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Competitive effects of mergers and of spectrum divestment remedies in mobile telecommunication markets[☆]

Duarte Brito [a,b,*], Helder Vasconcelos [c,d]

[a] *Nova School of Business and Economics, Universidade NOVA de Lisboa, Campus de Carcavelos, 2775-405 Carcavelos, Portugal*
[b] *Nova School of Science and Technology, Department of Applied Social Sciences (DCSA), Universidade NOVA de Lisboa, Campus da Caparica, 2829-516 Caparica, Portugal*
[c] *Faculdade de Economia, Universidade do Porto, CEF.UP, Portugal*
[d] *Compass Lexecon (Madrid), Spain*

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ABSTRACT

Motivated by recent mergers in mobile telecommunications markets, this paper investigates the merger induced effects on consumer surplus in a setting where: (i) the industry is modeled as a triopoly in which firms sell products that are both horizontally and vertically differentiated; (ii) the merging parties are able to pool their spectrum assets; and (iii) the joint management of pooled spectrum assets enables merging parties to offer a better quality service, for which customers are willing to pay more. From a merger policy perspective, our contribution is two-fold. First, we conclude that mergers may benefit consumers even in the absence of any cost-related efficiencies and establish under which circumstances this is more likely to occur. Second, our results also indicate that when the merger has a negative impact on consumer surplus, remedies based on reallocation of spectrum are not very likely to change this outcome. The reason is that the circumstances under which the merger is unlikely to benefit consumers are precisely those under which spectrum reallocation will be unable to fix merger-induced anticompetitive effects.

1. Introduction

Mobile communications markets are usually characterized by a limited number of operators. Despite being markets exhibiting high concentration, many mobile network operator mergers have been recently proposed and approved subject to remedies (or commitments by the merging parties).

Spectrum divestiture is a common remedy in mergers or acquisitions involving mobile network operators. For instance, the U.S. Department of Justice approved the 2020 T-Mobile and Sprint merger with the agreement to sell all of Sprint's 800 MHz portfolio. In Europe, approval of the 2014 Telefónica DE/ E-Plus merger required Telefónica to lease 10 MHz of paired spectrum in the 2.6 GHz band and 10 MHz of paired spectrum in the 2.1 GHz band. In a recent 2022 case, the Portuguese Competition Authority decided to open an in-depth investigation into the acquisition by Vodafone Portugal of Nowo Communications. Further, in its non-binding opinion regarding the same transaction, the sectoral regulator, ANACOM, defended that the merging party's commitments "should include the return of at least the spectrum held by Nowo which Vodafone could not bid for in the 2021 Auction".

☆ While the authors have advised different telecommunications companies in a number of antitrust matters (including mergers), the opinions expressed in this paper are the sole responsibility of the authors and cannot be attributed to Compass Lexecon or to its clients. The authors thank the editor, Erik Bohlin, and two anonymous referees for their helpful comments and suggestions. The usual disclaimer applies.

* Corresponding author at: Nova School of Business and Economics, Universidade NOVA de Lisboa, Campus de Carcavelos, 2775-405 Carcavelos, Portugal.
E-mail address: dmb@fct.unl.pt (D. Brito).

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In this paper, we discuss the competitive effects of a merger of this type. In addition to possible cost synergies (which we do not include in the analysis), the competitive effects in this type of mergers encompass the following issues: (i) the insiders may have an incentive to increase the prices of their services as some consumers that would switch to a competitor will now be switching to another brand of the merged firm and (ii) the aggregate amount of spectrum available to the insiders increases, which could allow these firms to offer a service of higher quality and for which consumers have a higher willingness to pay.

The impact of the merger on consumer welfare and social welfare is therefore ambiguous, as consumers may be charged a higher price for a higher quality service. In addition, spectrum divestiture may decrease the quality of the services provided by the merging firms, while increasing the quality of their competitors' offerings, also with ambiguous effects on consumer welfare.

In this paper, we model the industry as a concentrated oligopoly with firms competing on price and selling products that are both vertically differentiated (due to possible ownership of different amounts of spectrum) and horizontally differentiated (due to different brand preferences).

Within this theoretical structure and considering different scenarios with respect to the ex-ante distribution of spectrum between competitors, we establish the conditions under which a merger, absent cost efficiencies, increases or decreases consumer surplus and welfare, and characterize the optimal remedies, if any.

When consumer loyalty is relatively low and consumer valuation depends significantly on service quality (which is related to the quantity of spectrum available to the firm), the merger may benefit consumers. In that case, the competitive pressure exerted by the outsider is sufficient to make the prices increase less with the merger than the increase in consumer valuation due to higher quality. Our results also indicate that when the merger does not benefit consumers, remedies based on the reallocation of insiders' spectrum are not very likely to invert this outcome. The reason is simply that the circumstances under which the merger is unlikely to benefit consumers are precisely the same circumstances under which the reallocation of spectrum will have limited results in terms of possibly fixing merger-induced anticompetitive effects.

The next section discusses the related literature. Section 3 describes the model and Section 4 discusses the competitive effects of mergers for several alternative scenarios. Finally, Section 5 concludes. All proofs are presented in Appendix.

2. Related literature

This paper proposes a model to study the effects of mergers in mobile telecommunications markets. This type of mergers has been addressed in the literature in several ways.

Many recent articles conduct an empirical ex-post assessment of specific mergers. That is the case of the 2006 T-Mobile/tele.ring merger in Austria and the 2007 T-Mobile/Orange merger in the Netherlands, studied in Aguzzoni, Buehler, Martile, Kemp, and Schwarz (2018). Using a difference-in-difference approach, they conclude that the former merger did not lead to an increase in mobile tariff prices (when compared to the control countries considered in their study) whereas the latter was followed by an increase in the mobile tariff prices. Grajek, Gugler, Kretschmer, et al. (2019) also adopt a difference-in-difference approach to investigate differences in prices and investment levels between the firms involved in five different mergers (namely, the two above mentioned mergers along with the 2004 Telia/Orange merger in Denmark, the dutch KPN Mobile/Telfort merger and the greek Stet Hellas/Q-Telecom merger) and firms present in other markets where no merger took place. The obtained results turn out to differ significantly from case to case, which leads the authors to conclude that the specifics of each merger play an important role in the induced competitive effects of the merger at stake. The same study also reports a positive correlation between the effect of the merger on prices and investment levels. In a complementary paper that uses a different dataset, Genakos, Valletti, and Verboven (2018) conclude that, on average, a merger leads to an increase of 16.3% on prices and of 19.3% on investment per firm, leaving total industry investment unchanged. Further, Maier, Jørgensen, Lunde, and Toivanen (2021), apply the same methodology to the Norwegian 2005 merger between TeliaSonera and Chess, a mobile virtual network operator with a small market share, reporting little evidence of a price increase.

Other authors have simulated, ex-ante, the effects of a merger or other counterfactual scenario. That is the case of Grzybowski and Pereira (2007) who estimate a nested logit model to obtain the price elasticities of demand and the marginal costs which are then used to simulate the effects of a merger between the largest and the third mobile network operators in Portugal, obtaining, on average, prices increases in the range 7%–10% in the case of no cost efficiencies. Liang, Guiffard, Ivaldi, and Aimene (2022) also estimate a nested-logit model with firm level data from 5 countries spanning over seventy quarters in order to conduct counterfactual simulations of different alternatives of spectrum allocation in Germany.

Despite the number of recent empirical studies on mobile telecommunication markets, mergers in this specific industry have received limited attention in terms of economic theory, possibly because it has been considered that this industry's features may be captured by the general, non industry-specific, models that have been used for merger analysis. It should be highlighted, however, that this industry has an important distinctive feature: the use of an input, spectrum, that is positively related to quality and that, when pooled after a merger, may allow the merged entity to significantly increase the quality of service. To the best of our knowledge, this feature has not so far been explicitly considered in the theoretical models of merger analysis, although it has recently received some attention in empirical work. A case in point is Liang et al. (2022) that estimate a nested-logit regression of market share on firm's network quality, prices and log-share terms related to the nesting structure, using the product of coverage and the amount of spectrum owned by each operator as a proxy for the mobile network quality for consumers.

Another related strand of the literature, that we review in the remainder of this section, is the one which investigates the competitive effects of mergers in the absence of cost synergies, making use of theoretical models encompassing some characteristics which are typical of the industry we are analyzing in this paper. In a seminal paper in the merger literature, Perry and Porter (1985), firms are assumed to own an asset (a fraction of the industry's capital stock, which, just like spectrum is in fixed supply) that allows them to produce more efficiently: the higher the amount of this asset held by a firm, the lower its marginal cost. The industry is then modeled as an oligopoly with large and small firms, with each one of the former owning two times the amount of the asset held by each one of the latter, and a merger between two small firms allows them to pool their assets and become as efficient as a large firm. Within this theoretical structure, the authors show that the incentives to merge are higher than those obtained in Salant, Switzer, and Reynolds (1983) in which marginal cost is constant and unaffected by the merger. In the present paper, we follow a similar approach toward mergers but the asset in question (spectrum) allows merged firms to market a service with a higher quality, instead of lowering their cost.

In addition to the vertical differentiation between their services, we assume that competitors are also horizontally differentiated. Deneckere and Davidson (1985) showed that when firms compete in price and sell differentiated products, mergers are profitable: under price competition and differentiated products, prices are strategic complements, implying that the best response of the outsiders to the original price is to also increase prices, which naturally benefits the merging parties. Moreover, the effects of mergers when firms compete in the circular city model of horizontal differentiation have been addressed by Brito (2003) and Levy and Reitzes (1992) in the context of no vertical differentiation.[1]

It should be noted that horizontal differentiation models have been extensively applied to mobile telephony industries (see, for instance, Buehler (2015), Foros, Hanseny, and Vergéz (2020), Grønnevet, Hansen, and Reme (2016) or Jeanjean and Houngbonon (2017)). In terms of modeling, Jeanjean and Houngbonon (2017) is the closest paper to ours, as it also considers a Salop model with vertical differentiation. Jeanjean and Houngbonon (2017) assume that firms can make investments to increase quality or reduce their marginal costs in an investment stage that precedes price competition and, therefore, also feature vertical differentiation. However, the effect of mergers is not explicitly modeled. Motta and Tarantino (2021), on the other hand, study the effects of horizontal mergers when firms sell differentiated products and can also invest in cost reducing (or quality increasing) R&D. Motivated by recent high-profile mergers in the mobile telephony industry, but using a general model that may be applied to other industries, they find that in the absence of efficiency gains, a merger always reduces total investments and consumer surplus, a result that may be reversed when (the realistic scenario of) spectrum pooling is considered.

The assumption that more spectrum available allows a firm to offer a higher quality product, a key assumption in our model, is supported by several studies, which we discuss in turn.

Castells and Bahia (2019) report that download speeds have been positively impacted by the amount of spectrum holdings, in both developed and developing countries, which suggests network quality could be improved had more spectrum been released to mobile operators. In their study, it is concluded that a selection of 10 operators that had 20 MHz less than the median level would have experienced a significant increase in download speeds if they instead owned average spectrum levels. This view is shared by Liang et al. (2022) who consider that when the market is more concentrated and eventually prices are higher, the quantity of spectrum becomes less diluted, allowing the operators to offer higher quality of service to their customers. This then places the regulators before a trade-off between the quality and price offered to consumers.

Along similar lines, Kwon, Parkb, and Rheec (2017) discuss the effect of spectrum fragmentation, a concept that describes the degree of split for a given portion of spectrum.[2] By so doing, they conclude that when two 10 MHz LTE [4G] channels are simultaneously used to deliver data to one user, the productivity of these channels is the same as that of a 20 MHz LTE channel. However, when these two channels are used for different LTE service users, data rates slow down, as there is a decrease in the productivity of spectrum.

Further, Grønnevet et al. (2016) study the effects of radio spectrum sharing in a duopoly and model the relationship between spectrum and quality explicitly, by using a M/M/1 queuing model (but consider only the case of a symmetric duopoly in order to obtain closed form solutions, which would not be adequate to model a merger, as it lacks the existence of an outsider firm). The average waiting time on network i depends on its capacity s_i (available spectrum), number of subscribers, D_i, and data volume per customer, y_i, in the following way: $1/(s_i - D_i y_i)$. Following Bourreau, Kourandi, and Valleti (2015), they assume that quality q_i is the inverse of waiting time: $q_i = s_i - D_i y_i$. In a hypothetical situation in which the two networks pool their available spectrum and data volume, average waiting time is cut in half (i.e., quality is doubled).

In an application to 5G investments in mobile telecommunications, Foros et al. (2020) consider a Hotelling duopoly in which firms are able to make investments to increase consumer valuation for their services. When allowing firms to share their quality investments, they consider that consumer valuation for both services is given by $k_i + \theta k_j$ (where k_i denotes firm i's investment), with $\theta \in [0, 1]$ measuring the degree of sharing of investments, which is a variable chosen by firms. In the case of a full merger there would be a complete sharing of investments and consumer valuation would equal the sum of the pre-merger investment values.

Finally, how much consumers value additional quality of service is a question addressed in Liu, Prince, and Wallsten (2018), who find that the valuation that households' have of internet bandwidth is highly concave, with little added value beyond 100 Mbps.[3]

Our model, described in detail in the following section, combines several features from the above.

[1] As the effects of the merger do not propagate uniformly on the circle, it is challenging to solve the model with n firms, even in the absence of vertical differentiation.

[2] As defined by Kwon et al. (2017), spectrum fragmentation, is a concept that "describes the degree of split for a given portion of spectrum (...) which can affect productivity and technical efficiency in spectrum use as the telecommunication service paradigm evolves from voice oriented to data oriented services".

[3] For example, households are willing to pay about $2.34 per Mbps ($14 total) monthly to increase bandwidth from 4 Mbps to 10 Mbps, $1.57 per Mbps ($24) to increase from 10 to 25 Mbps, and only $0.02 per Mbps ($19) for an increase from 100 Mbps to 1000 Mbps.

3. The model

We assume that, at the outset, three firms (firm 1, 2 and 3) compete in prices and sell products or services (one sold by each firm) that are both horizontally and vertically differentiated. The merger is assumed to involve firm 1 (the acquirer) and firm 2 (the target), with firm 3 taking the role of the outsider.

With respect to vertical differentiation, we assume firms own or have access to an intangible input (spectrum assets) that is used to produce the service sold to final consumers. The quality of their service (related to network quality — download and upload speeds and latencies) depends on how much input they own. Before the merger, each firm is assumed to own a given quantity of the input and to incur on a constant marginal cost per consumer served, which is independent of the quality of its service. This is justified by the assumption that, in our model, the quality of service is entirely generated by the quantity of spectrum assets owned by each firm. These are immaterial goods that generate few operating costs when compared to material assets. In addition to the amount of spectrum owned by firms, vertical differentiation could arise from other sources, such as the quality of equipment, network design, network coverage, brand image, customer care, etc. However, we keep all these other factors equal across firms, to focus on spectrum impact.[4]

As far as horizontal differentiation is concerned, we assume firms are symmetrically located in a circle of length 1 in which consumers are uniformly distributed.[5] Although this is a model of localized competition in which firms compete directly only with the neighboring firms, with three competitors there is competition between all firms in the industry.

Consumers are assumed to always purchase one unit of the product, and to have a preferred firm or (brand) they would buy from if price and quality were equal for all active firms. Additionally, consumers are willing to pay more for products with a higher quality. More specifically, consumers are assumed to be homogeneous in terms of their valuation for quality and the marginal benefits of an increase in quality are assumed to be positive but decreasing with quality.[6]

The net surplus of a consumer when she purchases from firm i located at distance x from the consumer's location on the circle is given by $V + v_i - p_i - tx^2$ where: (i) V is the consumer's utility from subscribing a basic quality service, assumed to be sufficiently high so that the market is always fully covered (ii) v_i is the term in utility associated with firm i's quality of service, (iii) p_i denotes firm i's unit price and (iv) tx^2 is the transportation (or disutility) cost associated with distance x. The term $v_i - p_i$ measures the valuation for the quality of service, net of its price.

In order to discuss the competitive effects of the merger and of eventual remedies which might be required for its approval, we need to make further assumptions about v_i. In particular, we follow Liang et al. (2022) in using spectrum as a proxy for quality. Denoting by s_i the amount of spectrum owned by firm i, our proxy for quality, we thus assume that the utility term related to quality depends on s_i : $v_i = v(s_i)$. With respect to the functional form of $v(s_i)$, we assume that $v(0) = 0$, $\partial v / \partial s_i > 0$ and $\partial^2 v / \partial s_i^2 < 0$. Consumers value higher quality but the marginal valuation for quality is decreasing, which is in line with the findings reported by Liu et al. (2018).

To simplify the exposition, in what follows, we adopt a specific formulation that satisfies the above-mentioned properties, namely that $v(s_i) = \gamma \sqrt{s_i}$. We define $s_i = \alpha_i s$ and normalize $\alpha_1 = 1$, that is, we write the amount of spectrum owned by firm $i = 2, 3$ as a percentage α_i of the amount owned by firm 1. Without loss of generality, we also normalize s to be equal to 1. A low value for γ simply means that consumer valuation (or service quality) is not strongly affected by the amount of spectrum held by a firm. We also assume that $\alpha_i \in \left[\frac{1}{2}, 1\right]$. Firm 2 and 3 may have the same or less spectrum than firm 1, but not much less.

We further assume that firms do not relocate on the circle after the merger involving firm 1 and firm 2 and that the merged firm (firm $1+2$) keeps on selling the two products (or services or brands) that were previously independently marketed by the two insiders.

The merger (and any eventual remedies related to spectrum divestiture) is assumed to affect the values of v_1, v_2 and, possibly, v_3 and we refer to brand i's post-merger quality of service as v_i^M. We assume that after the merger, insiders pool their spectrum assets and improve the quality of their two brands. However, they may be forced to divest an exogenous amount of spectrum, βs, in favor of the outsider, firm 3. Thus, after the merger, spectrum holdings are $s_{1+2}^M = \left(1 + \alpha_2 - \beta\right) s$ and $s_3^M = \left(\alpha_3 + \beta\right) s$. With respect to remedies, we assume that $\beta < \frac{1+\alpha_2-\alpha_3}{2}$ so that the insiders do not end up with less spectrum than the outsider. Additionally, the merged firm cannot be forced to divest more than the spectrum owned by the target firm 2: $\beta < \alpha_2$. Thus, we assume that $\beta < \overline{\beta}(\alpha_2, \alpha_3) := \min\left\{\frac{1+\alpha_2-\alpha_3}{2}, \alpha_2\right\}$.

Hence, after a merger, the amount of spectrum of the insiders is combined, leading to a firm with a higher quality of service for its brands. Given that vertical differentiation in the model stems exclusively from spectrum ownership and that the merged firm 1 + 2 sells the same two products as before the merger, but now with more spectrum, we have $v_1^M = v_2^M = v_{1+2}^M$: as the only source of vertical differentiation is the amount of spectrum, no differences subsist between v_1^M and v_2^M after the merger.

This modeling approach yields a result similar to Grønnevet et al. (2016): when symmetric firms pool spectrum, quality is duplicated. Consumer utility, however, is not, due to the concavity of $v(s_i)$. Therefore, our approach can be seen as more conservative

[4] For instance, we abstain from considering differences in coverage which could be explained either by similar coverage obligations resulting from an eventual spectrum auction or, in its absence, by national roaming agreements.

[5] See of Salop (1979) and Vickrey (1964, 1999). We normalize the number of consumers to 1.

[6] In Foros et al. (2020) the expected utility from quality investment interacts with location. They allow for the possibility that consumers who have a strong preference for a given firm benefit more from its investment than consumers located further away.

when compared to the one in Grønnevet et al. (2016) with respect to the effects on subscriber utility resulting from pooling spectrum. The same can be said about Foros et al. (2020), as explained above.

The additivity assumption concerning the amount of spectrum owned by the merged entity might also be considered conservative for other reasons. For example, the frequency guard bands used to avoid interference could be impacted. Fewer operators could mean fewer frequencies wasted in guard bands and the additivity assumption could then be considered conservative.[7] Additionally, we are implicitly assuming that spectrum is a homogeneous input when, in fact, there are low-, mid-, and high-frequency bands. A higher frequency is generally associated with more capacity but with a degradation in propagation. As argued in the T-Mobile/Sprint merger (see Asker and Katz (2023)) when merging firms own complementary frequency bands, each spectrum band can, after the merger, be destined for the type of traffic for which it is better suited, increasing the capacity of a given spectrum portfolio.

Let $D_i(\mathbf{p}, \mathbf{v})$ denote the demand for firm i's product where $\mathbf{p} = (p_1, p_2, p_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$, let c denote the constant marginal cost and let π_i denote its profit. Then,

$$\pi_i(\mathbf{p}, \mathbf{v}) = (p_i - c)D_i(\mathbf{p}, \mathbf{v})$$

Let $\overline{x} = \sum_{i=1}^{3} \frac{x_i}{3}$ and $\sigma_{xy} = \sum_{i=1}^{3} \frac{(x_i-\overline{x})(y_i-\overline{y})}{3}$ be the simple (unweighted) average of variable x_i and covariance between x_i and y_i.

The demand for each brand and the general expression for consumer surplus, profits and social welfare are presented in Lemma 0.

Lemma 0. *The demand for each firm is given by*

$$D_i(\mathbf{p}, \mathbf{v}) = \frac{9}{2}\frac{v_i - p_i - (\overline{v} - \overline{p})}{t} + \frac{1}{3}$$

and, if $D_i(\mathbf{p}, \mathbf{v}) > 0$ for all i, consumer surplus, industry profits and social welfare are respectively given by:

$$CS(\mathbf{p}, \mathbf{v}) = V + \left(\overline{v} - \overline{p}\right) - \frac{1}{108}t + \frac{27}{4t}\sigma^2_{v_i - p_i}$$
$$\Pi(\mathbf{p}, \mathbf{v}) = \overline{p} + \frac{27}{2t}\left(\sigma_{p_i v_i} - \sigma^2_{p_i}\right) - c$$
$$SW(\mathbf{p}, \mathbf{v}) = V + \overline{v} - \frac{1}{108}t + \frac{27}{4t}\left(\sigma^2_{v_i} - \sigma^2_{p_i}\right) - c \quad \blacksquare$$

The expression for consumer surplus in Lemma 0 includes four terms: the utility from subscribing a basic quality service, V, the unweighted average of the consumer additional valuation from the quality of service net of price, $\overline{v} - \overline{p}$, the transportation costs if the indifferent consumers were equidistant from the firms, $-\frac{1}{108}t$, and a term related to the variance of $v_i - p_i$: when firms are not symmetric in terms of $v_i - p_i$, more consumers will purchase from the firms with higher $v_i - p_i$. Thus, the unweighted average additional valuation from the quality of service net of price underestimates the true value for the average consumer. Likewise, the average transportation cost will be higher. The last term, that increases with $\sigma^2_{v_i - p_i}$, accounts for this.

As the number of consumers is normalized to 1, industry profits correspond simply to the average price (weighted by each firm's market shares) net of the constant marginal cost c. If firms are asymmetric in terms of prices or quality, market shares are not uniformly distributed between the three firms and therefore the average price would differ from $\overline{p}$. For instance, if firms with a low price sold a higher quality product or service, their market share would be higher and this would lower the industry profits (a negative $\sigma_{p_i v_i}$) as the weighted average price would be lower than $\overline{p}$. As for social welfare, the interpretation is similar. The unit price is a mere transfer from consumers to firms that only impacts welfare indirectly, through the transportation (or disutility) costs. Therefore, social welfare is the difference between average consumer valuation and the constant marginal cost, net of the transportation costs.

With respect to the model parameters, we impose some constraints that ensure all firms are active. The results and assumptions below are presented in terms of the ratio γ/t, which we denote by γ'. This crucial parameter then measures the relative importance of horizontal and vertical differentiation. It decreases when transportation costs, as measured by t, are high and valuation for quality, as measured by γ, is low, that is, when horizontal differentiation (different locations) is relatively more relevant for consumers' choices than vertical differentiation (different quality of service). Assumption 1 ensures that all firms are active in the pre-merger and post-merger equilibrium.

Assumption 1.

$$\gamma' < \overline{\gamma}'(\alpha_2, \alpha_3) := \min\left\{\frac{1}{\max\left\{\frac{9}{5}\left(1 - 2\sqrt{\alpha_2} + \sqrt{\alpha_3}\right), \frac{9}{5}\left(1 + \sqrt{\alpha_2} - 2\sqrt{\alpha_3}\right)\right\}}, \frac{4}{9\left(\sqrt{1+\alpha_2} - \sqrt{\alpha_3}\right)}\right\}$$

As explained in the proofs of Lemmas 1 and 2, below, this assumption merely states that quality differences must be sufficiently small and/or transportation costs sufficiently high to ensure that all firms have a positive market share in equilibrium.

[7] We thank an anonymous referee for pointing this out to us.

3.1. Pre-merger equilibrium

In this section, we present the equilibrium when all firms are independent and for any admissible quality levels. The following Lemma presents the equilibrium prices, consumer surplus and profits.[8]

Lemma 1. *Under Assumption 1, before any merger, the equilibrium prices, profits, consumer surplus and social welfare are, respectively, given by:*

$$p_i(\mathbf{v}) = c + \frac{1}{9}t + \frac{3}{5}\left(v_i - \overline{v}\right)$$
$$\pi_i(\mathbf{v}) = \frac{\left(5t + 3^3(v_i - \overline{v})\right)^2}{3^3 5^2 t}$$
$$CS(\mathbf{v}) = V + \overline{v} - c - \frac{13}{108}t + \frac{27}{25}\frac{\sigma_v^2}{t}$$
$$SW(\mathbf{v}) = V + \overline{v} - \frac{1}{108}t + \frac{2^2 3^3}{5^2}\frac{\sigma_v^2}{t} - c$$

with $\sigma_v^2 = \frac{2}{9}\left(v_1^2 + v_2^2 + v_3^2 - v_1 v_2 - v_1 v_3 - v_2 v_3\right)$. ■

Prices increase with marginal cost and with the degree of horizontal product differentiation. Additionally, a higher own quality increases own price and a higher rivals' quality leads to a lower equilibrium price. Firms with a quality above the average will have a higher demand and also higher equilibrium prices and profits. For a given value of the unweighted average valuation for quality, consumer surplus increases with σ_v^2 as more consumers will purchase from the firms (or brands) with higher quality. These firms have also a higher price, but it does not increase as much as its quality: $\frac{\partial p_i(\mathbf{v})}{\partial v_i} < 1$.

3.2. Post-merger equilibrium

In this section we present the equilibrium after a merger between firms 1 and 2, for any post-merger quality levels. Post-merger equilibrium values are denoted by a superscript M. We assume that the valuation for quality of both insiders' services is the same after the merger (because the merged firms pools the inputs: $v_1^M = v_2^M = v_{1+2}^M$) and that the valuation for the quality of the outsider is given by v_3^M.

The following Lemma presents the equilibrium prices, consumer surplus and profits.

Lemma 2. *Under Assumption 1, after the merger between firms 1 and 2, the equilibrium prices, profits, consumer surplus and welfare are, respectively, given by*[9]*:*

$$p_1^M(\mathbf{v}^M) = p_2^M(\mathbf{v}^M) = c + \frac{5}{27}t + \frac{1}{3}\left(v_{1+2}^M - v_3^M\right)$$
$$p_3^M(\mathbf{v}^M) = c + \frac{4}{27}t + \frac{1}{3}\left(v_3^M - v_{1+2}^M\right)$$
$$\pi_3^M(\mathbf{v}^M) = \frac{\left(8t - 18\left(v_{1+2}^M - v_3^M\right)\right)^2}{972t};\ \pi_{1+2}^M(\mathbf{v}^M) = \frac{\left(5t + 9\left(v_{1+2}^M - v_3^M\right)\right)^2}{243t}$$
$$CS^M(\mathbf{v}^M) = V - c + \frac{14v_{1+2}^M + 13v_3^M}{27} + \frac{\left(v_{1+2}^M - v_3^M\right)^2}{6t} - \frac{175}{972}t$$
$$SW^M(\mathbf{v}^M) = V - c + \frac{2t\left(16v_{1+2}^M + 11v_3^M\right) + 45\left(v_{1+2}^M - v_3^M\right)^2}{54t} - \frac{11}{972}t \tag{1}$$

Post-merger equilibrium prices increase with own quality and decrease with the quality of the rival firm.

If the quality of the service provided by the merged firm and its competitor was equally valued by consumers ($v_{1+2}^M = v_3^M$), then the merged firm would set higher prices and would have a higher profit than that of the outsider. As before the merger, consumer surplus increases with average valuation for quality and with its variance.[10]

Note that, after the merger, consumer valuation net of price is larger for the insiders than for the outsider if and only if $v_{1+2}^M - p_1^M > v_3^M - p_3^M \Leftrightarrow t < 9\left(v_{1+2}^M - v_3^M\right)$. If $v_{1+2}^M = v_3^M$ this is impossible because the outsider increases its price less than the insiders. However, if $v_{1+2}^M > v_3^M$ and if t is sufficiently low this is possible.

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8 The average market price, average valuation for quality and aggregate transportation (or disutility) costs are reported in the proof.

9 The average market price, average valuation for quality and aggregate transportation (or disutility) costs are reported in the proof, as well as the way β impacts these variables.

10 As $\left(v_{1+2}^M - v_3^M\right) = 3\left(v_{1+2}^M - \overline{v}^M\right) = \frac{3}{2}\left(\overline{v}^M - v_3^M\right)$ a brand's price is higher if the valuation for quality is above the average. Variance $\sigma_{v^M}^2 = \frac{2}{9}\left(v_{1+2}^M - v_3^M\right)^2$also affects the same variables as before the merger.

4. Effects of the merger and of remedies

In this section we compare the pre-merger and the post-merger equilibrium in order to discuss the merger induced effects on consumer surplus and on social welfare. Given that there is a reduction in competition but that, at the same time, the insiders' quality will increase, the net effect on consumer surplus is ambiguous. The effects of remedies that consist in making insiders divest a portion of the spectrum they hold to the outsider are also addressed. To the extent that divestiture of the input increases the quality of the outsider at the expense of the quality of the insiders, divestiture will lower the insiders' prices while increasing the outsider's price (and quality). Its effects on consumer welfare are, therefore, also ambiguous.

4.1. Competitive effects of the merger with no remedies

In the absence of remedies, consumer surplus is affected by the merger in multiple ways.

First, there is an increase in the insiders' prices due to (i) the fact that by increasing the price of one brand it is creating demand for the other brand sold by firm 1 + 2 and (ii) the fact that consumer valuation for the insiders' brands increase when spectrum is combined. The price of the outsider may, however, increase or decrease. On the one hand, the rival brands have increased their prices and so, as prices are strategic complements, firm 3 should also increase its price. On the other hand, the quality of the rival brands has increased, lowering the demand for firm 3, that should lower its price to compensate for the lower relative quality it offers in the new market structure induced by the merger.

Second, the quality of the insiders' brands increases due to the merger.

With respect to these two effects, what is relevant for consumers is the difference between the valuation for quality and the equilibrium price, $v_i - p_i$. It is straightforward to show that $\Delta(v_1 - p_1) < \Delta(v_2 - p_2)$ and that all $\Delta(v_i - p_i)$ can be positive or negative, depending on $\gamma' = \gamma/t$. For high values of t (low values of γ) these variations are negative but the opposite will happen for low values of t (high values of γ). Let $\underline{\gamma_i'}$ be defined such that $\Delta(v_i - p_i) > 0$ if and only if $\gamma' > \underline{\gamma_i'}$. It is possible to show that $\overline{\gamma}' > \underline{\gamma_1'} > \underline{\gamma_3'} > \underline{\gamma_2'} > 0$ so that, say, $v_2 - p_2$ increases for more parameter values than $v_3 - p_3$ or $v_1 - p_1$ increases or, in other words, whenever $v_1 - p_1$ increases so do $v_2 - p_2$ and $v_3 - p_3$.

Third, transportation (or disutility) costs may increase or decrease. If, at the outset, firms were symmetric, the asymmetries brought about by the merger would necessarily increase transportation costs, but the opposite may also happen.

In general terms (i.e. before using the particular expressions assumed for v_i and v_i^M), the expression for the impact of the merger on consumer surplus is given by:

$$\Delta CS = v_3^M - \overline{v} - \left(t - 9\left(v_{1+2}^M - v_3^M\right)\right)\frac{29t + 9\left(v_{1+2}^M - v_3^M\right)}{486t} - \frac{27}{25}\frac{\sigma_v^2}{t}$$

If, for instance, firms are ex-ante symmetric, we have $v_3^M - \overline{v} = \sigma_v^2 = 0$ and consumer surplus increases if and only if $t < 9(v_{1+2}^M - v_1)$. When $\sigma_v^2 = 0$, it can be easily established that the merger always increases transportation costs and the average price. However, average consumer valuation due to higher quality also increases. The change in consumer surplus is the sum of these three effects, with the first two being negative and the third one positive in the consumers' perspective. The merger might be beneficial for consumers if the improvement in the insiders' quality of service is higher than the increase in price (and more than compensates for the higher transportation or disutility costs). If transportation costs are sufficiently small – smaller than $9(v_{1+2}^M - v_1)$ –, the competitive pressure from the outsider prevents the insiders from capturing all of the additional valuation for quality.

The condition for social welfare to increase is significantly weaker because industry profit increases with the merger.

The next Proposition establishes when the merger benefits consumers given the particular assumptions about v_i and v_i^M.

Proposition 1. *Let Assumption 1 hold.*

(a) The merger (with no remedies) increases consumer surplus if and only if

$$\gamma' > \gamma'_{CS}(\alpha_2, \alpha_3) := \frac{\begin{pmatrix} -2025\left(\sqrt{\alpha_2} + 1\right) + 900\sqrt{\alpha_3} + 3150\sqrt{\alpha_2 + 1} \\ -135\sqrt{\begin{matrix} 734\left(\alpha_2 + 1\right) + 9\alpha_3 + 566\sqrt{\alpha_2} + 150\sqrt{\alpha_3}\sqrt{\alpha_2 + 1} \\ -84\sqrt{\alpha_3}\left(\sqrt{\alpha_2} + 1\right) - 700\sqrt{\alpha_2 + 1}\left(\sqrt{\alpha_2} + 1\right) \end{matrix}} \end{pmatrix}}{891\left(\alpha_2 + \alpha_3 + 1\right) - 2916\left(\sqrt{\alpha_2} + \sqrt{\alpha_3} + \sqrt{\alpha_2}\sqrt{\alpha_3}\right) + 4050\sqrt{\alpha_3}\sqrt{\alpha_2 + 1}}$$

(b) The merger (with no remedies) increases social welfare if and only if

$$\gamma' > \gamma'_{SW}(\alpha_2, \alpha_3) := -\frac{\begin{pmatrix} 2025\left(\sqrt{\alpha_2} + 1\right) - 450\sqrt{\alpha_3} - 3600\sqrt{\alpha_2 + 1} \\ +135\sqrt{\begin{matrix} 934\left(\alpha_2 + 1\right) + 9\alpha_3 + 466\sqrt{\alpha_2} - 84\sqrt{\alpha_3}\left(\sqrt{\alpha_2} + 1\right) \\ -800\sqrt{\alpha_2 + 1}\left(\sqrt{\alpha_2} + 1\right) + 150\sqrt{\alpha_3}\sqrt{\alpha_2 + 1} \end{matrix}} \end{pmatrix}}{1539\left(\alpha_2 + \alpha_3 + 1\right) - 11\,664\left(\sqrt{\alpha_2} + \sqrt{\alpha_3} + \sqrt{\alpha_2}\sqrt{\alpha_3}\right) + 20\,250\sqrt{\alpha_3}\sqrt{\alpha_2 + 1}} \quad \blacksquare$$

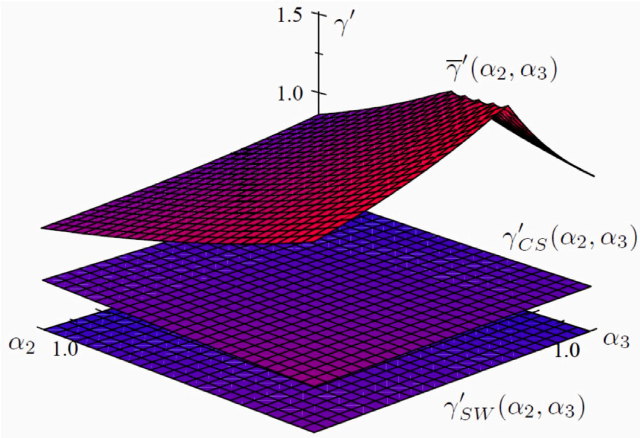

Fig. 1. Thresholds as a function of α_2 and α_3.

If γ' is sufficiently large, that is, if transportation costs are low and consumer valuation depends strongly on quality (which in turn depends on the amount of spectrum held by each firm), the merger with no remedies will benefit consumers. The intense price competition between the firms that results from low horizontal product differentiation means that prices will not increase too much. Further, the fact that insiders will benefit from a substantial increase in consumer valuation for their products will lead to an increase in consumer surplus.

In terms of social welfare, the condition is similar but, as the increase in price does not play a direct role because the market is assumed to be covered, it is easier to verify. Welfare decreases in extreme circumstances: when t is very high and/or γ is very low. Indeed, when $\gamma \to 0$, consumers do not value quality and firms are symmetric regardless of their spectrum holdings. Pre-merger transportation costs are thus minimized as, in equilibrium, consumers select the nearest firm. This is no longer true after the merger due to the equilibrium price asymmetry and transportation costs increase. The increase in quality from pooling spectrum is also irrelevant in terms of welfare when γ is low and the negative effect from higher transportation costs dominates. However, relatively low values of $\gamma' > 0$ may invert this, as illustrated in Fig. 1.

Fig. 1 presents the thresholds in Proposition 1 as well as $\overline{\gamma}'(\alpha_2, \alpha_3)$ for all admissible values of α_2 and α_3.[11]

4.2. Competitive effects of remedies

Recall that, in our analysis, remedies refer exclusively to a reallocation of spectrum. Remedies decrease v_{1+2}^M while increasing v_3^M and it is possible to establish that[12]:

(i) remedies decrease the post-merger average price, weighted by the corresponding market shares, $\overline{p}_w^M$.

(ii) remedies have an ambiguous effect on post merger transportation costs, T^M.

(iii) remedies have an ambiguous effect on the post-merger weighted average valuation for quality $\overline{v}_w^M$.

Average price always decreases with remedies. Remedies lower each of the insiders' prices by as much as they increase the outsider's price.[13] As insiders always have a market share larger than the outsider, average industry price decreases.[14] This price decrease is less pronounced when firms are more horizontally differentiated (i.e. when t is high) and less vertically differentiated (i.e. when $v_{1+2}^M - v_3^M$ is low) because the insiders' aggregate market share will be smaller in this case (their quality advantage is smaller and it is mitigated when transportation costs are high).

Transportation costs are minimized when the consumer who is indifferent between any pair of neighboring firms is located at the midpoint between the two firms. After a merger, it is not clear if the consumer indifferent between one of the insiders and the outsider is closer to the former or to the latter: the insiders set higher prices, which would make the indifferent consumer move closer to the closest insider (increasing the demand for the outsider) but insiders also have a higher quality service, which has the opposite effect. The imposition of remedies, by lowering the quality gap by more than the price difference, will make the consumer indifferent between the two firms become closer to the insider (increasing the demand for the outsider).[15] Depending on where this consumer is originally located, remedies may increase or decrease transportation costs. For instance, if the consumer indifferent between the two

11 In Fig. 1, $\gamma'_{SW}(\alpha_2, \alpha_3)$ is almost indistinguishable from 0.

12 See proof of Lemma 2.

13 This follows from $\frac{\partial p_1^M}{\partial(v_{1+2}^M - v_3^M)} = \frac{\partial p_2^M}{\partial(v_{1+2}^M - v_3^M)} = -\frac{\partial p_3^M}{\partial(v_{1+2}^M - v_3^M)} = \frac{1}{3}$.

14 Note that $D_1(\mathbf{v}^M) + D_2(\mathbf{v}^M) = \frac{(v_{1+2}^M - v_3^M)}{t} + \frac{10}{18} > \frac{1}{2}$.

15 This follows from $\frac{\partial(p_1^M - p_3^M)}{\partial(v_1^M - v_3^M)} = \frac{18}{27} < 1$.

firms is already at a distance smaller than 1/6 from the insider, marginal remedies will increase average transportation costs. This happens when $\left(v_{1+2}^M - v_3^M\right)$ is small and when t is large because the insiders' quality advantage is mitigated when transportation costs are high.

It should be noted that, in the consumers' perspective, if transportation costs increase, the price decrease more than compensates for this.[16]

As far as average quality is concerned, remedies lower the quality of insiders and increase the one of the outsider. If $\frac{\partial v_3^M}{\partial \beta} = -\frac{\partial v_{1+2}^M}{\partial \beta}$, then the average valuation for quality would decrease: $\frac{\partial \overline{v}^M}{\partial \beta} = -\frac{\partial v_3^M}{\partial \beta}\left(\frac{t+36\left(v_{1+2}^M - v_3^M\right)}{9t}\right) < 0$. Again, this happens because insiders always have a post-merger market share larger than 1/2. However, we have that $\frac{\partial v_3^M}{\partial \beta} > -\frac{\partial v_{1+2}^M}{\partial \beta}$ and average quality can either increase or decrease.[17] Higher transportation costs t and a lower $\left(v_1^M - v_3^M\right)$ decrease the weight associated with the negative effect.

The sign of the overall effect of marginal remedies on consumer surplus corresponds to the sum of these three effects and is given by:

$$\frac{\partial CS^M}{\partial \beta} = \frac{\partial v_3^M}{\partial \beta} + \frac{9\left(v_{1+2}^M - v_3^M\right) + 14t}{27t}\frac{\partial\left(v_{1+2}^M - v_3^M\right)}{\partial \beta} = (1-w)\frac{\partial v_3^M}{\partial \beta} + w\frac{\partial v_{1+2}^M}{\partial \beta} \lesseqgtr 0$$

with $w = \frac{9\left(v_{1+2}^M - v_3^M\right)+14t}{27t}$.

With the specific function for quality valuation, this is equal to

$$\frac{\partial CS^M}{\partial \beta} = \gamma \frac{9\gamma'\left(2\beta - \left(1+\alpha_2-\alpha_3\right)\right) + 13\sqrt{1+\alpha_2-\beta} - 14\sqrt{\alpha_3+\beta}}{54\sqrt{\alpha_3+\beta}\sqrt{1+\alpha_2-\beta}}$$

which is positive at $\beta = 0$ if $\gamma' < \frac{13\sqrt{1+\alpha_2}-14\sqrt{\alpha_3}}{9(1+\alpha_2-\alpha_3)}$. When β is such that the insiders and outsider are symmetric in terms of spectrum the derivative is negative.

Before presenting the optimal remedies, note that

$$\frac{\partial^2 CS^M}{\partial \beta \partial t} = \frac{\gamma^2}{t^2}\frac{\left(1+\alpha_2-\alpha_3\right)-2\beta}{6\sqrt{\alpha_3+\beta}\sqrt{1+\alpha_2-\beta}} > 0.$$

This means that remedies are more efficient (in the sense that the derivative of CS with respect to β is higher) when t is high. But it is possible to show that when t is high (and also when γ is low), the lower will be the merger induced impact on consumers surplus, ΔCS. In other words, the less the merger is consumer beneficial, the more effective the remedies are.[18]

If competition authorities condition the merger approval on the imposition of spectrum divestiture remedies and if the authorities' standard is the consumer surplus standard, the optimal remedies are presented in Proposition 2:

Proposition 2. *Let Assumption 1 hold:*

(a) Optimal remedies are

$$\beta^*(\alpha_2,\alpha_3,\gamma') = \begin{cases} 0 & \text{if } \gamma' > \frac{13\sqrt{1+\alpha_2}-14\sqrt{\alpha_3}}{9(1+\alpha_2-\alpha_3)} \\ \beta : \gamma' = \frac{13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}}{9((1+\alpha_2-\alpha_3)-2\beta)} & \gamma' < \frac{13\sqrt{1+\alpha_2}-14\sqrt{\alpha_3}}{9(1+\alpha_2-\alpha_3)} \end{cases}$$

with $\beta^* \in \left[0, \frac{169\alpha_2-196\alpha_3+169}{365}\right)$.

(b) Remedy β lowers consumer surplus (when compared to the merger with no remedies at all) if and only if

$$\gamma' > \tilde{\gamma}'(\alpha_2,\alpha_3,\beta) := \frac{13\sqrt{\alpha_3} - 13\sqrt{\alpha_3+\beta} + 14\sqrt{\alpha_2+1} - 14\sqrt{\alpha_2+1-\beta}}{9\left(\sqrt{\alpha_3}\sqrt{\alpha_2+1} - \sqrt{\alpha_3+\beta}\sqrt{\alpha_2+1-\beta}\right)}$$

(c) The merger with remedy β decreases consumer surplus if

$$\gamma' < \gamma_R'(\alpha_2,\alpha_3,\beta)$$

with the expression for $\gamma_R'(\alpha_2,\alpha_3,\beta)$ presented in Appendix. ■

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16 This follows from $\frac{\partial \overline{p}_w^M}{\partial \beta} + \frac{\partial T^M}{\partial \beta} = \frac{t+45\left(v_{1+2}^M - v_3^M\right)}{27t}\frac{\partial\left(v_{1+2}^M - v_3^M\right)}{\partial \beta} < 0$.

17 Lowering the spectrum available to one firm while increasing it in the same amount to another firm does not necessarilly mean that the decrease in consumer valuation for the former firm matches the increase for the latter. Given our assumptions on $v_i()$ and on β we have $\frac{\partial\left(\gamma\sqrt{\alpha_3+\beta}\right)}{\partial \beta} > -\frac{\partial\left(\gamma\sqrt{1+\alpha_2-\beta}\right)}{\partial \beta}$.

18 The same cannot be said about γ because the sign of $\frac{\partial^2 CS^M}{\partial \beta \partial \gamma} = \frac{18\frac{\gamma}{t}\left(2\beta-\left(1+\alpha_2-\alpha_3\right)\right)+\left(13\sqrt{1+\alpha_2-\beta}-14\sqrt{\alpha_3+\beta}\right)}{54\sqrt{\alpha_3+\beta}\sqrt{1+\alpha_2-\beta}}$ can be positive or negative.

Table 1
Parameterization of the four examples.

	Case	Figure(s)	α_2	α_3	$\overline{\gamma}'(\alpha_2,\alpha_3)$	$\overline{\beta}(\alpha_2,\alpha_3)$	$\gamma'_{SW}(\alpha_2,\alpha_3)$
Symmetric firms	(a)	(2(a)) and (2(a′))	1	1	$\overline{\gamma}'(1,1) \approx 1.0730$	1/2	8.3421×10^{-3}
Small outsider	(b)	(2(b))	1	1/2	$\overline{\gamma}'(1,\frac{1}{2}) \approx 0.62854$	3/4	9.0725×10^{-3}
Small target	(c)	(2(c)) and (2(c′))	1/2	1	$\overline{\gamma}'(\frac{1}{2},1) \approx 0.94841$	1/4	8.9285×10^{-3}
Large acquirer	(d)	(2(d))	1/2	1/2	$\overline{\gamma}'(\frac{1}{2},\frac{1}{2}) \approx 0.85859$	1/2	9.7751×10^{-3}

Table 2
Sign of effect on consumer surplus of the merger and remedies.

Area	*A*	*B*	*C*	*D*	*E*	*F*
Effect of the merger (with no remedies, $\beta = 0$)	+	+	−	−	+	−
Effect of the remedies (not necessarily $\beta = \beta^*$)	−	+	+	+	−	−
Effect of the merger with remedies	+	+	+	−	−	−

Proposition 2 states that optimal remedies depend on the original spectrum allocation, transportation costs and the relevance of spectrum for consumer valuation or quality.[19] It also states that for sufficiently high levels of γ' optimal remedies are zero (no spectrum reallocation). Finally, it states that even under optimal remedies, a merger may lower consumer welfare. Remedies that reallocate spectrum between the firms are not likely to be successful in leading to an increase in consumer surplus if consumers' valuation does not depend much on spectrum differences and if insiders have a high incentive to raise prices. This happens when γ is small and t is large or, in other words, when γ' is low.

Given these results there are several possibilities about the effect of the merger and eventual remedies, depending on the value taken by parameter γ' and on the ranking of the thresholds presented in Propositions 1 and 2.

For example, let α_2, α_3 and β be such that $0 < \gamma'_R < \gamma'_{CS} < \widetilde{\gamma}' < \overline{\gamma}'$. For $\gamma' \in [0, \gamma'_R]$, the merger is always consumer surplus detrimental, even under optimal remedies. For $\gamma' \in [\gamma'_R, \gamma'_{CS}]$, the merger decreases consumer surplus, but adequate remedies may invert this situation. For $\gamma' \in [\gamma'_{CS}, \widetilde{\gamma}']$, the merger increases consumer surplus but this increase could be higher if optimal remedies were implemented. For $\gamma' \in [\widetilde{\gamma}', \overline{\gamma}']$, the merger increases consumer surplus with no remedies and any remedies would have a negative effect on consumer welfare.

It is also possible that α_2, α_3 and β are such that $0 < \widetilde{\gamma}' < \gamma'_{CS} < \gamma'_R < \overline{\gamma}'$. When this is the case, then, for $\gamma' \in [0, \widetilde{\gamma}']$, the merger has a negative effect on consumer surplus, remedies increase welfare but are insufficient to invert the negative effect of the merger. For $\gamma' \in [\widetilde{\gamma}', \gamma'_{CS}]$, the merger has a negative effect on consumer surplus and remedies make it even worse. For $\gamma' \in [\gamma'_{CS}, \gamma'_R]$, the merger has a positive effect on consumer surplus but remedies invert this situation. Finally, for $\gamma' \in [\gamma'_R, \overline{\gamma}']$ the merger has a positive effect on consumer surplus which dominates the negative effect brought about by remedies.

In order to illustrate these possibilities and that the results are qualitatively similar for different values of α_2 and α_3 we now present several different cases as far as the original distribution of spectrum is concerned.

4.3. Examples

In this section, the relevant thresholds for γ' as well as the optimal remedies are illustrated for four particular cases. We have assumed that $\alpha_i \in \left[\frac{1}{2}, 1\right]$ for $i = 2, 3$ and the cases addressed here should be considered as the most extreme cases. For each case we present graphically the results in Propositions 1 and 2.[20] The Table 1 identifies these cases.

As mentioned in the previous section, depending on the values taken by α_2, α_3 and β there are several possibilities with respect to the effects on consumer surplus of the merger and of eventual remedies. These possibilities are summarized in Table 2. Labels A to F will be used in the figures to identify the areas in which the corresponding effects are verified.

Fig. 2(a) presents all the relevant thresholds for γ' as a function of β when firms are symmetric ex ante.

As Fig. 2(a) illustrates, we have $0 < \gamma'_R < \gamma'_{CS} < \widetilde{\gamma}' < \overline{\gamma}'$ for all admissible β. Hence, Fig. 2(a) presents four areas. In area A, when $\gamma' \in [\widetilde{\gamma}', \overline{\gamma}']$, the merger increases consumer surplus with no remedies and any remedy involving spectrum divestiture would have a negative effect on welfare. In areas identified with B, when $\gamma' \in [\gamma'_{CS}, \widetilde{\gamma}']$, the merger also increases consumer surplus but this increase could be higher if optimal remedies were implemented. In area C, when $\gamma' \in [\gamma'_R, \gamma'_{CS}]$, the merger decreases consumer surplus, but adequate remedies may invert this situation.[21] Finally, in area D, when $\gamma' < \gamma'_R$, the merger is always consumer surplus detrimental, even under optimal remedies.

It is striking that the parameter space that corresponds to area C is relatively small. This means that whenever the merger has a negative effect on welfare, it is relatively unlikely that remedies, even the optimal ones, will invert this situation. The explanation, however, is straightforward. The merger has a negative effect on welfare if transportation costs t are high when compared to how

[19] Corollary 1, presented in the Appendix establishes how γ', α_2 and α_3 affect optimal remedies.

[20] The figures do not present $\gamma'_{SW}(\alpha_2,\alpha_3)$ which is reported in Table 1 because the magnitude of this value is not comparable to the magnitude of the others.

[21] In Figs. 2(a) and 2(c) area C is too small to appear in the Figures as γ'_R is very close to γ'_{CS}. For that reason we present Figs. 2(a′) and 2(c′) with a different scaling in order to make this visible.

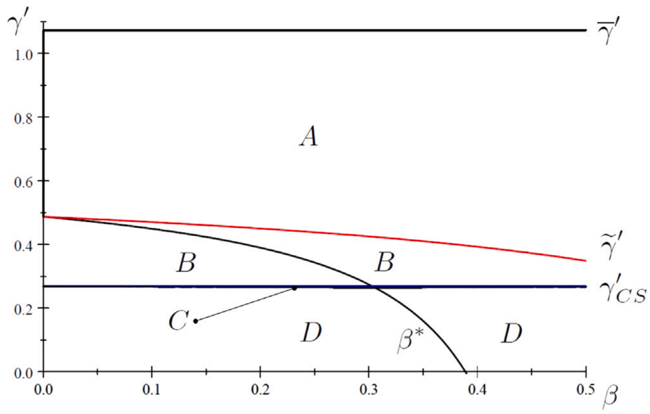


Fig. 2(a). Relevant thresholds for γ' as a function of β when firms are symmetric ex ante.

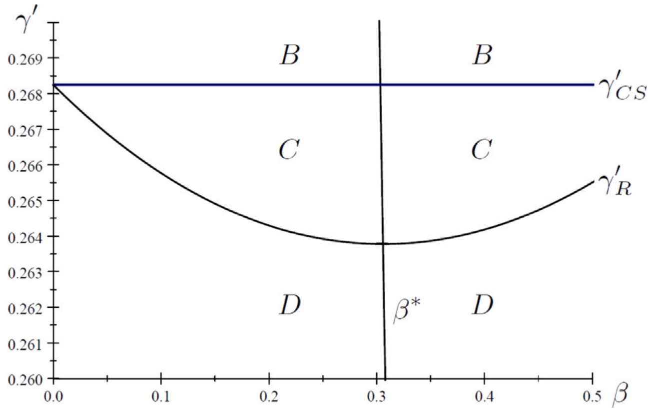


Fig. 2(a′). Relevant thresholds for $\gamma' \in [0.26, 0.27]$ as a function of β when firms are symmetric ex ante.

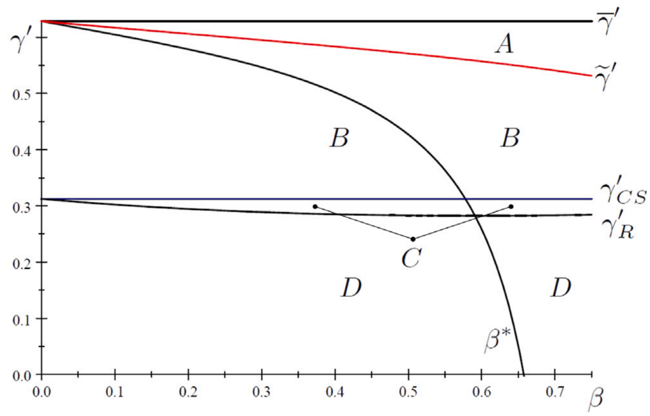


Fig. 2(b). Relevant thresholds for γ' as a function of β when there is a small outsider.

spectrum affects consumer valuation γ. In this case, with no significant increases in valuation for quality and with high price increases due to high consumer "loyalty", the merger will have a negative effect on consumer surplus. But precisely under these circumstances, remedies that reallocate spectrum (and hence, quality) have little scope to produce effects, as consumer valuation does not depend significantly on these.

Fig. 2(a′) reproduces Fig. 2(a) for $\gamma' \in [0.26, 0.27]$ so that γ'_R and γ'_{CS} and area C are visible.

Fig. 2(b) represents the relevant thresholds in case (b), when the outsider has a smaller amount of spectrum than the insiders:

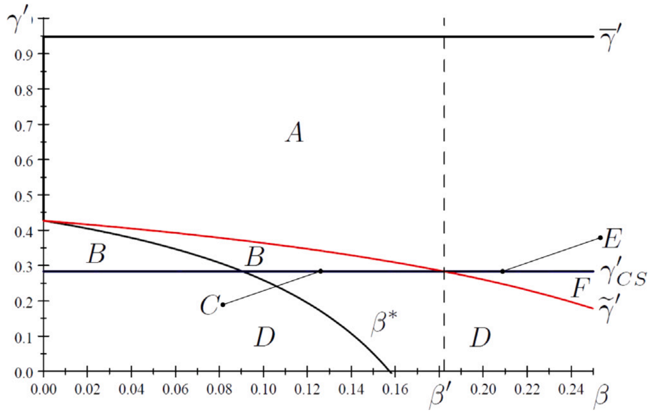


Fig. 2(c). Relevant thresholds for γ' as a function of β when there is a small target.

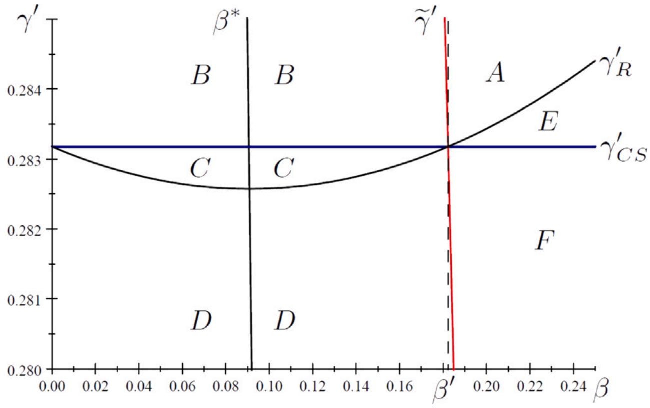


Fig. 2(c′). Relevant thresholds for $\gamma' \in [0.28, 0.285]$ as a function of β when there is a small target.

As $\gamma'_{CS}(1, \alpha_3)$ decreases with α_3, a lower α_3 makes it more likely that consumer surplus decreases with the merger. Essentially this is due to how the merger affects prices. The price increase due to the merger depends negatively on v_3. If the outsider has a lower consumer valuation for quality (i.e., a lower α_3), the insiders will have a higher incentive to increase price because their customer base is larger. Thus, there are less values for γ' such that the merger increases consumer surplus. The fact that the marginal valuation for quality is decreasing increases the scope for remedies to turn a consumer surplus decreasing merger into a beneficial one (a larger area *C* when compared to the previous case). This happens because remedies decrease the quality of the merged firm which was high (and therefore presented a low marginal valuation for quality) while increasing the pre-merger low quality of the outsider (for which the marginal valuation for quality is high).

In case (c), a higher α_2 makes it more likely that consumer surplus increases with the merger because the increase in quality from pooling the input is larger. When compared to the case in which insiders have the same quality of service before the merger, it is not clear at the outset whether the merger is more or less likely to benefit consumers. This happens because the incentives to increase price are lower (as firm 2 has less customers before the merger) and also because the increase in quality by pooling spectrum is also lower. For $\alpha_i > \frac{1}{2}$ it can be established that $\gamma'_{CS}(\alpha, 1) \leq \gamma'_{CS}(1, \alpha)$ meaning that for the same relative size, the merger is more likely to benefit consumers in the present case than in the previous one.

Fig. 2(c) represents the relevant thresholds in this case. The ranking of the relevant thresholds depends in this case on β. For $\beta < \beta' := 0.182\,34$, we have $0 < \gamma'_R < \gamma'_{CS} < \tilde{\gamma}' < \bar{\gamma}'$ whereas for $\beta > \beta'$, we have $0 < \tilde{\gamma}' < \gamma'_{CS} < \gamma'_R < \bar{\gamma}'$. In the latter case, there are two additional possibilities: (i) In area *E*, when $\gamma' \in [\gamma'_{CS}, \gamma'_R]$ the merger increases consumer surplus but the remedies in this area revert this situation; (ii) In area *F*, when $\gamma' \in [\tilde{\gamma}', \gamma'_{CS}]$, the merger decreases consumer surplus and remedies in this area reinforce this negative effect. When $\gamma' \in [\gamma'_R, \bar{\gamma}']$ and $\gamma' \in [0, \tilde{\gamma}']$ we have the same situation as in areas *A* and *D* respectively.

As expected, and for the reasons opposite those in the previous example, area *C* almost disappears from this figure. Fig. 2(c′) reproduces Fig. 2(c) for $\gamma' \in [0.28, 0.285]$ so that γ'_R and γ'_{CS} and areas *C* and *E* are now visible.

The two new possibilities that arise in this case (areas *E* and *F*) refer to cases in which remedies have a negative effect on consumer welfare. This is not surprising because in the case of a small target, the marginal valuation for the quality of the merged firm is relatively high, meaning that spectrum divestiture is more likely to have adverse effects.

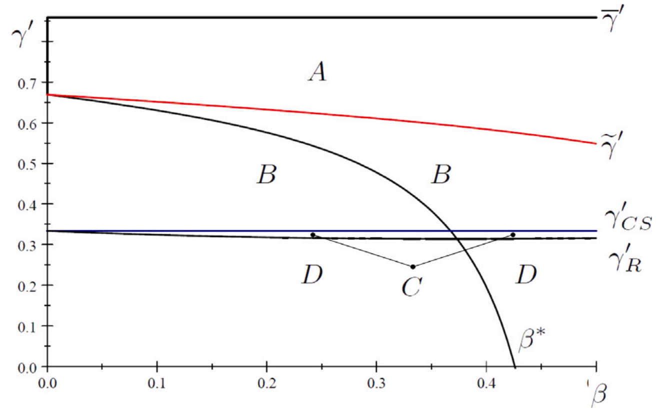


Fig. 2(d). Relevant thresholds for γ' as a function of β when there is a large acquirer.

Fig. 2(d) represents the relevant thresholds when the acquirer is the only large firm and combines features from cases (b) and (c).

5. Conclusions

In this paper, we consider a setting wherein by pooling their spectrum assets through a merger, insiders are able to produce a product or offer a service with a quality that is higher than the one each insider marketed before the merger. A case in point is the mobile telecommunications market, in which each firm is entitled to use a limited amount of spectrum, and that has witnessed several recent mergers. Before the merger, spectrum is more fragmented. Thus, by reducing fragmentation, the merger allows for an increase in the quality of the services provided by the merging parties.

In the absence of any eventual cost savings, the merger will lead to higher prices. However, even in the absence of cost efficiencies, the quality is also likely to increase due to the pooling of the insiders' spectrum holdings. The effect on consumer welfare is, therefore, a priori ambiguous.

In this type of merger, remedies involving the use of spectrum are ubiquitous. Insofar as quality is directly related to the amount of spectrum held by a firm, remedies that reallocate spectrum rights are also ambiguous: the quality of some firms' service may increase while the one of other firms may decrease.

To discuss the competitive effects of mergers in this context and also the effectiveness of spectrum divestiture remedies, we model the industry as a concentrated oligopoly (a triopoly) in which firms sell both horizontally and vertically differentiated products or services. We further assume that consumers have a positive but decreasing marginal valuation for quality and allow for different pre-merger spectrum allocations.

In this context, we conclude that mergers may benefit consumers even in the absence of any cost synergies. This happens when the degree of horizontal differentiation is relatively low and consumer valuation depends significantly on the quantity of spectrum available to the firm. When this is the case, competition pressure exerted by the third firm (the single outsider) is sufficient to make the merger-induced average price increase smaller than the increase in average quality. Our results also indicate that when the merger has a negative impact on consumer surplus, remedies based on reallocation of spectrum are not very likely of changing this outcome. The reason is that the circumstances under which the merger is unlikely to benefit consumers (namely when spectrum has a small impact on consumer valuation for the service) are precisely the circumstances under which the reallocation of spectrum will produce limited results. This should not be interpreted as meaning that whenever the merger hurts consumer surplus, then remedies are less effective. Indeed, when transportation costs are high and therefore the merger is more likely to hurt consumer welfare, it is also true that remedies are more capable of affecting consumer surplus. However, it is highly unlikely (in terms of the relevant parameter space in the model) that, as far as consumer-surplus-diminishing mergers are concerned, remedies, despite their higher effectiveness, will invert the (more) negative effects of the merger.

Concerning social welfare, it turns out that merger induced effects are likely to be positive except when the impact of additional spectrum holdings on consumer valuation is extremely low when compared to transportation (or disutility) costs. As we assume that the market is covered, equilibrium price effects are neutral in terms of social welfare: an increase in price corresponds to a pure transfer fro consumers' surplus to firms' profits. In addition, given that in our setting the merger increases the quality of the services provided by insiders at no additional cost, it is very unlikely that welfare will decrease. This may happen only when transportation (or disutility) costs are very high and/or the impact of quality on consumer valuation is very low.

This model is not without its limitations. We have assumed that the firms' only decision after the merger takes place is to set prices, meaning that the quality of their service is determined by past decisions (i.e. it is exogenous in our model). We are thus implicitly assuming that the majority of the quality defining investments took place before the decision to merge, which is certainly true as far as spectrum is concerned, at least until technological developments or contract duration leads to a new spectrum allocation. However, there are other factors affecting quality that we have abstained from including in the model. An alternative would be to have the firms making decisions about pricing and quality (or pricing and investment) after the merger took place, as in Motta and Tarantino (2021), but including the possibility of combining an asset (spectrum) that could lead to quality improvements. It should be noted that, although Motta and Tarantino (2021) conclude that in the absence of efficiency gains, the merger leads insiders to raise prices and reduce investments, thus reducing consumer surplus, the same need not happen (but may happen) when spectrum pooling is introduced. The reason for the reduction in investment is the lower demand that results from the merger induced price increase. When introducing spectrum pooling, it is possible that the demand for the merged firm increases despite the increase in price and this could lead to higher investment.

Another limitation is related to the existence of bundles. The model is written in terms of stand alone mobile telephony when, in many circumstances, these services are included in a bundle (such as quadruple play including (fixed and mobile) telephony, fixed and broadband internet access, and television). We consider, however, our results are likely to hold in cases when most consumers purchase bundles, when fixed telephony and broadband internet access are considered relatively homogeneous goods and when differences in the composition of TV channels packs, if any, are a form of horizontal differentiation.

Appendix

This appendix presents the proofs of the results in the paper. For convenience of exposition, some of the proofs results are presented in terms of parameter $t' = t/\gamma$ instead of $\gamma' = \gamma/t$.

Proof of Lemma 0. The consumer indifferent between firm 1 (located at 0) and 2 (located at 1/3) is located at x_{12} such that

$$v_1 - p_1 - tx_{12}^2 = v_2 - p_2 - t(\frac{1}{3} - x_{12})^2 \Leftrightarrow x_{12} = \frac{1}{6} + \frac{3}{2t}\left((v_1 - p_1) - (v_2 - p_2)\right)$$

The consumer indifferent between firm 2 and 3 (located at 2/3) is located at x_{23} such that

$$v_2 - p_2 - t(x_{23} - \frac{1}{3})^2 = v_3 - p_3 - t(\frac{2}{3} - x_{23})^2 \Leftrightarrow x_{23} = \frac{1}{2} + \frac{3}{2t}\left((v_2 - p_2) - (v_3 - p_3)\right)$$

The consumer indifferent between firm 3 and 1 is located at x_{31} such that

$$v_3 - p_3 - t(x_{31} - \frac{2}{3})^2 = v_1 - p_1 - t(1 - x_{31})^2 \Leftrightarrow x_{31} = \frac{5}{6} + \frac{3}{2t}\left((v_3 - p_3) - (v_1 - p_1)\right)$$

Thus, the demand faced by each firm is

$$D_1(\mathbf{p}, \mathbf{v}) = x_{12} + (1 - x_{31}) = \frac{3}{2}\frac{2(v_1 - p_1) - (v_2 - p_2) - (v_3 - p_3)}{t} + \frac{1}{3}$$
$$D_2(\mathbf{p}, \mathbf{v}) = x_{23} - x_{12} = \frac{3}{2}\frac{2(v_2 - p_2) - (v_1 - p_1) - (v_3 - p_3)}{t} + \frac{1}{3}$$
$$D_3(\mathbf{p}, \mathbf{v}) = x_{31} - x_{23} = \frac{3}{2}\frac{2(v_3 - p_3) - (v_1 - p_1) - (v_2 - p_2)}{t} + \frac{1}{3}$$

Letting y_i denote $v_i - p_i$, consumer surplus is given by $CS(\mathbf{p}, \mathbf{v}) =$

$$= V + \int_0^{x_{12}} (y_1 - tx^2)dx + \int_{x_{12}}^{x_{23}} (y_2 - t(\frac{1}{3} - x)^2)dx + \int_{x_{23}}^{x_{31}} (y_3 - t(\frac{2}{3} - x)^2)dx + \int_{x_{31}}^{1} (y_1 - t(1 - x)^2)dx$$
$$= V + \frac{2^2 3^3 t\left(\frac{y_1+y_2+y_3}{3}\right) + 162\left(y_1^2 + y_2^2 + y_3^2 - y_1y_2 - y_1y_3 - y_2y_3\right) - t^2}{2^2 3^3 t}$$
$$= V + \overline{v} - \overline{p} + \frac{27}{4}\frac{\sigma^2_{v_i - p_3}}{t} - \frac{1}{108}t$$

As for social welfare we have $SW(\mathbf{p}, \mathbf{v}) =$

$$= V - c + \int_0^{x_{12}} (v_1 - tx^2)dx + \int_{x_{12}}^{x_{23}} (v_2 - t(\frac{1}{3} - x)^2)dx + \int_{x_{23}}^{x_{31}} (v_3 - t(\frac{2}{3} - x)^2)dx + \int_{x_{31}}^{1} (v_1 - t(1 - x)^2)dx$$
$$= V - c + \frac{v_1 + v_2 + v_3}{3} +$$
$$\frac{27}{4t}\left(\frac{2}{9}\left(v_1^2 + v_2^2 + v_3^2 - v_1v_2 - v_1v_3 - v_2v_3\right) - \frac{2}{9}\left(p_1^2 + p_2^2 + p_3^2 - p_1p_2 - p_1p_3 - p_2p_3\right)\right) - \frac{1}{108}t$$
$$= V - c + \overline{v} + \frac{27}{4t}\left(\sigma^2_{v_i} - \sigma^2_{p_i}\right) - \frac{1}{108}t \quad \blacksquare$$

Proof of Lemma 1. First-order conditions for profit maximization of firm $i = 1, 2, 3$ are

$$\frac{\partial \pi_i}{\partial p_i} = \frac{18c + 2t - 36p_i + 9p_j + 9p_k + 18v_i - 9v_j - 9v_k}{6t} = 0$$

that result in

$$p_i = c + \left(\frac{1}{9}t + \frac{2}{5}v_i - \frac{1}{5}v_j - \frac{1}{5}v_k\right)$$

Corresponding quantities are

$$D_i(\mathbf{v}) = \left(\frac{1}{3} - \frac{9}{5}\frac{\overline{v} - v_i}{t}\right)$$

All quantities are positive if and only if $\overline{v} - v_{\min} < \frac{5}{27}t$, where $v_{\min} = \min_i v_i$. This is equivalent to

$$\frac{\gamma\sqrt{s} + \gamma\sqrt{\alpha_2 s} + \gamma\sqrt{\alpha_3 s}}{3} - \min_i \gamma\sqrt{\alpha_i s} < \frac{5}{27}t \Leftrightarrow \frac{t}{\gamma} > \max\left\{\frac{9}{5}\left(1 - 2\sqrt{\alpha_2} + \sqrt{\alpha_3}\right), \frac{9}{5}\left(1 + \sqrt{\alpha_2} - 2\sqrt{\alpha_3}\right)\right\}$$

or

$$\frac{\gamma}{t} < \frac{1}{\max\left\{\frac{9}{5}\left(1 - 2\sqrt{\alpha_2} + \sqrt{\alpha_3}\right), \frac{9}{5}\left(1 + \sqrt{\alpha_2} - 2\sqrt{\alpha_3}\right)\right\}}$$

which is implied by Assumption 1.

Average price, weighted by the corresponding market shares is

$$\begin{aligned}\overline{p}_w(\mathbf{v}) &= \sum_{i=1}^{3} p_i D_i(\mathbf{v}) = \sum_{i=1}^{3}\left(c + \left(\frac{1}{9}t + \frac{2}{5}v_i - \frac{1}{5}v_j - \frac{1}{5}v_k\right)\right)\left(\frac{1}{3} - \frac{9}{5}\frac{\overline{v} - v_i}{t}\right)\\ &= c + \frac{1}{9}t + \frac{81}{25}\frac{\sigma_v^2}{t}\end{aligned}$$

Average valuation for quality, weighted by the corresponding market shares is

$$\begin{aligned}\overline{v}_w(\mathbf{v}) &= \sum_{i=1}^{3} v_i D_i(\mathbf{v}) = \sum_{i=1}^{3} v_i\left(\frac{1}{3} - \frac{9}{5}\frac{\overline{v} - v_i}{t}\right)\\ &= \overline{v} + \frac{27}{5}\frac{\sigma_v^2}{t}\end{aligned}$$

Total transportation costs are

$$\begin{aligned}T(\mathbf{v}) &= \int_0^{\frac{1}{6}+\frac{3}{2t}(y_1-y_2)} tx^2 dx + \int_{\frac{1}{6}+\frac{3}{2t}(y_1-y_2)}^{\frac{1}{2}+\frac{3}{2t}(y_2-y_3)} t(\frac{1}{3} - x)^2 dx + \int_{\frac{1}{2}+\frac{3}{2t}(y_2-y_3)}^{\frac{5}{6}+\frac{3}{2t}(y_3-y_1)} t(\frac{2}{3} - x)^2 dx + \int_{\frac{5}{6}+\frac{3}{2t}(y_3-y_1)}^{1} t(1 - x)^2 dx\\ &= \frac{1}{108}t + \frac{27}{25}\frac{\sigma_v^2}{t}\end{aligned}$$

Consumer surplus is

$$CS(\mathbf{v}) = V - c + \frac{v_1 + v_2 + v_3}{3} + \frac{648\left(v_1^2 + v_2^2 + v_3^2 - v_1v_2 - v_1v_3 - v_2v_3\right) - 325t^2}{2700t}$$

and pre-merger profits are

$$\pi_i(\mathbf{v}) = \frac{\left(5t + 9\left(2v_i - v_j - v_k\right)\right)^2}{675t}.$$

Finally, social welfare is

$$\begin{aligned}SW(\mathbf{v}) &= V - c + \overline{v} + \frac{27}{4t}\left(\sigma_{v_i}^2 - \sigma_{p_i}^2\right) - \frac{1}{108}t\\ &= V - c + \frac{v_1 + v_2 + v_3}{3} + \frac{2592\left(v_1^2 + v_2^2 + v_3^2 - v_1v_2 - v_1v_3 - v_2v_3\right)}{2700t} - \frac{1}{108}t \quad \blacksquare\end{aligned}$$

Proof of Lemma 2. In this proof we omit the superscript M in variables p_i and v_i and assume initially that v_1^M may differ from v_2^M. After the merger between firms 1 and 2, first-order conditions are

$$\begin{aligned}\frac{\partial\left(\pi_1 + \pi_2\right)}{\partial p_1} &= \frac{9c + 2t - 36p_1 + 18p_2 + 9p_3 + 18v_1 - 9v_2 - 9v_3}{6t} = 0\\ \frac{\partial\left(\pi_1 + \pi_2\right)}{\partial p_2} &= \frac{9c + 2t + 18p_1 - 36p_2 + 9p_3 - 9v_1 + 18v_2 - 9v_3}{6t} = 0\\ \frac{\partial \pi_3}{\partial p_3} &= \frac{18c + 2t + 9p_1 + 9p_2 - 36p_3 - 9v_1 - 9v_2 + 18v_3}{6t} = 0\end{aligned}$$

that result in

$$\begin{aligned}p_1 &= c + \frac{5}{27}t + \frac{5}{12}v_1 - \frac{1}{12}v_2 - \frac{1}{3}v_3\\ p_2 &= c + \frac{5}{27}t - \frac{1}{12}v_1 + \frac{5}{12}v_2 - \frac{1}{3}v_3\\ p_3 &= c + \frac{4}{27}t - \frac{1}{6}v_1 - \frac{1}{6}v_2 + \frac{1}{3}v_3\end{aligned}$$

Equilibrium demands are

$$D_1(\mathbf{v}) = \frac{99v_1 - 63v_2 - 36v_3 - 4t}{72t} + \frac{1}{3} = \frac{63\left(v_1 - v_2\right) + 36(v_1 - v_3)}{72t} + \frac{5}{18}$$
$$D_2(\mathbf{v}) = \frac{99v_2 - 63v_1 - 36v_3 - 4t}{72t} + \frac{1}{3} = \frac{63\left(v_2 - v_1\right) + 36\left(v_2 - v_3\right)}{72t} + \frac{5}{18}$$
$$D_3(\mathbf{v}) = \frac{18v_3 - 9v_1 - 9v_2 + 2t}{18t} + \frac{1}{3} = \frac{9\left(v_3 - v_1\right) + 9\left(v_3 - v_2\right)}{18t} + \frac{8}{18}$$

Assuming without loss of generality that $v_1 \geq v_2$, then $D_1(\mathbf{v}) > D_2(\mathbf{v})$. All quantities are positive if

$$D_2(\mathbf{v}) = \frac{-4t - 63v_1 + 99v_2 - 36v_3}{72t} + \frac{1}{3} > 0 \Leftrightarrow \overline{v} - v_3 > \frac{162\left(v_1 - v_3\right) - 20t}{297}$$
$$D_3(\mathbf{v}) = \frac{2t - 9v_1 - 9v_2 + 18v_3}{18t} + \frac{1}{3} > 0 \Leftrightarrow \overline{v} - v_3 < \frac{8}{27}t$$

Note that the first inequality can be written as $\frac{9}{20}(7v_1 - 11v_2 + 4v_3) < t$. After the merger, even with remedies, we have assumed that $v_1 = v_2 \geq v_3$. Thus, this always holds. The second inequality is equivalent to

$$\frac{2\gamma\sqrt{\left(1 + \alpha_2 - \beta\right)s} + \gamma\sqrt{\left(\alpha_3 + \beta\right)s}}{3} - \gamma\sqrt{\left(\alpha_3 + \beta\right)s} < \frac{8}{27}t \Leftrightarrow$$
$$\frac{\gamma}{t} < \frac{4}{9\left(\sqrt{1 + \alpha_2 - \beta} - \sqrt{\alpha_3 + \beta}\right)}$$

which is implied by Assumption 1 for any β.

Average price, weighted by the corresponding market shares is

$$\overline{p}_w^M(\mathbf{v}) = \sum_{i=1}^{3} p_i^M D_i(\mathbf{v}^M) = c + \frac{41}{243}t + \frac{18t\left(v_{1+2}^M - v_3^M\right) + 162\left(v_{1+2}^M - v_3^M\right)^2}{243t}$$

Average valuation for quality, weighted by the corresponding market shares is

$$\overline{v}_w^M(\mathbf{v}) = \sum_{i=1}^{3} v_i^M D_i(\mathbf{v}^M) = v_3^M + \frac{5t\left(v_{1+2}^M - v_3^M\right) + 9\left(v_{1+2}^M - v_3^M\right)^2}{9t}$$

Total transportation costs are

$$T^M(\mathbf{v}^M) = \frac{11t^2 - 36t\left(v_{1+2}^M - v_3^M\right) + 162\left(v_{1+2}^M - v_3^M\right)^2}{972t}$$

Corresponding consumer surplus is

$$CS^M = V - c + \frac{288t\left(7v_1 + 7v_2 + 13v_3\right) + 81\left(31v_1^2 + 31v_2^2 + 16v_3^2 - 46v_1v_2 - 16v_1v_3 - 16v_2v_3\right) - 1400t^2}{7776t}$$

Profits are

$$\pi_{1+2}^M = \frac{720t\left(v_1 + v_2 - 2v_3\right) + 2511\left(v_1^2 + v_2^2\right) - 1296v_3\left(v_1 + v_2 - v_3\right) - 3726v_1v_2 + 400t^2}{3888t}$$
$$\pi_3^M = \frac{\left(8t - 9\left(v_1 + v_2 - 2v_3\right)\right)^2}{972t}$$

Finally, social welfare is given by

$$SW^M = V - c + \frac{288t\left(8v_1 + 8v_2 + 11v_3\right) + 81\left(101v_1^2 + 101v_2^2 + 80v_3^2 - 122v_1v_2 - 80v_1v_3 - 80v_2v_3\right)}{7776t} - \frac{11}{972}t$$

Making $v_1^M = v_2^M = v_{1+2}^M$ yields the result.

With respect to how the consumer surplus component change with remedies β we have:

(i) remedies decrease the average price:

$$\frac{\partial\overline{p}_w^M}{\partial\beta} = \frac{2}{27}\frac{18\left(v_{1+2}^M - v_3^M\right) + t}{t}\frac{\partial\left(v_{1+2}^M - v_3^M\right)}{\partial\beta} < 0;$$

(ii) remedies have an ambiguous effect on transportation costs

$$\frac{\partial T^M}{\partial\beta} = \frac{9\left(v_{1+2}^M - v_3^M\right) - t}{27t}\frac{\partial\left(v_{1+2}^M - v_3^M\right)}{\partial\beta} \lesseqgtr 0;$$

(iii) remedies have an ambiguous effect on average valuation for quality:

$$\frac{\partial \overline{v}^M}{\partial \beta} = \frac{\partial v_3^M}{\partial \beta} + \frac{18\left(v_{1+2}^M - v_3^M\right) + 5t}{9t}\frac{\partial\left(v_{1+2}^M - v_3^M\right)}{\partial \beta} \lessgtr 0$$
$$= \left(\frac{4t - 18\left(v_{1+2}^M - v_3^M\right)}{9t}\right)\frac{\partial v_3^M}{\partial \beta} + \frac{18\left(v_{1+2}^M - v_3^M\right) + 5t}{9t}\frac{\partial v_{1+2}^M}{\partial \beta} \lessgtr 0$$

Assumption 1 implies that $9\left(v_{1+2}^M - v_3^M\right) - 4t < 0$. This condition does not allow us to sign the second and third effects. A higher $v_{1+2}^M - v_3^M$ and lower t make both effects more likely to be positive. Note that $\frac{18\left(v_{1+2}^M - v_3^M\right)+t}{t}$, $\frac{9\left(v_{1+2}^M - v_3^M\right)-t}{27t}$ and $\frac{18\left(v_{1+2}^M - v_3^M\right)+5t}{9t}$ all decrease with t. ■

Proof of Proposition 1. Before the merger, we have $v_1 = \gamma\sqrt{s}$, $v_2 = \gamma\sqrt{\alpha_2 s}$, $v_3 = \gamma\sqrt{\alpha_3 s}$ and after the merger, we have $v_{1+2}^M = \gamma\sqrt{s}\sqrt{1+\alpha_2-\beta}$ and $v_3^M = \gamma\sqrt{s}\sqrt{\alpha_3+\beta}$ with $s = 1$.

(a) Pre-merger consumer surplus (divided by γ^2) is given by

$$\frac{CS(\mathbf{v})}{\gamma^2} = \frac{1}{\gamma^2}\left(V - c + \frac{v_1+v_2+v_3}{3} + \frac{648\left(v_1^2+v_2^2+v_3^2-v_1v_2-v_1v_3-v_2v_3\right)-325t^2}{2700t}\right)$$
$$= \frac{V-c}{\gamma^2} + \frac{648\left(1+\alpha_3+\alpha_2-\sqrt{\alpha_3}-\sqrt{\alpha_2}-\sqrt{\alpha_3}\sqrt{\alpha_2}\right)+900\left(\sqrt{\alpha_3}+\sqrt{\alpha_2}+1\right)t'-325(t')^2}{2700t}$$

where $\frac{t}{\gamma}$ is denoted by t'.

Assuming remedies β, post-merger consumer surplus (divided by γ^2) can be written as

$$\frac{CS^M(\mathbf{v}^M,\beta)}{\gamma^2} = \frac{V-c}{\gamma^2} + \frac{\left(\begin{array}{c}162\left(\alpha_3+\alpha_2+1\right)-324\sqrt{\alpha_3+\beta}\sqrt{\alpha_2-\beta+1}-175(t')^2\\ +t'\left(504\sqrt{\alpha_2-\beta+1}+468\sqrt{\alpha_3+\beta}\right)\end{array}\right)}{972t}$$

The normalized change in consumers surplus is

$$\Delta CS = \left(CS^M(\mathbf{v}^M,\beta) - CS(\mathbf{v})\right)/(\gamma^2) =$$
$$-\frac{\left(\begin{array}{c}725(t')^2+t'\left(4050\left(\sqrt{\alpha_3}+\sqrt{\alpha_2}+1\right)-6300\sqrt{\alpha_2-\beta+1}-5850\sqrt{\alpha_3+\beta}\right)\\ +891\left(\alpha_3+\alpha_2+1\right)+4050\sqrt{\alpha_3+\beta}\sqrt{\alpha_2-\beta+1}-2916\left(\sqrt{\alpha_3}+\sqrt{\alpha_2}+\sqrt{\alpha_3}\sqrt{\alpha_2}\right)\end{array}\right)}{12150t}$$

With no remedies, $\beta = 0$, this simplifies to

$$\Delta CS = -\frac{\left(\begin{array}{c}725(t')^2+t'\left(4050\left(\sqrt{\alpha_3}+\sqrt{\alpha_2}+1\right)-6300\sqrt{\alpha_2+1}-5850\sqrt{\alpha_3}\right)\\ +891\left(\alpha_3+\alpha_2+1\right)+4050\sqrt{\alpha_3}\sqrt{\alpha_2+1}-2916\left(\sqrt{\alpha_3}+\sqrt{\alpha_2}+\sqrt{\alpha_3}\sqrt{\alpha_2}\right)\end{array}\right)}{12150t}$$

The two roots are

$$t_{CS}^{\prime +} = \frac{9\left(4\sqrt{\alpha_3}+14\sqrt{\alpha_2+1}-9\sqrt{\alpha_2}-9\right)}{29} + \frac{27}{145}\sqrt{f(\alpha_2,\alpha_3)}$$
$$t_{CS}^{\prime -} = \frac{9\left(4\sqrt{\alpha_3}+14\sqrt{\alpha_2+1}-9\sqrt{\alpha_2}-9\right)}{29} - \frac{27}{145}\sqrt{f(\alpha_2,\alpha_3)}$$

with

$$f(\alpha_2,\alpha_3) = 9\alpha_3 + 734\left(\alpha_2+1\right) - 84\sqrt{\alpha_3}\left(\sqrt{\alpha_2}+1\right) + 566\sqrt{\alpha_2} + 150\sqrt{\alpha_3}\sqrt{\alpha_2+1} - 700\sqrt{\alpha_2+1}\left(\sqrt{\alpha_2}+1\right)$$

and the change in consumers surplus is positive inside the roots:

$$t_{CS}^{\prime -} < t' < t_{CS}^{\prime +}$$

It is possible to show numerically that $t_{CS}^{+} > 0$ and $t_{CS}^{-} < \underline{t'}$ where $\underline{t'}$ is the minimum value of t' implied by Assumption 1. Thus, consumer surplus increases with the merger if $\underline{t'} < t' < t_{CS}^{+}$ and decreases if $t' > t_{CS}^{+}$, which, in terms of γ' corresponds to $\gamma_{CS}' < \gamma' < \overline{\gamma'}$ with

$$\gamma_{CS}' = \frac{1}{t_{CS}^{\prime +}}$$

$$= \frac{\begin{pmatrix} -2025\left(\sqrt{\alpha_2}+1\right)+900\sqrt{\alpha_3}+3150\sqrt{\alpha_2+1} \\ -135\sqrt{\begin{matrix} 734\left(\alpha_2+1\right)+9\alpha_3+566\sqrt{\alpha_2}+150\sqrt{\alpha_3}\sqrt{\alpha_2+1} \\ -84\sqrt{\alpha_3}\left(\sqrt{\alpha_2}+1\right)-700\sqrt{\alpha_2+1}\left(\sqrt{\alpha_2}+1\right)\end{matrix}}\end{pmatrix}}{891\left(\alpha_2+\alpha_3+1\right)-2916\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+\sqrt{\alpha_2}\sqrt{\alpha_3}\right)+4050\sqrt{\alpha_3}\sqrt{\alpha_2+1}}$$

(b) Pre-merger social welfare (divided by γ^2) is given by

$$\frac{SW(\mathbf{v})}{\gamma^2} = \frac{1}{\gamma^2}\left(V-c+\frac{v_1+v_2+v_3}{3}+\frac{2592\left(v_1^2+v_2^2+v_3^2-v_1v_2-v_1v_3-v_2v_3\right)}{2700t}-\frac{1}{108}t\right)$$
$$= \frac{V-c}{\gamma^2}+\frac{-25(t')^2+900t'\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+1\right)+2592\left(\alpha_2+\alpha_3-\sqrt{\alpha_2}-\sqrt{\alpha_3}-\sqrt{\alpha_2}\sqrt{\alpha_3}+1\right)}{2700t}$$

where $\frac{t}{\gamma}$ is denoted by t'.

Assuming remedies given by βs, post-merger social welfare (divided by $\gamma^2 s$) can be written as

$$\frac{SW^M(\mathbf{v}^M,\beta)}{\gamma^2} = \frac{1}{\gamma^2}\left(V-c+\frac{2t\left(16v_1+11v_3\right)+45\left(v_1-v_3\right)^2}{54t}-\frac{11}{972}t\right)$$
$$= \left(\frac{V-c}{\gamma^2}+\frac{\begin{matrix}-11(t')^2+36t'\left(16\sqrt{1+\alpha_2-\beta}+11\sqrt{\beta+\alpha_3}\right)+ \\ 810\left(\alpha_2+\alpha_3-2\sqrt{\beta+\alpha_3}\sqrt{1+\alpha_2-\beta}+1\right)\end{matrix}}{972t}\right)$$

The normalized change in social welfare is

$$\Delta SW = \left(SW^M(\mathbf{v}^M,\beta)-SW(\mathbf{v})\right)/(\gamma^2) =$$
$$\frac{\begin{pmatrix}-25(t')^2-450t\left(9\sqrt{\alpha_2}+9\sqrt{\alpha_3}-16\sqrt{1+\alpha_2-\beta}-11\sqrt{\beta+\alpha_3}+9\right) \\ -81\left(19\left(\alpha_2+\alpha_3+1\right)+250\sqrt{\beta+\alpha_3}\sqrt{1+\alpha_2-\beta}-144\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+\sqrt{\alpha_2}\sqrt{\alpha_3}\right)\right)\end{pmatrix}}{12150t}$$

With no remedies, $\beta=0$, this simplifies to

$$\Delta SW = \frac{\begin{pmatrix}-25(t')^2-450t'\left(9\sqrt{\alpha_2}-2\sqrt{\alpha_3}-16\sqrt{\alpha_2+1}+9\right) \\ -81\left(19\left(\alpha_2+\alpha_3+1\right)+250\sqrt{\alpha_3}\sqrt{\alpha_2+1}-144\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+\sqrt{\alpha_2}\sqrt{\alpha_3}\right)\right)\end{pmatrix}}{12150t}$$

The two roots are

$$t'^{+}_{SW} = -9\left(9\sqrt{\alpha_2}-2\sqrt{\alpha_3}-16\sqrt{\alpha_2+1}+9\right)+\frac{27}{5}\sqrt{g(\alpha_2,\alpha_3)}$$
$$t'^{-}_{SW} = -9\left(9\sqrt{\alpha_2}-2\sqrt{\alpha_3}-16\sqrt{\alpha_2+1}+9\right)-\frac{27}{5}\sqrt{g(\alpha_2,\alpha_3)}$$

with

$$g(\alpha_2,\alpha_3) = 934\left(\alpha_2+1\right)+9\alpha_3+466\sqrt{\alpha_2}+150\sqrt{\alpha_3}\sqrt{\alpha_2+1}-4\left(\sqrt{\alpha_2}+1\right)\left(21\sqrt{\alpha_3}+200\sqrt{\alpha_2+1}\right)$$

and the change in social welfare is positive inside the roots:

$$t'^{-}_{SW} < t' < t'^{+}_{SW}$$

It is possible to show numerically that $t'^{-}_{SW} < \underline{t'} < t'^{+}_{SW}$. Thus, social welfare increases with the merger if $\underline{t'} < t' < t'^{+}_{SW}$ and decreases if $t' > t'^{+}_{SW}$ which means that, in terms of γ', the condition for social welfare to improve is $\gamma'_{SW} < \gamma' < \overline{\gamma'}$ with

$$\gamma'_{SW} = \frac{1}{t'^{+}_{SW}}$$
$$= -\frac{\begin{pmatrix}2025\left(\sqrt{\alpha_2}+1\right)-450\sqrt{\alpha_3}-3600\sqrt{\alpha_2+1}+ \\ 135\sqrt{\begin{matrix}934\left(\alpha_2+1\right)+9\alpha_3+466\sqrt{\alpha_2}-84\sqrt{\alpha_3}\left(\sqrt{\alpha_2}+1\right) \\ -800\sqrt{\alpha_2+1}\left(\sqrt{\alpha_2}+1\right)+150\sqrt{\alpha_3}\sqrt{\alpha_2+1}\end{matrix}}\end{pmatrix}}{1539\left(\alpha_2+\alpha_3+1\right)-11\,664\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+\sqrt{\alpha_2}\sqrt{\alpha_3}\right)+20\,250\sqrt{\alpha_3}\sqrt{\alpha_2+1}} \quad \blacksquare$$

Proof of Proposition 2. From Assumption 1,

$$t' > \underline{t'} := \max\left\{\frac{9}{5}\left(1-2\sqrt{\alpha_2}+\sqrt{\alpha_3}\right), \frac{9}{5}\left(1-2\sqrt{\alpha_3}+\sqrt{\alpha_2}\right), \frac{9}{4}\left(\sqrt{\alpha_2+1}-\sqrt{\alpha_3}\right)\right\}$$

(a) From Lemma 2, post-merger consumer surplus is given by

$$CS = V - c + \frac{\begin{array}{c}288t\left(7v_1^M+7v_2^M+13v_3^M\right)+\\ 81\left(31v_1^{M2}+31v_2^{M2}+16v_3^{M2}-46v_1^Mv_2^M-16v_1^Mv_3^M-16v_2^Mv_3^M\right)-1400t^2\end{array}}{7776t}$$

which, with $v_1^M = v_2^M$, simplifies to

$$\begin{aligned}CS &= V - c + \frac{2^2 3^2 t\left(14v_1^M+13v_3^M\right)+2*3^4\left(v_1^M-v_3^M\right)^2-5^2 7t^2}{2^2 3^5 t}\\ &= V-c+\frac{14v_1^M+13v_3^M}{3^3}+\frac{\left(v_1^M-v_3^M\right)^2}{2\times 3t}-\frac{5^2 7}{2^2 3^5}t\end{aligned}$$

The marginal impact of divestitures on consumer surplus is given by

$$\frac{\partial CS^M}{\partial \beta} = \frac{1}{3}\left(\frac{14}{3^2}\frac{\partial v_1^M}{\partial \beta}+\frac{13}{3^2}\frac{\partial v_3^M}{\partial \beta}+\left(\frac{v_1^M-v_3^M}{t}\right)\left(\frac{\partial v_1^M}{\partial \beta}-\frac{\partial v_3^M}{\partial \beta}\right)\right)$$

Under the assumption that $v_1^M = \gamma\sqrt{1+\alpha_2-\beta}$ and $v_3^M = \gamma\sqrt{\alpha_3+\beta}$, the first-order condition (FOC) for consumer surplus maximization is

$$\frac{\partial CS^M}{\partial \beta} = \gamma^2\frac{9\left(2\beta-1+\alpha_3-\alpha_2\right)+t'\left(13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}\right)}{54t\sqrt{\beta+\alpha_3}\sqrt{1+\alpha_2-\beta}} = 0$$

With the assumption that the outsider cannot have more spectrum than the insiders:

$$1+\alpha_2-\beta > \alpha_3+\beta \Leftrightarrow 2\beta-1-\alpha_2+\alpha_3 < 0 \Leftrightarrow \beta < \frac{1+\alpha_2-\alpha_3}{2}$$

one needs that $\left(13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}\right) > 0$ for the FOC to hold. Otherwise, the derivative is always negative.

This means that the divestiture that satisfies the FOC must verify:

$$13^2(1+\alpha_2-\beta)-14^2(\beta+\alpha_3) > 0 \Leftrightarrow \beta < \frac{169\alpha_2-196\alpha_3+169}{365}$$

with $\frac{169\alpha_2-196\alpha_3+169}{365} < \frac{1+\alpha_2-\alpha_3}{2}$.

The corresponding second-order condition (SOC) is

$$\frac{\partial^2 CS^M}{\partial \beta^2} = \gamma^2\frac{9\left(\alpha_2+\alpha_3+1\right)^2-t'\left(13\left(1+\alpha_2-\beta\right)^{\frac{3}{2}}+14\left(\beta+\alpha_3\right)^{\frac{3}{2}}\right)}{108t\left(\sqrt{\beta+\alpha_3}\right)^3\left(\sqrt{1+\alpha_2-\beta}\right)^3} < 0$$

The divestiture β that verifies the FOC is such that

$$t' = -\frac{9\left(2\beta-1+\alpha_3-\alpha_2\right)}{\left(13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}\right)}$$

Plugging this into the SOC we obtain

$$9\left(\alpha_2+\alpha_3+1\right)^2+\frac{9\left(2\beta-1+\alpha_3-\alpha_2\right)\left(13\left(1+\alpha_2-\beta\right)^{\frac{3}{2}}+14\left(\beta+\alpha_3\right)^{\frac{3}{2}}\right)}{\left(13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}\right)} < 0 \Leftrightarrow$$

$$9\frac{\begin{pmatrix}13\sqrt{1+\alpha_2-\beta}\left(\left(2\alpha_2+2\alpha_3+1\right)+\left(\alpha_2+\alpha_3\right)^2+\left(1-\beta+\alpha_2\right)\left(2\beta-\alpha_2+\alpha_3-1\right)\right)\\ -14\sqrt{\beta+\alpha_3}\left(\left(\alpha_2+\alpha_3+1\right)^2-\left(\beta+\alpha_3\right)\left(2\beta-\alpha_2+\alpha_3-1\right)\right)\end{pmatrix}}{13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}} < 0$$

If $\left(\left(2\alpha_2+2\alpha_3+1\right)+\left(\alpha_2+\alpha_3\right)^2+\left(1-\beta+\alpha_2\right)\left(2\beta-\alpha_2+\alpha_3-1\right)\right) < 0$ the SOC holds.

If $\left(\left(2\alpha_2+2\alpha_3+1\right)+\left(\alpha_2+\alpha_3\right)^2+\left(1-\beta+\alpha_2\right)\left(2\beta-\alpha_2+\alpha_3-1\right)\right) > 0$ it holds if and only if

$$\begin{aligned}&13\sqrt{1+\alpha_2-\beta}\left(\left(2\alpha_2+2\alpha_3+1\right)+\left(\alpha_2+\alpha_3\right)^2+\left(1-\beta+\alpha_2\right)\left(2\beta-\alpha_2+\alpha_3-1\right)\right)\\ &< 14\sqrt{\beta+\alpha_3}\left(\left(\alpha_2+\alpha_3+1\right)^2-\left(\beta+\alpha_3\right)\left(2\beta-\alpha_2+\alpha_3-1\right)\right)\end{aligned}$$

As both sides of the inequality are positive, this is equivalent to:

$$13^2(1+\alpha_2-\beta)\left(\left(2\alpha_2+2\alpha_3+1\right)+\left(\alpha_2+\alpha_3\right)^2+\left(1-\beta+\alpha_2\right)\left(2\beta-\alpha_2+\alpha_3-1\right)\right)^2$$
$$-14^2(\beta+\alpha_3)\left(\left(\alpha_2+\alpha_3+1\right)^2-\left(\beta+\alpha_3\right)\left(2\beta-\alpha_2+\alpha_3-1\right)\right)^2<0$$

which can be simplified to:

$$\left(\beta+\alpha_3\right)\left(\alpha_2+1-\beta\right)f(\beta,\alpha_2,\alpha_3)<0$$

with

$$f(\beta,\alpha_2,\alpha_3)=\begin{pmatrix}1460\beta^3-12\beta^2\left(169\alpha_2-196\alpha_3+169\right)+3\beta\left(311\alpha_2-419\alpha_3+311\right)\left(1+\alpha_2-\alpha_3\right)\\ -\left(4+4\alpha_2-\alpha_3\right)\left(98\alpha_2-74\alpha_3+49\alpha_2^2+169\alpha_3^2-74\alpha_2\alpha_3+49\right)\end{pmatrix}$$

We now show that $f(\beta,\alpha_2,\alpha_3)$ is always negative. We start by showing that $f(\beta,0,\alpha_3)<0$ and then that $\frac{\partial f(\beta,\alpha_2,\alpha_3)}{\partial\alpha_2}<0$.

(i) $f(\beta,0,\alpha_3)=-\left(4-5\beta-\alpha_3\right)\left(292\beta^2+4\left(103\alpha_3-43\right)\beta-74\alpha_3+169\alpha_3^2+49\right)$

As $(4\left(103\alpha_3-43\right))^2-4*292*(-74\alpha_3+169\alpha_3^2+49)=-27\,648\left(\alpha_3+1\right)^2<0$ there are no real roots in β in $\left(292\beta^2+4\left(103\alpha_3-43\right)\beta-74\alpha_3+169\alpha_3^2+49\right)$ and the second term is always positive. The first term is positive if $\beta<\frac{4-\alpha_3}{5}$ which is ensured by $\beta<\frac{169\alpha_2-196\alpha_3+169}{365}$ if $\frac{169\alpha_2-196\alpha_3+169}{365}-\frac{4-\alpha_3}{5}<0$ which is true for $\alpha_i>\frac{46}{123}=0.373\,98$.[22]

(ii) $\frac{\partial f(\beta,\alpha_2,\alpha_3)}{\partial\alpha_2}=-6\left(338\beta^2-\beta\left(311\alpha_2-365\alpha_3+311\right)+196\alpha_2-115\alpha_3+98\alpha_2^2+125\alpha_3^2-115\alpha_2\alpha_3+98\right)$

As $\left(311\alpha_2-365\alpha_3+311\right)^2-4*338*(196\alpha_2-115\alpha_3+98\alpha_2^2+125\alpha_3^2-115\alpha_2\alpha_3+98)=-35\,775\left(\alpha_2+\alpha_3+1\right)^2<0$ there are no real roots in β and this expression is always positive.

Therefore, the optimal remedy β^* is implicitly given by

$$t'=\frac{-9\left(2\beta-1+\alpha_3-\alpha_2\right)}{\left(13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}\right)}$$

This expression increases in β, which must be positive. Therefore, one needs that $t'>\frac{9(1+\alpha_2-\alpha_3)}{13\sqrt{1+\alpha_2}-14\sqrt{\alpha_3}}$ for the optimal remedies to be positive.

(b) Remedies do not increase consumer surplus if and only if

$$CS^M(\mathbf{v}^M,\beta)-CS^M(\mathbf{v}^M,0)\leq 0$$

which, given the expression for $CS^M(\mathbf{v}^M,\beta)$ presented in Proposition 1 is equivalent to

$$f(\alpha_2,\alpha_3,\beta)t'\geqslant 9\left(\sqrt{\alpha_3}\sqrt{\alpha_2+1}-\sqrt{\alpha_3+\beta}\sqrt{\alpha_2+1-\beta}\right)$$

with $f(\alpha_2,\alpha_3,\beta)=\left(13\sqrt{\alpha_3}-13\sqrt{\alpha_3+\beta}+14\sqrt{\alpha_2+1}-14\sqrt{\alpha_2+1-\beta}\right)$

Recall that optimal remedies are positive if $\left(13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}\right)>0$. If it is not the case it is clear that any remedy will lower consumer surplus.

Assuming $\left(13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}\right)>0$, then $\frac{\partial f(\alpha_2,\alpha_3,\beta)}{\partial\beta}=-\frac{13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}}{2\sqrt{\beta+\alpha_3}\sqrt{1+\alpha_2-\beta}}<0$ and $f(\alpha_2,\alpha_3,0)=0$. Thus, $f(\alpha_2,\alpha_3,\beta)<0$ for any $\beta>0$.

Remedies β do not increase consumer surplus if and only if

$$t'<\widetilde{t'}(\alpha_2,\alpha_3,\beta):=\frac{9\left(\sqrt{\alpha_3}\sqrt{\alpha_2+1}-\sqrt{\alpha_3+\beta}\sqrt{\alpha_2+1-\beta}\right)}{\left(13\sqrt{\alpha_3}-13\sqrt{\alpha_3+\beta}+14\sqrt{\alpha_2+1}-14\sqrt{\alpha_2+1-\beta}\right)}$$

with

$$\lim_{\beta\to 0}\widetilde{t'}(\alpha_2,\alpha_3,\beta)=\frac{9\left(1+\alpha_2-\alpha_3\right)}{13\sqrt{\alpha_2+1}-14\sqrt{\alpha_3}}$$

In terms of γ' the condition above corresponds to

$$\gamma'>\widetilde{\gamma'}(\alpha_2,\alpha_3,\beta):=\frac{\left(13\sqrt{\alpha_3}-13\sqrt{\alpha_3+\beta}+14\sqrt{\alpha_2+1}-14\sqrt{\alpha_2+1-\beta}\right)}{9\left(\sqrt{\alpha_3}\sqrt{\alpha_2+1}-\sqrt{\alpha_3+\beta}\sqrt{\alpha_2+1-\beta}\right)}$$

(c) The merger with remedies increases consumer surplus if and only if

$$CS^M(\mathbf{v}^M)-CS(\mathbf{v})>0\Leftrightarrow$$

22 This is ensured by the assumption that $\alpha_i\in\left[\frac{1}{2},1\right]$.

$$\frac{14v_1^M+13v_3^M}{3^3}+\frac{\left(v_1^M-v_3^M\right)^2}{2\times 3t}-\frac{5^2 7}{2^2 3^5}t-$$
$$(\frac{v_1+v_2+v_3}{3}+\frac{2^3 3^4\left(v_1^2+v_2^2+v_3^2-v_1v_2-v_1v_3-v_2v_3\right)-5^2 13t^2}{2^2 3^3 5^2 t})>0$$

With $v_1=\gamma$, $v_2=\gamma\sqrt{\alpha_2}$, $v_3=\gamma\sqrt{\alpha_3}$, $v_1^M=\gamma\sqrt{1+\alpha_2-\beta}$ and $v_3^M=\gamma\sqrt{\alpha_3+\beta}$ this simplifies to:

$$\frac{\gamma^2\begin{pmatrix}-725(t')^2+\left(5850\sqrt{\beta+\alpha_3}+6300\sqrt{1-\beta+\alpha_2}-4050\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+1\right)\right)t'\\ -891\left(\alpha_2+\alpha_3+1\right)+2916\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+\sqrt{\alpha_2}\sqrt{\alpha_3}\right)-4050\sqrt{\beta+\alpha_3}\sqrt{1-\beta+\alpha_2}\end{pmatrix}}{12\,150t}$$

This is an inverted parabola in t' with roots

$$t_R'^{\pm}:=-\frac{9}{29}\left(9\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+1\right)-14\sqrt{1+\alpha_2-\beta}-13\sqrt{\beta+\alpha_3}\right)\pm$$
$$\frac{27}{145}\sqrt{\begin{pmatrix}734\left(\alpha_2+1\right)-75\beta+659\alpha_3+850\sqrt{\beta+\alpha_3}\sqrt{\alpha_2-\beta+1}+566\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+\sqrt{\alpha_2}\sqrt{\alpha_3}\right)\\ -\left(700\sqrt{1+\alpha_2-\beta}+650\sqrt{\beta+\alpha_3}\right)\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+1\right)\end{pmatrix}}$$

Thus, if $t'>t_R'^{+}$ consumer surplus decreases. It should be noted that in the four cases addressed in the text we have $t_R^{-}<\underline{t}'$. It then follows that $\gamma_R'=\frac{1}{t_R'^{+}}=$

$$\frac{3150\sqrt{1+\alpha_2-\beta}+2925\sqrt{\beta+\alpha_3}-2025\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+1\right)}{891\left(\alpha_2+\alpha_3+1\right)+4050\sqrt{\beta+\alpha_3}\sqrt{1+\alpha_2-\beta}-2916\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+\sqrt{\alpha_2}\sqrt{\alpha_3}\right)}+$$
$$-\frac{135\sqrt{\begin{pmatrix}734\left(\alpha_2+1\right)+850\sqrt{\beta+\alpha_3}\sqrt{1+\alpha_2-\beta}+566\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+\sqrt{\alpha_2}\sqrt{\alpha_3}\right)\\ -75\beta+659\alpha_3-700\sqrt{1+\alpha_2-\beta}\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+1\right)-650\sqrt{\beta+\alpha_3}\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+1\right)\end{pmatrix}}}{891\left(\alpha_2+\alpha_3+1\right)+4050\sqrt{\beta+\alpha_3}\sqrt{1+\alpha_2-\beta}-2916\left(\sqrt{\alpha_2}+\sqrt{\alpha_3}+\sqrt{\alpha_2}\sqrt{\alpha_3}\right)} \quad\blacksquare$$

Corollary 1. *Optimal remedies, if positive, increase with t', may increase or decrease in α_2 and always decrease in α_3.*

Proof of Corollary 1. The first-order condition that defines remedies implicitly is

$$\gamma^2\frac{9\left(2\beta-1+\alpha_3-\alpha_2\right)+t'\left(13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}\right)}{54t\sqrt{\beta+\alpha_3}\sqrt{1+\alpha_2-\beta}}=0$$

As second-order conditions hold, the sign of $\frac{\partial\beta}{\partial\alpha_3}$ is equal to the sign of

$$\frac{\partial\left(\gamma^2\frac{9(2\beta-1+\alpha_3-\alpha_2)+t'\left(13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}\right)}{54t\sqrt{\beta+\alpha_3}\sqrt{1+\alpha_2-\beta}}\right)}{\partial\alpha_3}=\gamma^2\frac{9\left(\alpha_2+\alpha_3+1\right)-13t'\sqrt{1+\alpha_2-\beta}}{108t\left(\sqrt{\beta+\alpha_3}\right)^3\sqrt{1+\alpha_2-\beta}}$$

which as the same sign as $9\left(\alpha_2+\alpha_3+1\right)-13t'\sqrt{1+\alpha_2-\beta}$.

Evaluated at $t'=\frac{-9(2\beta-1+\alpha_3-\alpha_2)}{\left(13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}\right)}$ this simplifies to

$$18\frac{13\left(\beta+\alpha_3\right)\sqrt{1+\alpha_2-\beta}-7\sqrt{\beta+\alpha_3}\left(\alpha_2+\alpha_3+1\right)}{13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}}$$

The denominator is always positive (otherwise optimal remedies are not positive) and therefore the expression is negative if

$$13\sqrt{1+\alpha_2-\beta}\left(\beta+\alpha_3\right)<7\sqrt{\left(\beta+\alpha_3\right)}\left(\alpha_2+\alpha_3+1\right)\Leftrightarrow$$
$$13^2(1+\alpha_2-\beta)\left(\beta+\alpha_3\right)^2<7^2\left(\beta+\alpha_3\right)\left(\alpha_2+\alpha_3+1\right)^2\Leftrightarrow$$
$$-169\beta^2+169\beta\left(\alpha_2-\alpha_3+1\right)-49\left(\left(\alpha_2+1\right)^2+\alpha_3^2\right)+71\alpha_3\left(\alpha_2+1\right)<0$$

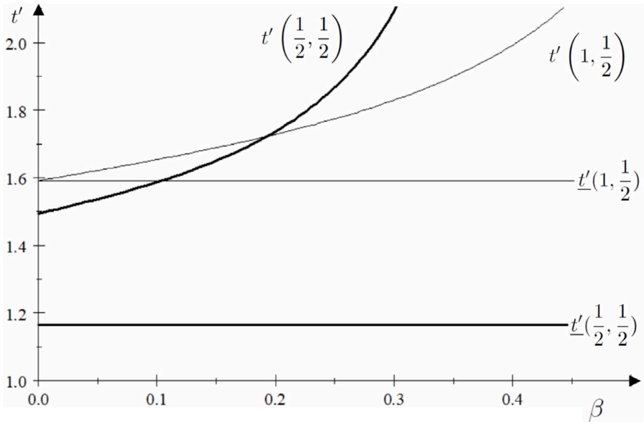
This is always true because the polynomial does not have any real roots in β.

With respect to α_2 an example is sufficient. Let $\alpha_3=\frac{1}{2}$ and $\alpha_2=\frac{1}{2}$ or $\alpha_2=1$.

The following figure presents

$$t'\left(\alpha_2,\alpha_3\right)=\frac{-9\left(2\beta-1+\alpha_3-\alpha_2\right)}{\left(13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}\right)}$$

for these two cases. Recall that in the first case $\underline{t}'(1, \frac{1}{2}) = \max\left\{\frac{18}{5}\left(1-\sqrt{\frac{1}{2}}\right), \frac{9}{4}\left(\sqrt{2}-\sqrt{\frac{1}{2}}\right)\right\} = \frac{9}{8}\sqrt{2}$ and in the second case $\underline{t}'(\frac{1}{2}, \frac{1}{2}) := \frac{9}{8}\sqrt{2}\left(\sqrt{3}-1\right)$.



This illustrates that depending on the value of t' an increase in α_2 may increase or decrease optimal remedies.
Finally, the sign if $\frac{\partial \beta}{\partial t'}$ is equal to the sign of

$$\frac{\partial\left(\gamma^2 \frac{9(2\beta-1+\alpha_3-\alpha_2)+t'\left(13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}\right)}{54t\sqrt{\beta+\alpha_3}\sqrt{1+\alpha_2-\beta}}\right)}{\partial t'} = \gamma^2\frac{13\sqrt{1+\alpha_2-\beta}-14\sqrt{\beta+\alpha_3}}{54t\sqrt{\beta+\alpha_3}\sqrt{1+\alpha_2-\beta}} > 0. \quad \blacksquare$$

References

Aguzzoni, L., Buehler, B., Martile, L., Kemp, R., & Schwarz, A. (2018). Ex-post analysis of mobile telecom mergers: The case of Austria and the netherlands. *De Economist*, *166*(1), 63–87.
Asker, J., & Katz, M. (2023). *The sprint/t-mobile merger, in antitrust economics at a time of upheaval: Competition policy cases on two continents*.
Bourreau, M., Kourandi, F., & Valleti, T. (2015). Net neutrality with competing internet platforms. *The Journal of Industrial Economics*, *63*(1), 30–73.
Brito, D. (2003). Preemptive mergers under spatial competition. *International Journal of Industrial Organization*, *21*(10), 1601–1622.
Buehler, B. (2015). Do international roaming alliances harm consumers? *The Journal of Industrial Economics*, *63*(4), 642–672.
Castells, P., & Bahia, K. (2019). The impact of spectrum prices on consumers, GSMA intelligence. Available at https://www.gsma.com/spectrum/wp-content/uploads/2019/09/Impact-of-spectrum-prices-on-consumers.pdf.
Deneckere, R., & Davidson, C. (1985). Incentives to form coalitions with bertrand competition. *Rand Journal of Economics*, *16*(4), 473–486.
Foros, Ø., Hanseny, B., & Vergéz, T. (2020). Product quality investment co-operation and sharing among downstream rivals: An application to mobile telecommunications, norwegian competition authority, 7/2020. Available at https://konkurransetilsynet.no/wp-content/uploads/2020/10/7_2020_product-quality-investment1.pdf.
Genakos, C., Valletti, T., & Verboven, F. (2018). Evaluating market consolidation in mobile communications. *Economic Policy*, *33*(93), 45–100.
Grajek, M., Gugler, K., Kretschmer, T., et al. (2019). Static or dynamic efficiency: Horizontal merger effects in the wireless telecommunications industry. *Review of Industrial Organization*, *55*, 375–402.
Grønnevet, G., Hansen, B., & Reme, B.-A. (2016). Spectrum policy and competition in mobile data. *Information Economics and Policy*, *37*, 34–41.
Grzybowski, L., & Pereira, P. (2007). Merger simulation in mobile telephony in Portugal. *Review of Industrial Organization*, *31*(3), 205–220.
Jeanjean, F., & Houngbonon, G. (2017). Market structure and investment in the mobile industry. *Information Economics and Policy*, *38*, 12–22.
Kwon, Y., Parkb, D., & Rheec, H. (2017). Spectrum fragmentation: Causes, measures and applications. *Telecommunications Policy*, *41*, 447–459.
Levy, D., & Reitzes, J. (1992). Anticompetitive effects of mergers in markets with localized competition. *The Journal of Law, Economics and Organization*, *8*(2), 427–440.
Liang, J., Guiffard, J.-B., Ivaldi, M., & Aimene, L. (2022). Welfare cost of mobile spectrum (mis)allocation. http://dx.doi.org/10.2139/ssrn.4228058, Available at SSRN: https://ssrn.com/abstract=4228058 or.
Liu, Y.-H., Prince, J., & Wallsten, S. (2018). Distinguishing bandwidth and latency in households' willingness-to-pay for broadband internet speed. *Information Economics and Policy*, *45*, 1–15.
Maier, N., Jørgensen, J., Lunde, A., & Toivanen, O. (2021). Ex-post analysis of the TeliaSonera-chess 2005 merger. *De Economist*, *169*, 141–178.
Motta, M., & Tarantino, E. (2021). The effect of horizontal mergers, when firms compete in prices and investments. *International Journal of Industrial Organization*, *78*, Article 102774.
Perry, M., & Porter, R. (1985). Oligopoly and the incentive for horizontal merger. *The American Economic Review*, *75*(1), 219–227.
Salant, S., Switzer, S., & Reynolds, R. (1983). Losses from horizontal merger: The effects of an exogenous change in industry structure on cournot-Nash equilibrium. *Quarterly Journal of Economics*, *98*(2), 185–199.
Salop, S. (1979). Monopolistic competition with outside goods. *Bell Journal of Economics*, *10*, 141–156.
Vickrey, W. (1964). *Microstatics, harcourt*. New York: Brace and World.
Vickrey, W. (1999). Spatial competition, monopolistic competition, and optimum product diversity. *International Journal of Industrial Organization*, *17*, 953–963.