An Analysis of the Stability Characteristics of Celo

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DRAFT version 0.13

Abstract

This document analyzes the stability of the Celo protocol over a set of simulated market scenarios.

Contents

1 Introduction

The Celo protocol [5] defines a decentralized payments system in which participants use coins pegged to a local fiat currency or a local basket of goods. In this paper, we investigate the stability characteristics of Celo stable value assets through a series of simulations under various market conditions. For simplicity, we focus on the case of a single stable value asset, called the Celo Dollar, that is pegged to the US Dollar.

1.1 Stability Mechanism

The Celo protocol, at base, has two assets: Celo Dollars, an elastic-supply stable value asset, and Celo Gold, a fixed-supply variable value asset.[1]

To maintain stability of Celo Dollars, the protocol continuously adjusts Celo Dollar supply to match Celo Dollar demand at the price peg. When the market price of Celo Dollars is greater than \$1, the protocol expands the supply of Celo Dollars by creating new Celo Dollars and selling them on the open market in exchange for Celo Gold, which it then deposits into a reserve and diversifies. The protocol continues to expand supply in this manner until the market price reaches the \$1 peg.

When the market price of Celo Dollars is less than \$1, the protocol contracts the supply of Celo Dollars by buying Celo Dollars on the open market using the assets in the reserve, and burning the Celo Dollars that it bought. The protocol continues to contract supply in this manner until the market price reaches the \$1 peg. Section 4 gives a more detailed description of the implemented expansion and contraction mechanism that aims to achieve the above dynamics.

1.2 Stability Risks

The primary risk to Celo Dollar stability is a scenario in which there is a contraction in demand for Celo Dollars greater than the total value of the reserves. In such a scenario, the protocol would be unable to contract supply enough to meet decreased demand.

A secondary risk is a scenario in which there exists enough value in the reserves to handle a contraction in demand, but not enough market liquidity to sell the amount of crypto assets quickly enough to handle the contraction.[2]

To model the likelihood of either of these risks, we would need to model the demand for Celo Dollars, the value of the reserves, and the flows of Celo currency through the expansion and contraction mechanism. We do so in the next three sections.

2 Demand: A Stochastic Anchor Point Model

We model demand for Celo Dollars via a two-step process, in which we use a Geometric Brownian Motion (GBM) model to sample demand quantity Q_t at a series of time points t, and then derive full demand curves from these anchor points by modeling reasonable price elasticity parameters at these anchor points.

2.1 Stochastic Anchor Points for the Demand Curve

We use Geometric Brownian Motion to generate stochastic demand quantities Q_t for Celo Dollars at the price of one US Dollar, using the following stochastic differential equation:

$$dQ_t = \mu Q_t \, dt + \sigma Q_t \, dW_t \tag{1}$$

where W_t is a Brownian Motion, μ is the drift rate and σ is the volatility parameter. The solution to this equation is:

$$Q_t = Q_0 \exp\left[(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t\right] \tag{2}$$

[1] In actuality, the Celo protocol allows for many stable value assets and many reserve assets [5]. However, the analysis of the base case is generalizable to the multiple-asset case.

[2] A similar risk occurs if the protocol cannot buy crypto assets quickly enough to handle an expansion in demand.

with $t > 0$ where Q_0 is the starting value of the demand quantity.[3] By the properties of log-normal distributions, the expected value of $\mathrm{E}[Q_t]$ that results from equation (2) is

$$\mathrm{E}[Q_t] = Q_0 \exp(\mu t) \tag{3}$$

which implies an average arithmetic return of

$$\mathrm{E}\left[\frac{Q_t}{Q_0} - 1\right] = \frac{\mathrm{E}[Q_t]}{Q_0} - 1 = \exp(\mu t) - 1. \tag{4}$$

Because $\log(1 + x) \approx x$ for small x, it holds that

$$\exp(\mu t) - 1 \approx \mu t \tag{5}$$

for small μt, but in general, the average arithmetic return over a short period of length t is slightly larger than μ.

2.2 Modeling Demand Shocks

Our base model for stochastic elements can lead to strong changes in the Celo Dollar demand and/or extreme cryptomarket downturns over longer periods. It does not, however, generate extreme instant movements like an instant rise in Celo Dollar demand of 20% or an instant market drop of 20%.

We modify the base model to include demand shocks using the Merton jump diffusion model [9]. More precisely, we extend equation (2) by an additional factor that captures jumps for which the occurrence frequency is governed by a Poisson process, i.e.

$$Q_t = Q_0 \exp\left[(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t\right] \prod_{j=1}^{N_t} \exp\left(Y_j^d\right) \tag{6}$$

where N_t is a Poisson process with an average number of jumps per annum of λ_d and Y^d has a normal distribution with parameters μ_d and σ_d such that

$$Y^d \sim \mathcal{N}\left(\mu_d, \sigma_d^2\right). \tag{7}$$

In our simulations, we choose the parameters such that jumps have a mean of zero and a standard deviation of 10%, i.e.

$$\mu_d = 0 \quad \text{and} \quad \sigma_d = 0.1 \tag{8}$$

We assume an average of one demand jump per annum ($\lambda_d = 1$). Figure 1 shows on a logarithmic scale that the stochastic process for the demand described by equation (6) leads to an extremely wide range of simulated demand developments given the considered parameter settings. Figure 2 and Figure 3 show what this implies on a linear scale by differentiating between cases of positive and negative overall demand growth over the 30 year period.

[3] The term $-\frac{1}{2}\sigma^2$ is often referred to as the "convexity adjustment" as it results from the convexity of the natural logarithm in the derivation of the solution to the GBM SDE. Including this correction is important as the expected value of the process at some time step t will otherwise depend not only on the drift but also on the volatility parameter.

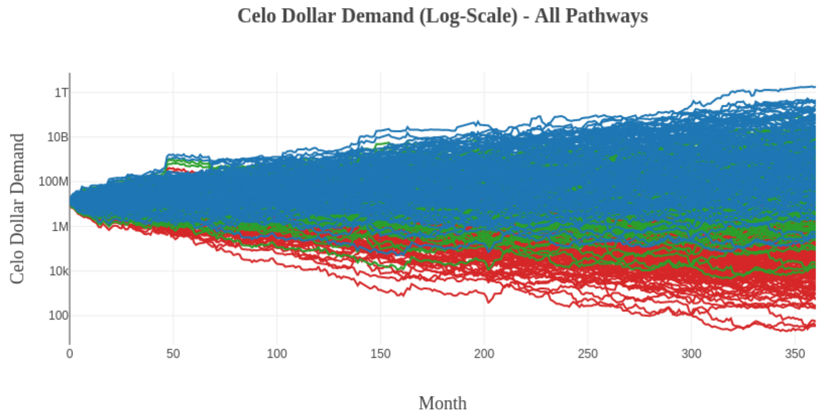

Figure 1: This figure shows a random sample of simulation paths across simulated Celo Dollar demand scenarios (-10%, 10% and 20% expected growth) on a logarithmic scale. The graph shows that we consider an extremely wide range of Celo Dollar demand developments.

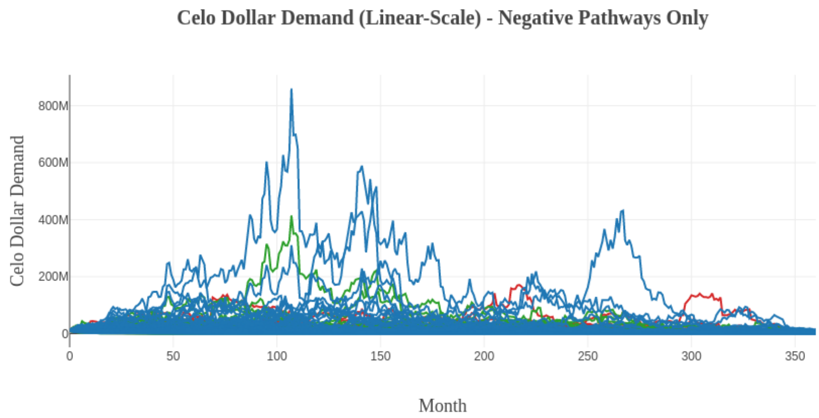

Figure 2: This figure shows simulation paths for a random sample of Celo Dollar demand scenarios (-10%, 10% and 20% expected growth) on a linear scale that resulted in a decline in demand over the 30 year period.

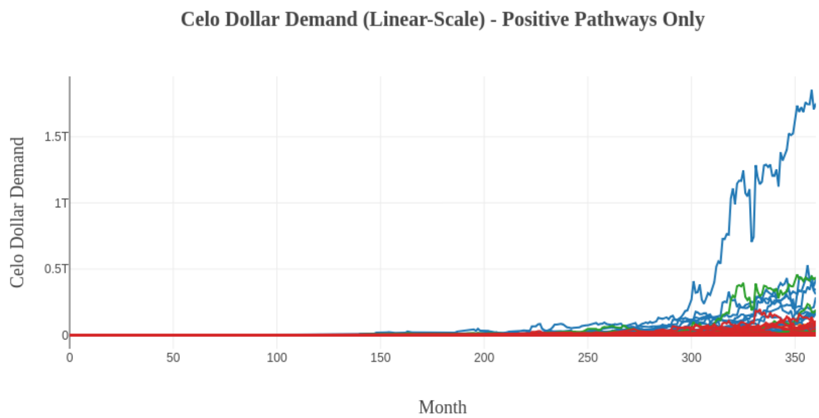

Figure 3: This figure shows simulation paths for a random sample of Celo Dollar demand scenarios (-10%, 10% and 20% expected growth) on a linear scale that resulted in an increase in demand over the 30 year period.

2.3 Shape of the Demand Curve

Once we have stochastic anchor points Q_t for the demand curve at 1 US Dollar as defined by equation (6), we create the demand curve by assuming that it has a price elasticity of $-\gamma$ at those anchor points, such that a price decrease by $x\%$ leads to an approximate increase in the demand quantity of $\gamma x\%$.[4] Thus, the demand function is given by

$$q_t = Q_t \frac{\gamma}{p_t} \tag{9}$$

where Q_t is the demand at the \$1 price peg generated by the stochastic model, and q_t is the demand at price p_t. This demand curve also gives the price sensitivity of Celo Dollars in our model in cases where supply cannot match demand – see, for example, Figure 4, a scenario in which the demand for Celo Dollars (at 1 US Dollar) at time step t is $Q_t = 20M$, at $\gamma = 1$. If Celo Dollar supply is 20M coins (green line), then our simulation maintains the peg. If Celo Dollar supply is only 19M (for example due to constrained liquidity), our simulation depegs at a Celo Dollar price of \$1.05 (orange line).

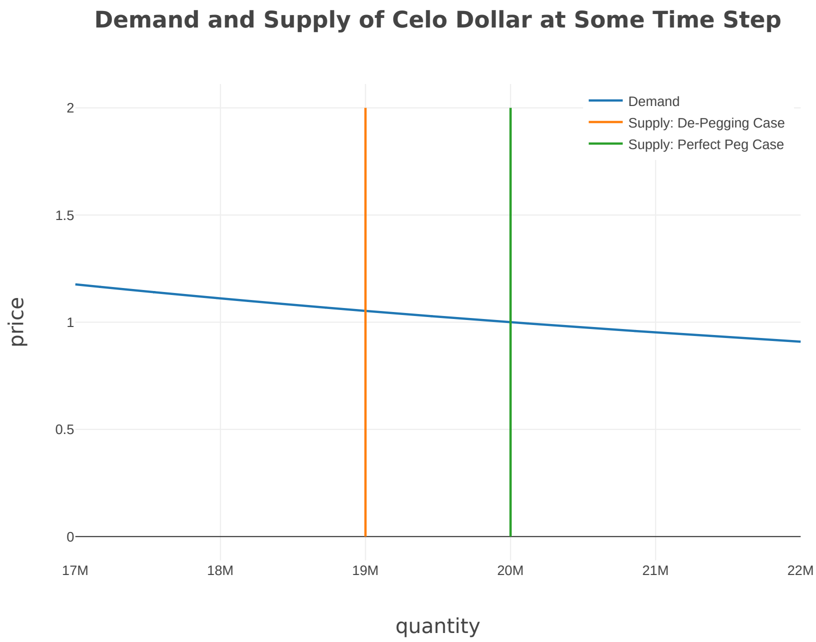

Figure 4: This Figure shows an example demand curve for Celo Dollars in our model. In this, case a demand for $Q_t = 20$M coins is generated via a stochastic process. The rest of the demand curve is then constructed around this anchor by assuming the parametric form of the demand curve described by equation (9). This setup allows us to model the price of Celo Dollars in cases where the protocol is unable to expand or contract the supply sufficiently.

3 Supply: Pricing the Reserve

There are two constraints to adjusting Celo Dollar supply: the reserve (the supply of Celo Dollars cannot contract by more than the value of the reserve), and market liquidity (the supply of Celo Dollars cannot expand or contract more quickly than the market's willingness to sell or buy reserve assets through the expansion and contraction mechanism).

The primary risk to stability is the case in which there is a contraction in demand that is larger in magnitude than the value of the reserves. This can only happen in cases where the price of reserve assets dips substantially such that the reserve is undercollateralized prior to the demand contraction. To model the probability of this, we need to model the price of the assets in reserve.

[4]This is an initial modeling assumption; in future work we will explore how sensitive our stability results are with respect to changes in the elasticity of demand.

3.1 Initial Reserve Value

The initial reserve may be bootstrapped by a limited private sale of Celo Gold. Of the proceeds of this offering, some portion is used to purchase a basket of diversified crypto assets that get committed to the reserve. Further, a fixed amount of Celo Gold is held back and initially committed to the reserve.

This bootstrapping leads to an initial overcollateralization of outstanding Celo Dollars. Even before any Celo Dollars are in existence, the reserve has both Celo Gold and non-Gold assets in reserve from the limited private offering. Any Celo Dollars that are then purchased into existence are additionally backed by new reserve assets used for the purchase.

For the simulations, we assume an initial non-Gold reserve size of 30mm.

3.2 Reserve Composition

The reserve consists of Celo Gold and a diversified basket of non-Gold crypto assets, and is periodically rebalanced to achieve a target ratio of Celo Gold and non-Gold assets[5].

For modeling purposes, we assume an equal weighting of N reserve candidates for the non-Gold portion of the reserve, with monthly rebalancing. We chose this approach – as opposed to more passive methods like market-cap weighted indexing, or more active methods like those described by Markowitz ([8], optimize risk/return), Kelly ([6], optimize log-utility), or Michaud ([10], include estimation risk in defining portfolio optimality) – because market-cap weighted indexing would lead to high single-asset concentrations and methods like Kelly, Markowitz or Michaud are highly dependent on input parameters. In finance literature, portfolios based on a naive 1/N heuristic are standard benchmarks to more sophisticated allocation methods, and in our case, a portfolio constructed in this manner would provide a comprehensible lower-bound in our stability analysis.

3.3 Pricing the non-Gold Portion of the Reserve

We use multivariate GBM to model the value of the non-Gold portion of the reserve. Doing so allows us to model several different scenarios, from boom phases to bust phases, by using different parameters for the average mean and average volatility for each of the N assets. In addition, we use the DeMiguel approach [1] to introduce a correlation structure between reserve assets, and analyze various levels of correlations in the reserves. We model the change in value dX_t^i of asset i at time t by the following equation:

$$dX_t^i = \mu_i X_t^i \, dt + \sigma_i X_t^i \, dW_t^i \tag{10}$$

where μ_i is the drift rate of asset i and σ_i is its volatility parameter, and where the increments of the respective Brownian Motions W_t^i are multivariate normal, i.e.

$$\left[W_t^1 - W_s^1, W_t^2 - W_s^2, ..., W_t^N - W_s^N\right] \sim N\left((t-s)\mu, (t-s)\Sigma\right) \tag{11}$$

where μ is a vector with the drift parameters μ_i as elements and Σ is a covariance matrix with the variances σ_i^2 on the diagonal and non-zero off-diagonal elements in case of a correlation between the respective assets.

To model the covariance matrix Σ, we assume that the returns of the reserve assets follow a single factor structure, as per [1]:

$$R_t = \beta f_t + \epsilon_t \tag{12}$$

$$\text{with} \quad f_t \sim \mathcal{N}\big(\mu_f, \sigma_f^2\big) \quad \text{and} \quad \epsilon_t \sim \mathcal{N}(0, \Sigma_\epsilon)$$

where μ_f and σ_f^2 influence the average of the drift parameters and the average of the variance of the assets respectively. Σ_ϵ gives the covariance matrix of the error terms and is assumed to be diagonal with volatilities drawn from a uniform distribution with a specific support $[\sigma_{\epsilon,l}, \sigma_{\epsilon,u}]$. The elements of β are spread evenly between 0.5 and 1.5.

DeMiguel [1] chose the parameters μ_f, σ_f, $\sigma_{\epsilon,l}$ and $\sigma_{\epsilon,u}$ such that the resulting moments are aligned with empirically observed equity returns. For our purposes, we would like to be able to choose different sets of parameters that capture a range of possible future scenarios. We do so by

[5] The target ratio in our model is 1:1, so that 50% of the reserve is held as Celo Gold, and 50% is held as a diversified basket of cryptocurrencies. Additionally, the reserve has lower and upper bounds of 20% and 80% of all existing Gold coins.

adjusting the DeMiguel approach such that it allows us to specify an average mean return and an average volatility, and then extrapolate the mean return vector and covariance matrix given those inputs. The implementation used in this version of the stability analysis assumes $N = 10$ assets in the reserve.

3.3.1 Modeling Reserve Asset Price Shocks

To model jumps in the reserve assets, we extend the Merton model described in section 2.2 to the multivariate GBM by adding an idiosyncratic jump component Y^i with

$$Y^i \sim \mathcal{N}\left(\mu_c, \sigma_c^2\right) \tag{13}$$

for each asset i as well as market wide jump component Y^m with

$$Y^m \sim \mathcal{N}\left(\mu_m, \sigma_m^2\right) \tag{14}$$

that affects all cryptoassets. This yields the following process

$$X_t^i = X_0^i \exp\left[(\mu_i - \frac{1}{2}\sigma_i^2)t + \sigma_i W_t^i\right] \prod_{j=1}^{N_t} \exp\left(Y_j^i\right) \prod_{k=1}^{M_t} \exp\left(Y_j^m\right) \tag{15}$$

where N_t and M_t are Poisson processes with an average annual number of jumps of λ_c and λ_m respectively.

For the purposes of our simulations, we choose the parameters such that all jumps have a mean of zero and a standard deviation of 20%, i.e.

$$\mu_c = \mu_m = 0 \quad \text{and} \quad \sigma_c = \sigma_m = 0.2 \tag{16}$$

And we assume 5 idiosyncratic crpytoasset jumps and 2 cryptomarket-wide jumps per annum, i.e. ($\lambda_c = 5$ and $\lambda_m = 2$). Figure 5 shows on a logarithmic scale that the stochastic process for the cryptomarket described by equation (15) leads to an extremely wide range of simulated cryptomarket developments given the considered parameter settings. Figure 6 and Figure 7 show what this implies on a linear scale by differentiating between cases of positive and negative overall cryptomarket growth over the 30 year period.

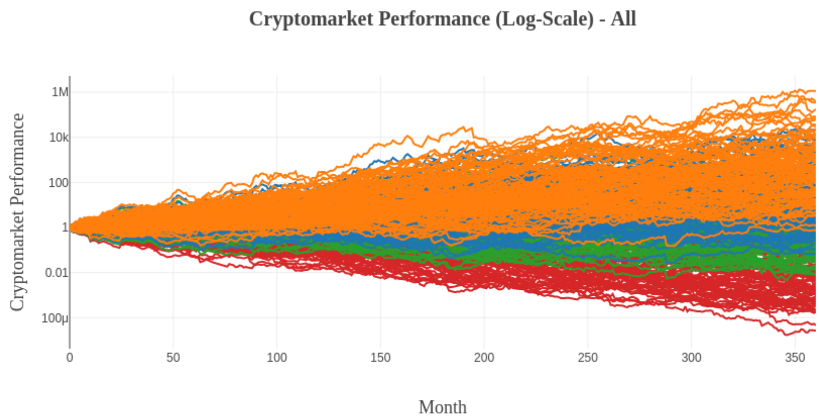

Figure 5: This figure shows how the hypothetical investment of 1 USD into an equally weighted cryptomarket portfolio would have evolved over a random sample of simulated cryptomarket growth scenarios on a logarithmic scale. Random paths that belong to the simulation runs with the same assumed average growth rates (-20%, 0%, 10% and 20% are investigated) share the same color. As the figure shows, we investigate an extremely wide range of cryptomarket developments.

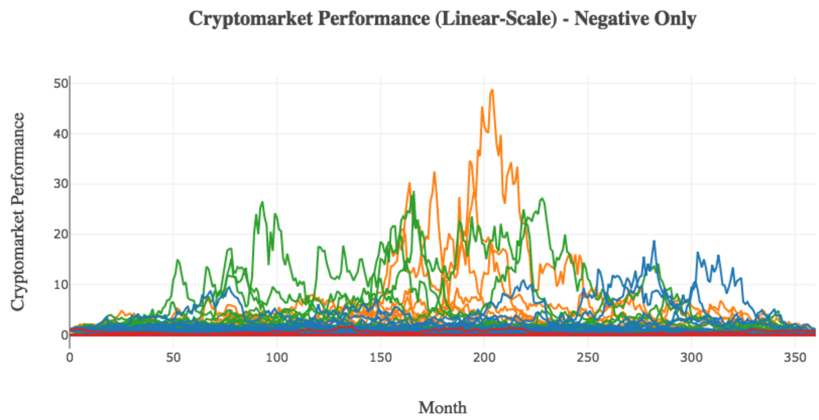

Figure 6: This figure shows, on a linear scale, a random sample of those simulation runs that resulted in an overall decline of the cryptomarket over the 30 year period. Random paths that belong to the simulation runs with the same assumed average growth rates (-20%, 0%, 10% and 20% are investigated) share the same color.

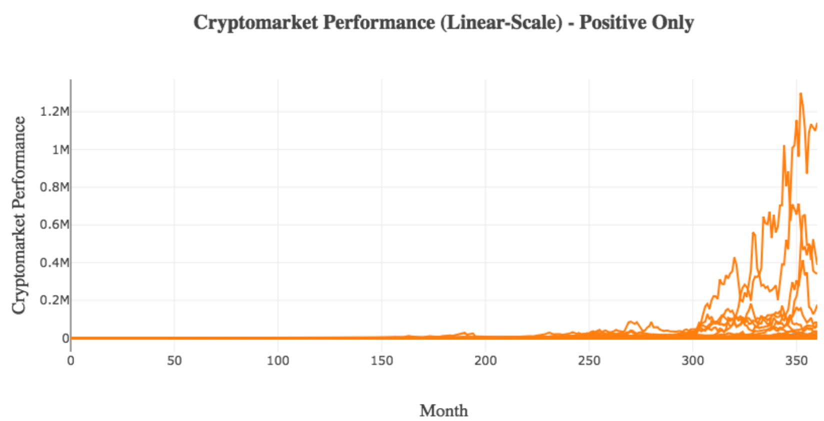

Figure 7: This figure shows, on a linear scale, a random sample of those simulation runs that resulted in an overall increase over the 30 year period. Random paths that belong to the simulation runs with the same assumed average growth rates (-20%, 0%, 10% and 20% are investigated) share the same color.

3.4 Pricing the Celo Gold Portion of the Reserve

To price the Celo Gold portion of the reserve, we must come up with a pricing model for Gold. The pricing model we derive in this section is an effort to demonstrate potential outcomes in a stability analysis, and are not intended to show or suggest that Celo Gold will appreciate in value.

In this model, we assume that there are two components to the value of Celo Gold. The first, which we call the expansion value, is based on the protocol-directed purchases of Celo Gold when the demand for Celo Dollars increases. The second, which we call the utility value, is based on the fact that transaction fees on the Celo network are denominated in Celo Gold.

We also assume that the market participants, which in aggregate hold all floating Gold at time point 0, expect their Gold ownership to be diluted over time - for example by block rewards and reserve transactions. More precisely, we assume that the ownership fraction ω_t can be described as

$$\omega_t = \omega_0 \exp(-\nu t) \tag{17}$$

with $\omega_0 = 0$, $t > 0 < \nu$ and where ν denotes the fraction of annual ownership dilution.

3.4.1 Expansion Value

The *expansion value*, qualitatively, derives from the fact that, on expansion in demand for Celo Dollars, the protocol will purchase Celo Gold.

More precisely, assume market participants at $t = 0$ expect a growth rate of Celo Dollar demand Q_t of $\hat{\mu} \geq 0$, and that the demand at 1 US Dollar at $t = 0$ equals the supply, i.e. $Q_0 = S_0$. Then:

$$Q_t = Q_0 \exp(\hat{\mu} t)\,. \tag{18}$$

implies

$$S_t = S_0 \exp(\hat{\mu} t)\,. \tag{19}$$

The expected instantaneous expansion of supply is thus

$$dS_t = \hat{\mu} S_t dt. \tag{20}$$

In this model, the present value generated to today's Gold holders generated through expected expansions, V_e, can be calculated by integrating over the product of the expected fractional ownership and the discounted expansion amounts

$$V_e = \int_0^\infty \omega_t \exp(-rt)\, \hat{\mu} S_0 \exp(\hat{\mu} t)\, dt. \tag{21}$$

where r is the discount rate. Evaluating this integral

$$V_e = \frac{\hat{\mu} S_0}{\hat{\mu} - r - \nu} \left[\exp\left[(\hat{\mu} - r - \nu)\, t\right]\right]_0^\infty \tag{22}$$

under the assumption $r + \nu > \hat{\mu}$ gives

$$V_e = \frac{\hat{\mu}}{r + \nu - \hat{\mu}} S_0. \tag{23}$$

If an annual stability fee of size s is introduced, then this increases the necessary expansion rate from $\hat{\mu}$ to $\hat{\mu} + s$ and thus leads to an expansion value of Celo Gold of

$$V_e = \frac{\hat{\mu} + s}{r + \nu - \hat{\mu} - s} S_0 \tag{24}$$

The derivation of this result can be seen as a variation of the Gordon Growth model [3]. If one for example assumes positive Celo Dollar demand growth of 5%, a 10% ownership dilution, a 0.5% stability fee and a discount rate of 25%, than a multiplier $\frac{V_e}{S_0} = 0.1864$ results. If we assume zero growth in Celo Dollar demand, i.e. $\hat{\mu} = 0$, equation (24) would reduce to

$$V_e = \frac{s}{r + \nu - s} S_0 \tag{25}$$

which leads to a ratio of $\frac{V_e}{S_0} = 0.0145$.

3.4.2 Utility Value

The *utility value* of Celo Gold derives from the fact that transaction fees are paid in Celo Gold[6]. If Celo Dollar holders pay a transaction fee f for each transaction, the incremental fee is:

$$dF_t = vfS_t dt. \tag{26}$$

where v is the annual velocity of Celo Dollars.

Just as in the calculation of the expansion value, the utility value follows from integrating over the product of the expected fractional ownership and the discounted expected future inflows:

$$V_u = \int_0^\infty \omega_t \exp(-rt)\, vfS_0 \exp(\hat{\mu}t)\, dt. \tag{27}$$

Evaluating this integral

$$V_u = \frac{vfS_0}{\hat{\mu} - r - \nu} \left[\exp\left[(\hat{\mu} - r - \nu)\, t\right]\right]_0^\infty \tag{28}$$

under the assumption that $r + \nu > \hat{\mu}$ results in a utility value of:

$$V_u = \frac{vf}{r + \nu - \hat{\mu}} S_0. \tag{29}$$

3.4.3 Deriving Price

The total value V_t of Celo Gold in float is calculated as the sum of the expansion and the utility value:

$$V_t = V_e + V_u. \tag{30}$$

Once we have the total value V_t of Celo Gold in float at time t, we can compute the price p_t of a single Celo Gold coin as follows:

$$p_t = \frac{V_t}{q_t}. \tag{31}$$

where q_t is the quantity of Celo Gold in float (not in the reserve). In our model, we know the number of coins in the reserve and the float at any time t, and so we can use p_t to give a value of the Gold portion of the reserve at time t.

3.4.4 Deriving Intra-Time Step Price

In addition to deriving the price of Celo Gold at each time step, we can also derive the average price of Celo Gold between time steps. This is useful in modeling the amount of Gold that the protocol needs to purchase or sell at a given time step to handle a contraction or expansion during that time step.

A naive estimate would be to assume that all gold purchased or sold during a time step is purchased at the price of gold at the previous time step. However, in the case, for example, of a large purchase during a time step, it is likely that the price would be affected by the purchase.

To model this, we take the Celo Dollar amount Δ generated by buying back floating Celo Gold as the integral over the pricing function above. Let the start quantity of floating Celo Gold at time step t to be q_s and the end quantity of floating Celo Gold at time step t to be q_e, then

$$\Delta = \int_{q_e}^{q_s} \frac{V}{x} dx = V\left(\log(q_s) - \log(q_e)\right). \tag{32}$$

Rearranging the above formula gives

$$q_e = q_s \exp\left(-\frac{\Delta}{V}\right) \tag{33}$$

as the end quantity of Celo Gold during a change in supply of Δ Celo Dollars.

[6]We may choose to have transaction fees paid in Celo Dollars. In that case, value derived from transaction fees will be a component of the expansion value much like the case of stability fees, since Celo Dollar-denominated transaction fees would increase the demand for Celo Dollars. Stability dynamics in either case will remain the same in our models; we model the Celo Dollar-denominated transaction fee scenario explicitly in a future version of this paper.

Figure 8: This figure illustrates the price/quantity relationship of Celo Gold. The floating Gold value chosen for demonstration purposes in this figure is $V = 100{,}000$. The resulting change in Celo Dollar supply, denoted by Δ, is the area under the curve from $q_e = 1,000$ to $q_s = 2,000$ in this example.

3.5 Additional Mechanisms to Bolster the Reserve

In addition to the natural price dynamics of the reserve, two additional mechanisms serve to bolster the reserve in times of volatility.

3.5.1 Block Reward Distribution Scheme

When the reserve ratio is below a certain threshold, block rewards in excess of miner rewards are distributed to the reserve. In our model, we assume a reserve ratio threshold of 2, and that a constant fraction of 60% of block rewards are distributed to miners. This implies that 40% of block rewards are, depending on the current reserve ratio, either distributed as incentives or to the reserve.

3.5.2 Celo Gold Transfer Fees

An additional mechanism for avoiding low reserve ratios is a transfer fee on Celo Gold transactions. This fee is paid on all transactions involving Celo Gold during times in which the reserve ratio is below a threshold of two, and thus encourages long-term holding of the reserve coin. All proceeds from this fee go to the reserve. This fee could be adjusted dynamically as a function of the reserve ratio to generate higher reserve inflows when the reserve ratio is low. In this version of the analysis, we simply assume a constant and conservatively estimated trading volume of Gold of 20% per annum and a constant transfer fee of $\tau = 0.5\%$ per Celo Gold transaction.

4 Expansion and Contraction Mechanism

At a high level, the Celo expansion and contraction mechanism allows users to create new Celo Dollars by sending 1 US Dollar worth of Celo Gold to the reserve, or to burn Celo Dollars by redeeming them for 1 US Dollar worth of Gold. This mechanism, referred to as decentralized one-to-one mechanism (DOTO) for the rest of this article, creates incentives such that when demand for the Celo Dollar rises and the market price is above the peg, an arbitrage profit can be achieved by buying 1 US Dollar worth of Celo Gold on the market, exchanging it with the protocol for one Celo Dollar, and selling that Celo Dollar for the market price. Similarly, when demand for the Celo Dollar falls and the market price is below the peg, an arbitrage profit can be achieved by purchasing Celo Dollars at the market price, exchanging it with the protocol for 1 US Dollar worth of Celo Gold, and selling the Celo Gold to the market. These actions drive the market price of the Celo Dollar back towards 1 US Dollar without the need for the protocol to estimate the optimal expansion or contraction amounts.

There is one major drawback to a direct implementation of the DOTO mechanism described above: in order for a user to send 1 US dollar worth of Celo Gold to the reserve, or redeem 1 US dollar worth of Celo Gold from the reserve, the protocol needs an oracle to give the exact price of Celo Gold in US Dollars. In cases where the Celo Gold to US Dollar oracle value is imprecise (in other words, if Celo Gold is trading on the market at a different price than what the oracle says), arbitrage opportunities exist even if the Celo Dollar is perfectly pegged[7]. These unintended arbitrage opportunities can lead to unintended supply adjustments and reserve depletion. The next section describes the implementation of the above mechanism that mitigates this potential.

4.1 Constant-Product Decentralized One-to-One Mechanism (CP-DOTO)

To address the risk of imprecise oracle values for the Celo Gold to US Dollar exchange rate in the DOTO mechanism, the protocol uses a constant-product market-maker model, inspired by Uniswap (see [11]), to dynamically adjust the on-chain exchange rate in response to on-chain exchange activity. For that purpose, two wallets controlled by the protocol, one containing Celo Dollars and one containing Celo Gold, are initialized whenever the oracle value is updated. Let G_0 denote the number of Celo Gold coins and D_0 the number of Celo Dollar coins in the respective wallets at initialization. The central equation for the constant-product market-maker model fixes the following relationship:

$$G_0 \times D_0 = G_t \times D_t \qquad \forall \quad 0 \leq t < T \tag{34}$$

where T denotes the point in time of the next oracle value update.

Given this, it can be shown (see [11]) that the price for an infinitesimal amount of Celo Gold in Celo Dollar units in the period $0 \leq t < T$ is

$$P_t = \frac{D_t}{G_t}. \tag{35}$$

Whenever the oracle price of Celo Gold is updated, the protocol initializes wallet quantities D_0 and G_0 that lead to a on-chain price P_0 which equals the current oracle rate.

If the oracle price is correct, the exchange rate quoted by the constant-product market-maker will be equal to that of the market, and no arbitrage opportunity will exist if the Celo Dollar is pegged. If the oracle price is incorrect, the two rates will differ, and an arbitrage opportunity will exist even in the absence of a Celo Dollar depeg. As arbitrageurs exploit this opportunity the constant-product market-maker model will dynamically adjust the quoted exchange rate until the arbitrage opportunity ceases to exist. This limits the depletion potential of the Celo expansion and contraction mechanism in the case of an incorrect oracle price.

In this analysis, we take the conservative approach of assuming that no external market makers or other market participants are willing to compensate short-term fluctuation of the Celo Dollar at market places on their own account. The short-term price fluctuations resulting in this simulation analysis are thus a conservative estimate of the short-term stability of the Celo Dollar.

[7] To give a concrete example, if the oracle says the price of Celo Gold is $1.50, and Celo Gold is trading on the open market for $2, then people have an incentive to redeem their Celo Dollars for Celo Gold from the reserve at $1.50 and then sell the Celo Gold on the open market for $2. Further, in that scenario, nobody would buy Celo Dollars from the reserve, because they will need to pay $2 worth of Celo Gold to buy 1.50 Celo Dollars.

5 Simulation Results

Given the above models for Celo Dollar demand, value of the reserve, and the expansion and contraction mechanism, we simulated a range of market scenarios and analyzed the stability of Celo Dollars in these scenarios.

In our simulations, we explored every permutation of the parameter settings in Table 1, and simulated 1,000 paths for each permutation, each path with daily time steps over a period of 30 years, for a total of 24,000 30-year simulations which gives more than 259 million simulated days overall.

Parameter Settings

Parameter			
	Symbol	Settings	Explanation
Demand	μ	$-10\%, 10\%, 20\%$	Drift rate of demand
	σ	$20\%, 40\%$	Volatility of demand
	μ_d	0%	Average demand jump size
	σ_d	10%	Volatility of demand jump
	λ_d	1	Demand jumps per annum
	γ	1	Price elasticity of demand
Cryptomarket	μ_f	$-20\%, 0\%, 10\%, 20\%$	Drift rate of reserve assets
	σ_f	50%	Volatility of reserve assets
	μ_c	0%	Average idiosyncratic reserve jump size
	σ_c	20%	Volatility of idiosyncratic reserve shock
	λ_c	5	Idiosyncratic reserve jumps per annum
	μ_m	0%	Average market wide reserve jump size
	σ_m	20%	Volatility of market wide reserve shock
	λ_m	2	Market wide reserve jumps per annum
Protocol & Macro	v	20	Annual velocity of Celo Dollars
	f	0.2%	Transaction fee
	s	0.5%	Stability fee
	τ	0.5%	Gold transfer fee
	r	25%	Discount rate
	$\hat{\mu}$	5%	Market expected Celo Dollar Growth
	N	10	Number of non-Gold reserve assets
	ν	10%	Ownership dilution per annum

Table 1: This table gives an overview over the main parameters of the simulation and the respective considered settings. When choosing parameters, we chose quite conservative values to achieve a lower bound on stability, assuming highly volatile markets and demand growth, low market expectations for Celo Dollar growth, and a low initial reserve ratio.

Table 2 provides a summary of the different permutations and some key stability metrics: average price (avg_price), average daily absolute deviations from the target price (avg_abs_dev), the 5% and 95% percentiles of the observed prices ($x\%\_perc$ respectively) and the percentage of days with depegs of more than 1 cent and 10 cent ($depeg\_1cent$ and $depeg\_10cent$ respectively). The results with respect to the average absolute deviation are visualized in Figure 9.

6 Conclusion

The purpose of this analysis was to gain a better understanding of the stability-related mechanics and overall stability of the Celo protocol. We found that in its current version, the Celo stability mechanism limits volatility to a reasonably narrow band over a range of market scenarios. However, it is important to note that any simulation is only as good as its assumptions, and no simulation can model all possible scenarios. In this simulation, we used a reasonable set of assumptions to derive the model and set the parameters, but as in any model, there are other reasonable assumptions that would lead to other models and other sets of parameters.

Future versions of the analysis will continue to refine the Celo Dollar demand and reserve asset pricing models, for example by including the stochastic volatility model of [4] that allows for volatility clustering, and block bootstrap methods (see for example [2] and [7]) that allow us to base the stochastic demand on historical crypto asset data without making strong distributional assumptions. Further, in future versions we will model large scale attacks, and include additional stability features of the protocol that we did not include in the current version, for example by introducing multiple stable value assets.

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Appendix I: Simulation Result Table

Simulation Results

Setting ID	Growth Parameters				Stability Metrics					
	μ_{demand}	σ_{demand}	μ_{market}	σ_{market}	*avg_price*	*avg_abs_dev*	5%_*perc*	95%_*perc*	*depeg*_1*cent*	*depeg*_10*cent*
#0	20%	20%	20%	50%	1.00115	0.00996	0.98154	1.02036	0.42962	0.00072
#1	20%	20%	10%	50%	1.00127	0.01044	0.98075	1.02139	0.45339	0.00076
#2	20%	20%	0%	50%	1.00147	0.01121	0.97951	1.02296	0.48856	0.00086
#3	20%	20%	-20%	50%	1.00178	0.01243	0.97751	1.02549	0.53672	0.00107
#4	10%	20%	20%	50%	1.00051	0.00979	0.98133	1.01959	0.42061	0.00069
#5	10%	20%	10%	50%	1.00055	0.01009	0.98076	1.02021	0.43609	0.00073
#6	10%	20%	0%	50%	1.00061	0.01076	0.97958	1.02154	0.46777	0.00082
#7	10%	20%	-20%	50%	1.00077	0.01232	0.97678	1.02464	0.53175	0.00105
#8	-10%	20%	20%	50%	0.99932	0.00967	0.98066	1.01843	0.41425	0.00069
#9	-10%	20%	10%	50%	0.99931	0.00976	0.98049	1.01859	0.41897	0.00070
#10	-10%	20%	0%	50%	0.99919	0.01016	0.97973	1.01914	0.43340	0.00122
#11	-10%	20%	-20%	50%	0.99102	0.01955	0.96165	1.02214	0.51992	0.01921
#12	20%	40%	20%	50%	1.00062	0.01601	0.96845	1.03326	0.62102	0.00062
#13	20%	40%	10%	50%	1.00065	0.01647	0.96753	1.03422	0.63153	0.00065
#14	20%	40%	0%	50%	1.00070	0.01741	0.96575	1.03611	0.65159	0.00072
#15	20%	40%	-20%	50%	1.00079	0.02006	0.96072	1.04131	0.69914	0.00102
#16	10%	40%	20%	50%	1.00020	0.01586	0.96835	1.03262	0.61729	0.00061
#17	10%	40%	10%	50%	1.00020	0.01614	0.96782	1.03317	0.62363	0.00062
#18	10%	40%	0%	50%	1.00019	0.01680	0.96648	1.03447	0.63796	0.00068
#19	10%	40%	-20%	50%	1.00003	0.01967	0.96091	1.03984	0.69070	0.00134
#20	-10%	40%	20%	50%	0.99937	0.01574	0.96794	1.03162	0.61446	0.00060
#21	-10%	40%	10%	50%	0.99936	0.01583	0.96777	1.03182	0.61671	0.00061
#22	-10%	40%	0%	50%	0.99933	0.01605	0.96728	1.03223	0.62183	0.00062
#23	-10%	40%	-20%	50%	0.99433	0.02247	0.96051	1.03551	0.66235	0.00941

Table 2: This table shows the Celo Dollar stability metrics for the scenarios we simulated. *avg_price* shows the average price over all simulation steps and simulation runs and *avg_abs_dev* gives the average absolute deviation from the target price of one. 5%_*perc* and 95%_*perc* give the respective percentiles of the observed prices. *depeg*_1*cent* and *depeg*_10*cent* give the fraction of days on which depegs of more than 1 cent and more then 10 cent occurred.

Appendix II: Simulation Result Visualization

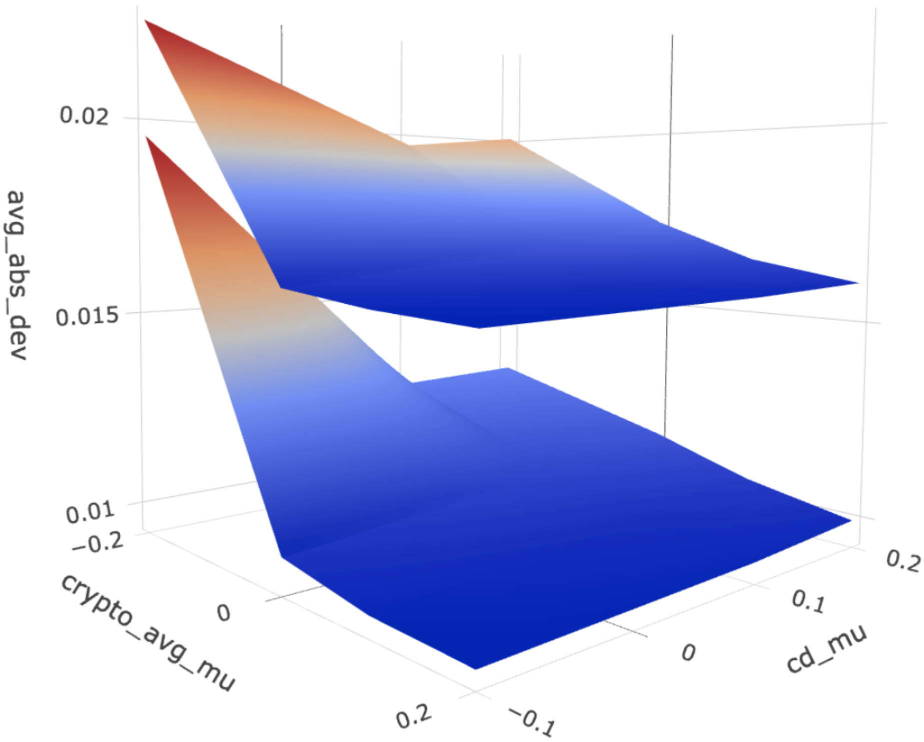

Figure 9: This figure visualizes the simulation results shown in Table 2 with respect to the stability metric of absolute average price deviation of the Celo Dollar. The respective average growth rates for the cryptomarket and the Celo Dollar demand growth are given on the horizontal axis. The lower and upper surface show results for $\sigma_{demand} = 20\%$ and $\sigma_{demand} = 40\%$ respectively.