{x^2} = \sqrt{\frac{25}{4}}$$

$$x = \frac{5}{2}, -\frac{5}{2}$$

$$4x^2 + 4y^2 = 74$$
$$4\left(\frac{5}{2}\right)^2 + 4y^2 = 74$$
$$\cancel{25} + 4y^2 = 74$$
$$-\cancel{25} \qquad -25$$
$$\frac{\cancel{4}y^2}{\cancel{4}} = \frac{49}{4}$$
$$y^2 = \frac{49}{4}$$
$$\sqrt{y^2} = \sqrt{\frac{49}{4}}$$
$$y = \frac{7}{2}, -\frac{7}{2}$$

$$4x^2 + 4y^2 = 74$$
$$4\left(-\frac{5}{2}\right)^2 + 4y^2 = 74$$
$$\cancel{25} + 4y^2 = 74$$
$$-\cancel{25} \qquad -25$$
$$\frac{4y^2}{4} = \frac{49}{4}$$
$$y^2 = \frac{49}{4}$$
$$\sqrt{y^2} = \sqrt{\frac{49}{4}}$$
$$y = \frac{7}{2}, -\frac{7}{2}$$

$$\left(\frac{5}{2}, \frac{7}{2}\right), \left(\frac{5}{2}, -\frac{7}{2}\right), \left(-\frac{5}{2}, \frac{7}{2}\right), \left(-\frac{5}{2}, -\frac{7}{2}\right)$$

* If we multiply the 1st equation by 4 and multiply the 2nd equation by 5, we could eliminate the “y^2”.
  * So, multiply the the 1st equation by 4 and the 2nd equation by 5.
  * Now write down one of the equations with the other equation under it.
  * This will leave you with x squared equaling a number.
  * Then take the square root of both sides.
  * Remember your answer will be both positive and negative.